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R[illegible] ARITHMETICK:
OR,
The GROUND of ARTS;

TEACHING

The perfect Work and Practice of ARITHME-TICK, both in whole Numbers and Fractions, after a more easie and exact form then in former time hath been set forth.

Made by Mr *ROBERT RECORD*, Dr in Physick.

Afterwards augmented by Mr. JOHN DEE.

And since enlarged with a third part of RULES of PRACTICE, abridged into a briefer method then hitherto hath been published; with divers necessary Rules incident to the Trade of Merchandise, with Tables of the valuation of all Coyns, as they are currant at this present time, by *JOHN MELLIS*.

And now diligently perused, corrected, illustrated and inlarged; with an *Appendix* of figurative Numbers, and the Extraction of their Roots, according to the method of *Christian Urstitius*; with Tables of Board and Timber-measure, and new Tables of Interest, after 10, 8, and 6 *per* 100; with the true value of Annuities to be bought or sold present, respited or in Reversion: the first calculated by *R. C.* but corrected; and the latter diligently calculated by *RO. HARTWELL, Philomath.*

Scientia non habet inimicum nisi ignorantem.

Fide —— *sed* —— *Vide.*

LONDON,
Printed by *James Flesher*, and are to be sold by *Robert Boulter*, at the Turks-head in *Bishopsgate-street*, near the great *James*. 1668.

THat which my friend hath well begun
For very love to common-weal,
Need not all whole to be new done,
But now increase I do reveal.

Something herein I once redrest,
And now again for thy behoof
Of zeal I do, and at request,
Both mend and adde, fit for all proof.

Of Numbers use the endless might
No wit nor language can express:
Apply and try both day and night,
And then this truth thou wilt confess.

The Book's Verdict.

TO please or displease sure I am,
But not of one sort to every man:
To please the best sort would I fain,
The froward displease shall I certain.
Yet wish I well, though not with hope
All ears or mouths to please or stop.

June 17

To the most mighty Prince, EDWARD the VIth, by the Grace of God, King of *England*, *France*, and *Ireland*, &c.

THE Excellency of Man's nature is such, as it is by God's divine favour (most mighty Prince) not onely created in highness of degree far above all other corporal things, but by perfection, Reason and search of wit, much approaching toward the Image of God, as not onely the Holy Scriptures testifie, but also those natural Philosophers which exactly did consider the nature of Man, and namely the far reach and infinite compass of the words of the Mind, were inforced to confess, that Man scarcely was able to know himself. And if he would duely ponder the nature of himself, he would find it so strange, that it might seem to him a very miracle. And thereof sprang that saying, *Magnum miraculum est Homo, maximum miraculum Sapiens homo.* For undoubtedly, as Man is one of the greatest miracles that ever God wrought, so a Wise man is plainly the greatest.

And therefore was it that some did account the Head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacy of Reason, Wit, Memory, Imagination, and such other Powers and works of the Mind, which can more easily conceive any thing, in a manner, then understand it self. Amongst all the creatures of God, it findeth none more difficult to be perceived then the said Powers of it self, whereby it doth conceive and judge, as it may be well conjectured by the diversity of opinions that the wisest Philosophers did utter touching the Spirit of man, and the substance of it, whereof I now intend to make no rehersal: but who so listeth to reade the rest,

forth, not onely in *Aristotle's* … in *Galen's* Book called *Historia Phi-losophica*, and again in *Plutarch's* Work *De Philosophorum placitis*, whose words are also repeated by *Eusebius* in his 15. Book Τῆς ἐυαγγελικῆς προπαρασκευῆς, unto whom I remit them that have desire to understand the intricate difficulty of knowing our own selves as touching our best part, and that part whereby we deserve to bear the name of Men.

This matter seemed so obscure and difficult to know, that *Galen*, who for his excellent wisedom and judgment in natural works is called of many men *a Miracle in Nature*, yet in searching the nature and substance of the Spirit of man, he not onely confesseth himself ignorant, but counteth it plain temerity to attempt to find it. So far above the hope of man's knowledg is that part whereby man doth know and judge of things. And although the ignorant sort (which hate all things that they know not) do little esteem the profoundness of a man's Spirit, and Reason, the chief power and faculty of it: yet as there is a kind of fear & obedience of unreasonable beasts unto man by the working power of God; so is there in those small-reasoned persons a certain kind of reverence toward Wisedome and Reason, which they do shew oftentimes, and by power of persuasion are enforced to obey Reason, will they nill they. And hereby it came to pass, that the rudeness of the first age of Man was brought unto some more civil trade; as it is well declared by *Cicero* in the beginning of his first Book *De Inventione Rhetorica*, where he saith thus: *Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabant. Nondum divinæ Religionis, non humanæ ratio colebatur. Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non jus æquabile quid utilitatis haberet acceperat: ita propter errorem atq; inscitiam cæca ac temeraria dominatrix animi cupiditas ad se explendum viribus corporis abutebatur perniciosissimis satellitibus. Quo tempore quidam magnus (videlicet) vir & sapiens cognovit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam posset elicere, & præcipiendo meliorem reddere. Qui dispersos homines in agris & in tectis sylvestribus abditos ratione quadam compulit in unum locum & congregavit; & eos in unamquamque rem inducens utilem atque honestam, primò propter insolentiam reclamantes, deinde propter rati-*

onem

onem atque orationem studiosius audiêntes, ex feris & immanibus mites reddidit & mansuetos.

The long repetition of *Tullie's* words will seem tedious to them that love but little, and care much less for, the knowledge of Reason; but unto your Majesty (I dare say) it is a delectable remembrance, and unto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily reade.

This sentence of *Cicero* am I loath to translate into English, partly for that unto your Majesty it needeth no translation, but especially knowing how far the grace of *Tullie's* Eloquence doth excell any English-man's tongue, and much more exceedeth the baseness of my barbarous style: yet for the fruit of my sentence, I had rather unto my meer English Country-men utter the rudeness of my translation, then to defraud them of the benefit of so good a Lesson, trusting they will so learn to love Reason, that they will also gladly and greedily embrace all good Sciences that may help to the just furniture of the same, when they consider that informed Reason was the onely instrument, or at least the chiefest means, to bring men into civil regiment, from barbarous manners and beastly conditions. *For the time was* (saith *Tully*) *that men wandred abroad in the fields up and down like beasts, and used no better order in feeding then they: so that by Reason's rule they wrought nothing, but most of their doings did they atchieve by force of strength. At this time there was no just regard of Religion towards God, nor of duty towards man. No man had seen right use of Marriage, neither did any man know their own children from other, nor no man had felt the commodity of just Laws: so that through errour and ignorance, wilfull lust, like a blind and heady ruler, abused bodily strength as a most mortal minister for the satisfying of his desire. At that time was there one which not onely in power but also in wisedome was great, and he considered how that in the mind of men was both apt instruments and great occasion to the due accomplishment of most weighty affairs, if a man could apply them to use, and by teaching of Rules frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fields, and lay lurking in wild cottages and woods; and bringing them into one common society, did trade them to all such things as either were profitable or honest, although not without repining at the first,*

by reason that they had not been so accustomed before. Yet at length through reason and persuasion of words they obeyed him more diligently; and so of a wild and cruel people he made them courteous and gentle.

Thus hath *Tully* set forth the efficacy of Reason and Persuasion, how it was able to convert wild people to a mildness, and to change their furious cruelness into gentle courtesie. Were it not now a great reproach in this our time, (when Knowledge reigneth so largely) that men should shew themselves less obsequious to Reason? Unless it may be thought, that now every man having sufficient knowledge of himself, needeth not to hearken to the persuasion of others.

Indeed he that thinketh himself wise will not esteem the reason of any other, be he never so wise; so that of such a one it may well be said, He that thinketh himself wiser then he is, may justly be counted a double fool. Wherefore such men are not to be permitted in open audience to talk, but must be put to silence, and be made to give ear to reason; which reason consisteth not in the multitude of words heaped rashly together, and applied for one purpose, but reason is the expressing of a just matter with witty persuasions, furnished with learned knowledg. Such knowledg had *Moses*, being expert in all the learning of the *Ægyptians*, as the Scriptures declare, and therefore was able to persuade the stubborn people of the *Jews*, although not without pain. Such knowledg and such reasons did *Druys* shew, which was the first Law-maker of all the West part of *Europe*. Like reason and wisedome did *Zamolxis* amongst the *Goths*, *Lycurgus* unto the *Lacedæmonians*, *Zaleucus* to the *Locrians*, *Solon* to the *Athenians*, and *Dunwallo Mulmutius* two thousand years past amongst the old *Britans* of this Realm. And hereby came it to pass that their Laws continued long, till more perfect reason altered many of them, and wilful power oppressed most of then.

Druys was [so]n to King [S]*arron*, and [su]cceeded [hi]m in his [k]ingdome.

At the beginning, when these wise men perceived how hard it was to bring the rude people to understand reason, they judged the best means to attain this honest purpose to depend of learning in every kind; for by learning (as *Ovid* saith) *Pectora mitescunt, asperitasque fugit*; Stout stomachs do wax mild, and sharp fierceness is exil'd. Therefore, as *Berosus* doth testifie, *Sarron*, that was the third King over all this West part of *Europe*, for to bring the people from beastly rage to manly reason, did erect Schools [o]f liberal Arts, which took such good effect, that his name

continued

continued in that ſort famous above two thouſand years after: for *Diodorus Siculus*, which was in the time of *Julius Cæsar*, maketh mention of the learned men of the *Goths* or *Celts*, and nameth them *Sarronides*, that is to ſay, *Sarron his Scholars* and *followers*.

Amongſt theſe Arts that then were taught, ſome did inform the tongue, and make them able both to utter aptly their mind, and alſo to perſuade, (as Grammar, Logick, and Rhetorick) although not ſo curiouſly as in this time: ſome other did appertain to the juſt order of partition of Lands, the true uſing of Weights, Meaſures, and Reckonings of all ſorts of Bargains, and for order of Building, and ſundry other uſes; thoſe were Arithmetick and Geometry. Again, to encourage men to the honour of God, they taught Aſtronomy, whereby the wonderful works of God were ſo manifeſtly ſet forth, that no man's tongue nor pen can in like ſort expreſs his infinite Power, his unſpeakable Wiſedom, and his exceeding Goodneſs toward man, whereby he doth bountifully provide for man all neceſſaries, not onely to live, but alſo to live pleaſantly. And ſo was their confidence in God's Providence ſtrongly ſtayed, knowing his Goodneſs to be ſuch, that he would help man as he could; and his Power to be ſo great, that he would doe nothing but that that was beſt. Beſides theſe Sciences they taught alſo Muſick, which moſt commonly they did apply partly to religious ſervices, to draw men to delight therein, and partly to Songs made of the manners of men, in praiſe of Vertue, and diſcommendation of Vice, whereby it came to paſs that no man would diſpleaſe them, nor doe any thing evil that might come to their hearing: for their Songs made evil men more abhorred in that time then any Excommunication doth in this time. The poſterity of theſe Muſicians continue yet both in *Wales* and *Ireland*, called *Bards* unto this day, by the ancient name of *Bardus* the firſt Founder.

This *Bard[us]* *Druidius*, the 5. King of the *Celt[s]* reigned 60 years, and died 1832 year[s] before *Chriſt*.

And as theſe Sciences did increaſe, ſo did Vertue increaſe thereby. Again, as thoſe Sciences did decay, ſo Vertue loſt her eſtimation, and conſequently was little in uſe: whereof to make a full declaration were a thing meet for a Prince to hear, but it would require a peculiar Treatiſe. Wherefore at this preſent I count it ſufficient lightly to have touched this matter in general words, and to ſay no more of the particularity thereof, but onely touching one of thoſe Sciences, that is Arithmetick, by which not onely juſt partition of Lands was made, but alſo touching

Buying and Selling, all Assises, Weights and Measures were devised, and all Reckonings and Accounts driven; yea by proportion of it were the true orders of Justice limited, as *Aristotle* in his Ethicks doth declare, and the degrees of Estates in the Commonwealth established: and although that Proportion be called Geometricall, and not Arithmeticall, yet doth that Proportion appertain to the Art of Arithmetick, and in Arithmetick is taught the progression of such proportions, and all things thereto belonging. Wherefore I may well say, that seeing Arithmetick is so many waies needfull unto the first planting of a Commonwealth, it must needs be as much required to the preservation of it also: for by the same means is any Commonwealth continued, by which it was erected and established. And I shall in small matters in appearance, but indeed very weighty, put one example or two. What shall we say for the Statutes of this Realm, which be the onely stay of good order, in a manner, now? As touching the Measuring of ground by length and breadth, there is a good and an ancient Statute made by Art of Arithmetick; and now it shall be to little use, if by the same Art it be not practised and tried. For the Assise of Bread and Drink, the two most common and most necessary things for sustentation of man, there was a goodly Ordinance in the Law made, which by ignorance hath so grown out of knowledge and use, that few men do understand it, and therefore the Statute-books are wonderfully corrupted, and the Commons cruelly oppressed: notwithstanding some men have written that it is too doubtful a matter to execute those Assises by those Stat. by reason they depend of the Stand. of the Coin, which is much chang'd from the state of that time when those Statutes were made. Thus shall every man reade (that listeth) in the Abridgment of the Statutes, in the Title of *Weights and Measures*, in the 7. number of the *English* Book, where he should have translated a good Ordinance which is set forth in the *French* Book: but no marvel if the Abridgment doth omit it, seeing the great Book of Statutes doth omit the same Statute, as it hath done many other very good Laws. And this is the fruit of ignorance, to reject & condemn all that it understandeth not, although they use some cloaks for it, but such cloaks as, being allowed, might serve to repell all good Laws; which God forbid.

Again, there is an ancient Order for Assise for Fire-wood and Coals, which was renewed not many years past; and now how avarice and ignorance doth canvass that Statute, it is too pitifull to talk of, and more miserable to feel. Far-

Farthermore, for the Statute of Coinage, and the Standard thereof, if the people understood rightly the Statute, they should not nor would (as they often doe) gather an excuse for their folly thereby. But, as I said, these Statutes by wisedom and good knowledg of Arithmetick were made, & by the same must they be continued; and let not ignorance any more meddle with the use of them, then it did with the making of them. Oh, in how miserable case is that Realm, where the ministers and interpreters of the Law are destitute of all good Sciences, which be the keys of the Laws! How can they either make good Laws, or maintain them, that lack the true knowledge whereby to judge them? And happy may that Realm be accounted, where the Prince himself is studious of Learning, and desireth to understand Equity in all Laws. Therefore most happy are we the loving Subjects of your Majesty, which may see in your Highness not onely such towardness, but also such knowledg of divers Arts, as seldom hath been seen in any Prince of such years; whereby we are inforced to conceive this hope certainly, that he which in those years seeketh Knowledge when Knowledge is least esteemed, and at such an age can discern them to be enemies both to his Royal Person and to his Realm, which labour to withdraw him from Knowledge to excessive Pastime, and from reasonable Study to idle or noysome Pleasures, he must needs, when he cometh to more mature years, be a most prudent Prince, a most just Governour, and a right Judge, not onely of his Subjects commonly, but also of the ministers of his Laws, yea and of the Laws themselves, and be able to conceive the true equity and exact understanding of all his Laws and Statutes, to the comfort of his good Subjects, and the confusion and reproach of them that labour to obscure or prevent the equity of the same Laws and Statutes. How some of these Statutes may be applied to use as well in this our time as in any other time, I have peculiarly declared in this Book; & some other I have omitted for just considerations, till I may offer them first unto your Majesty, to weigh them as to your Highness shall seem good: for many things in them are not to be published without your Highness knowledge and approbation; namely, because in them is declared all the rates of Allays for all Standards from one ounce upward, with other mysteries of Mint-matters, and also most part of the varieties of Coyns that have been currant in this your Majestie's Realm by the space almost of 600 years last past, and many of them that were currant in the time that the *Romans* ruled here.

All

All which, with the ancient deſcription of *England* and *Ireland*, and my ſimple cenſure of the ſame, I have almoſt compleated, to be exhibited to your Highneſs: In the mean ſeaſon moſt humbly beſeeching your Majeſty to accept this ſimple Treatiſe, not worthy to be preſented to ſo High a Prince: but my lowly requeſt to your Majeſty is, that this amongſt other of my Books may paſs under the protection of your Highneſs, whom I beſeech God moſt earneſtly and daily, according to my duty, to advance in all Honour and Princely Regality, and to encreaſe in all Knowledge, Juſtice, and godly Policy. Amen.

Your Majeſtie's moſt obedient

Subject and Servant,

Robert Record.

To

To the loving READERS,

The Preface of M^r *Robert Record.*

Sore oftimes have I lamented with my self the unfortunate condition of England, *seeing so many great Clerks to arise in sundry other parts of the World, and so few to appear in this our Nation; whereas for pregnancy of natural wit (I think) few Nations do excell* English*-men. But I cannot impute the cause to any other thing, then to the contempt or misregard of Learning. For as* English*-men are inferiour to no men in mother Wit, so they pass all men in vain Pleasures, to which they may attain with great pain and labour; and are slack to any never so great commodity, if there hang of it any painfull study or travelsome labour.*

Howbeit yet all men are not of that sort, though the most part be, the more pity it is: but of them that are so glad, not onely with painful study and studious pain to attain Learning, but also with as great study and pain to communicate their Learning to others, and make all England *(if it might be) partakers of the same, the most part are such that uneath they can support their own necessary charges, so that they are not able to bear any charges in doing of that good that else they desire to doe.*

But a greater cause of lamentation is this, that when Learned men have taken pains to doe things for the aid

of

of the unlearned, ſcarce ſhall they be allowed for their well-doing, but derided and ſcorned, and ſo utterly diſcouraged to take in hand any like enterpriſe again. So that if any be found (as here are ſome) that do favour Learning and learned wits, and can be contented to further Knowledge, yea onely with their word, ſuch perſons, though they be rare, yet ſhall they encourage Learned men to enterpriſe ſomething at the leaſt that England *may rejoyce of. And I have good hope that* England *will (after ſhe hath taken ſome ſure taſte of Learning) not onely bring forth more favourers of it, but alſo ſuch Learned men, that ſhe ſhall be able to compare with any Realm in the World. But in the mean ſeaſon, where ſo few regarders of Learning are, how greatly they are to be eſteemed that do favour and further it, my pen will not ſuffice at full to declare.*

Therefore, gentle Reader, whereas I do upon moſt juſt occaſion judge, yea and know aſſuredly, that there be ſome men in this Realm which both love and alſo much deſire to further good Learning, and am not well able to write their condign praiſe for the ſame; I think it better with ſilence to overpaſs it, then either ſay too little of it, or provoke againſt them the malice of ſuch other which doe nothing themſelves that is praiſe-worthy, and therefore cannot abide to hear the praiſe of any other man's good deed.

And conſidering their great favour unto Learning, though I my ſelf be not worthy to be reckoned in the number of great Learned men, yet am I bold to put my ſelf in Preſs with ſuch ability as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man to help my Country-men, and will not ceaſe daily, (as much as my ſmall ability

will

will suffer me) *to endite some such thing as shall be to the instruction, though not of Learned men, yet at the least of the vulgar sort, whose argument alwaies shall be such as it shall delight all learned wits, though they do not learn any great thing out of it.*

But to speak of this present Book of Arithmetick, I dare not, nor will I set it forth with any words, but remit it to the judgment of all gentle Readers, and namely such as love good Learning, beseeching them so to esteem it as it doth seem worthy, and so either to accept the thing for it self, or at the least to allow my good endeavour. But I perceive I need not use any perswasions unto them whose gentle nature and favourable mind is ready to receive thankfully and interpret to the best all such enterprises attempted for so good an end, though the thing do not alwaies satisfie mens expectation. This considered did bolden me to publish abroad this little Book of the Art of Numbring, which if you shall receive favourably, you shall encourage me to gratifie you hereafter with some greater thing.

And as I judge some men of so loving a mind to their native Countrey, that they would much rejoyce to see it prosper in good Learning and witty Arts; so I hope well of all the rest of English*-men, that they will not be unmindful of his due praise by whose means they are helped and further'd in any thing. Neither ought they to esteem this thing of so little value as many men of little discretion oftentimes do. For whoso setteth small price by the witty device and knowledge of Numbering, he little considereth it to be the chief point (in a manner) whereby Men differ from all brute Beasts: for as in all other things (almost) Beasts are partakers with us;*

so

so in Numbring we differ clean from them, and in a manner peculiarly, sith that in many things they excell us again.

The *Fox* in Crafty wit exceedeth most men,
A *Dog* in Smelling hath no man his peer :
To foresight of weather if you look then,
Many Beasts excell Men ; this is clear.
The Wittiness of *Elephants* doth Letters attain ;
But what Cunning doth there in the *Bee* remain ?
The *Emmet* foreseeing the hardness of winter,
Provideth victuals in the time of summer.
The *Nightingale*, the *Linet*, the *Thrush*, the *Lark*,
In Musical harmony pass many a Clark.
The *Hedghog* of Astronomy seemeth to know,
And stoppeth his cave where the wind will blow.
The *Spider* in Weaving such art doth show,
No man can him mend, nor follow, I trow.
When a house will fall, the *Mice* right quick
Flee thence before : can man doe the like ?

Many things else of the Wittiness of Beasts and Birds might I here say, save that another time of them I intend to write, wherein they excell in a manner all Men, as it is daily seen. But in Number was there never Beast found so cunning, that could know or discern one thing from many, as by daily experience you may well consider. When a Bitch hath many Whelps, or a Hen many Chickens, and likewise of other whatsoever they be, take from them all their young saving onely one, and you shall perceive plainly that they miss none, though they will resist you in taking them away, and will seek them again if they may know where they be, but else they will never

never misse them truly; but take away that one that is left, and then will they cry and complain; and restore to them that one, then are they pleased again. So that of Number this may I justly say, it is the onely thing almost that separateth Man from Beasts. He therefore that shall contemn Number declareth himself as brutish as a Beast, and unworthy to be counted in the fellowship of Men. But I trust there is no man so foully overseen, though many right smally do it regard.

Therefore will I now stay to write against such, and return again to this my Book, which I have written in the form of a Dialogue, because I judge that to be the easiest way of Instraction, when the Scholar may ask every doubt orderly, and the Master may answer to his Question plainly. Why th[e] Authour wrote in Dialogu[e] wise.

Howbeit I think not the contrary, but as it is easier to make another man's Work then to make the like, so there will be some that will find fault because I writ in a Dialogue: but as I conjecture those shall be such as do not, cannot, or will not perceive the reason of right Teaching, and therefore are unmeet to be answered unto, for such men with no reason will be satisfied.

And if any man object, that other Books have been written of Arithmetick *already so sufficiently that I needed not now to put pen to the Book, except I will condemn other mens writings: to them I answer, That as I condemn no man's diligence, so I know that no one man can satisfie every man; and therefore like as many do esteem greatly other Books, so I doubt not but some will like this my Book above any other* English Arithmetick *hitherto written, and namely such as shall lack Instructors, for whose sake I have so plainly set forth the Examples as no Book that I have seen hath done hitherto;*

therto : which things shall be great ease to the rude Readers.

Therefore (gentle Reader) though this Book can be but small aid to the Learned sort, yet unto the simple Ignorant (which needeth most help) it may be a good furtherance and mean unto Knowledge.

And though unto the King His Majesty privately I do it dedicate, yet I doubt not (such is his Clemency) but that he can be content, yea, and much desirous, that all his loving Subjects shall take the use of it, and imploy the same to their most profit. Which thing if I perceive that they thankfully doe, and receive with as good will as it was written, then will I shortly with no less kindness set forth such Introductions into Geometry *and* Cosmography *as I have at times promised, and as hitherto in* English *hath not been enterprised; wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.*

I will say no more, but let every man judge as he shall see cause. And thus for this time I will stay my Pen, committing you all to that true Fountain of perfect Number, which wrought the whole World by number and measure : *He is Trinity and Unity and Glory.* Amen.

March 1822 Portland

Here followeth a Table of the whole Contents of this Book.

The Contents of the First Dialogue.

The Second Dialogue containeth

The Contents of the Second Part, touching Fractions.

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The

The Contents.

 Italians

A Col-

A Collection of such Tables as are contained in this Treatise.

Figures and Notes of Numbers.

Before the Introduction of Arithmetick it were very good to have some understanding and knowledge of these Figures and Notes.

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&c.	&c.	*&c.*	&c.	&c.	*&c.*

A Di-

Edward Dennis

A Dialogue between the Master and the Scholar, teaching the *Art* and Use of *Arithmetick* with Pen.

The Scholar speaketh.

SIR, such is your authority in mine estimation, that I am content to consent to your saying, and to receive it as truth, though I see none other reason that doth lead me thereunto: whereas else in mine own conceit it appeareth but vain, to bestow any time privately in learning of that thing that every Child may and doth learn at all times and hours, when he doth any thing himself alone, and much more when he talketh or reasoneth with others.

Master. Lo, this is the fashion and chance of all them that seek to defend their blind ignorance, that when they think they have made strong reason for themselves, then have they proved quite contrary. For if numbering be so common (as you grant it to be) that no man can doe any thing alone, and much less talk or bargain with others, but he shall still have to doe with number: this proveth not number to be contemptible and vile, but rather right excel-

lent and of high reputation, sith it is the ground of all mens affairs, in that without it no tale can be told, no communication without it can be continued, no bargaining without it can duly be ended, or no business that a man hath justly completed. These commodities, if there were none other, are sufficient to approve the worthiness of number. But there are other innumerable, far passing all these, which declare number to exceed all praise. Wherefore in all great works are Clerks so much desired? Wherefore are Auditors so richly fed? What causeth Geometricians so highly to be inhaunced? Why are Astronomers so greatly advanced? Because that by number such things they find, Which else would far excell mans mind.

Scholar. Verily, Sir, if it be so that these men by numbring their cunning do attain, at whose great works most men do wonder, then I see well I was much deceived, and numbring is a more cunning thing then I took it to be.

Master. If number were so vile a thing as you did esteem it, then need it not to be used so much in mens communication. Exclude number, and answer to this question, How many years old are you?

Scholar. Mum.

Master. How many days in a week? How many weeks in a year? What lands hath your Father? How many men doth he keep? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if number want, you answer all by mums. How many miles to London?

Scholar.

Scholar. A peak full of plums.

Master. Why, thus you may see what rule number beareth, and that if number be lacking it maketh men dumb, so that to most questions they must answer, mum.

Scholar. This is the cause, Sir, that I judged it so vile, because it is so common in talking every while; For plenty is no dainty, as the common saying is.

Master. No, nor store is no sore: perceive you this? The more common that the thing is, being needfully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light and plain, so the most part is difficult and not easie to attain. The easier part serveth all men in common, and the other requireth some learning. Wherefore, as without numbring a man can do almost nothing, so with the help of it you may attain to all things.

Scholar. Yea, sir? why then it were best to learn the Art of numbring first of all other learning, and then a man need learn no more, if all other come with it.

Mast. Nay, not so: but if it be first learned, then shall a man be able (I mean) to learn, perceive and attain to other Sciences, which without it he could never get.

Scholar. I perceive by your former words, that Astronomy and Geometry depend much on the help of numbring; but that other Sciences, as Musick, Physick, Law, Grammar, and such like, have any help of Arithmetick, I perceive not.

Master. I may perceive your great Clerkliness by the ordering of your Sciences: but I will let that pass now, because it toucheth not the matter that

I will-

I intend; and I will shew you how Arithmetick doth profit in all these, somewhat grosly, according to your small understanding; omitting other reasons more substantial.

Musick. First (as you reckon them) Musick hath not onely great help of Arithmetick, but is made, and hath its perfectness of it: for all Musick standeth by number and proportion. And in Physick, beside the calculation of critical days, with other things which I omit, how can any man judge the pulse rightly, that is ignorant of the proportion of numbers.

Physick.

Law. And as for the Law, it is plain, that the man that is ignorant of Arithmetick, is neither meet to be a Judge, neither an Advocate, nor yet a Proctor. For how can he well understand another mans cause, appertaining to distribution of goods, or other debts, or of summes of mony, if he be ignorant of Arithmetick? This oftentimes causeth right to be hindred, when the Judge either delighteth not to hear of a matter that he perceiveth not, or cannot judge for lack of understanding: this cometh by ignorance of Arithmetick.

Grammar. Now, as for Grammar, methinks you should not doubt in what it needeth number, sith you have learned that Nouns of all sorts, Pronouns, Verbs and Participles are distinct diversly by numbers: besides the variety of Nouns of Number, and Adverbs. And if you take away number from Grammar, then is all the quantity of Syllables lost. And many other waies doth number help Grammar. Whereby were all kinds of Meters found and made? was it not by number?

But

But how needfull Arithmetick is to all parts of Philosophy, they may soon see that do reade either Aristotle, Plato, or any other Philosophers writings. For all their examples almost and their probations depend of Arithmetick. It is the saying of Aristotle, that he that is ignorant of Arithmetick is meet for no Science. And Plato his Master wrote a little sentence over his School-house door, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing he would have all his Scholars expert in Geometry, much rather he would have the same in Arithmetick, without which Geometry cannot stand. Philosophy.

And how needfull Arithmetick is to Divinity, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the Commodities of Arithmetick in civil acts, as in governance of Common-weals in time of peace, and in due provision and order of Armies in time of war, for numbring of the Host, summing of their wages, provision of Victuals, viewing of Artillery, with other Armour, beside the cunningest point of all for casting of ground, for encamping of men, with such other like; and how many waies also Arithmetick is conducible for all private Weals, of Lords and all Possessioners, of Merchants and all other Occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receivers, Stewards, Bailifs, and such like, whose Offices without Arithmetick are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmetick, it were Divinity. Army.

enough

enough to make a very great Book.

Scholar. No, no, sir, you shall not need: For I doubt not but this that you have said were enough to perswade any man to think this Art to be right excellent and good, and so necessary for man, that (as I think now) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so far changed since, by hearing these few Commodities in general? by likelihood you would be far changed if you knew all the particular Commodities.

Scholar. I beseech you, Sir, reserve those Commodities that rest yet behind unto their place more convenient: and if ye will be so good as to utter at this time this excellent treasure, so that I may be somewhat inriched thereby, if ever I shall be able, I will requite your pain.

Master. I am very glad of your request, and will doe it speedily, sith that to learn it you be so ready.

The duty of a Scholar.

Scholar. And I to your authority my wit do subdue: Whatsoever you say, I take it for true.

Master. That is too much, and meet for no man, to be believed in all things, without shewing of reason. Though I might of my Scholar some credence require; Yet except I shew reason, I do it not desire. But now sith you are so earnestly set this Art to attain, best it is to omit no time, lest some other passion cool this great heat, and then you leave off before you see the end.

Perseverance in study.

Scholar. Though many there be so unconstant of mind, that flitter and turn with every wind, which often begin, and never come to the end; I am none

none of this sort, as I trust you partly know. For by my good will what I once begin, Till I have it fully ended, I would never blin.

Master. So have I found you hitherto indeed, and I trust you will increase, rather then go back. For better it were never to assay, Then to shrink and flyin the mid way. But I trust you will not doe so; therefore tell me briefly, What call you the Science that you desire so greatly.

Scholar. Why, sir, you know.

Master. That maketh no matter, I would hear whether you know, and therefore I ask you. For great rebuke it were to have studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetrick, and some Augrime.

Master. And what do these names betoken?

Schol. That, if it please you, of you would I learn.

Master. Both names are corruptly written; Arsemetrick for Arithmetick, as the Greeks call it, and Augrime for Algorism, as the Arabians found it; which doth betoken the Science of Numbring: for Arithmos in Greek is called Number, and of it cometh Arithmetick, the Art of Numbring. So that Arithmetick is a Science or Art teaching the manner and use of Numbring. This Art may be wrought diversly, with Pen, or with Counters. But I will first shew you the working with the Pen, and then the other in order. Ἀριθμητική

Scholar. This will I remember. But how many things are to be learned to attain this Art fully?

Master. There are reckoned commonly seven parts or works of it;

Numeration,

Numeration, Addition, Subtraction, Multiplication, Division, Progression, and Extraction of roots: to these some men adde Duplation, Triplation, and Mediation. But as for these three last they are contained under the other seven: For Duplation and Triplation are contained under Multiplication, as it shall appear in their place; and Mediation is contained under Division, as I will declare in his place also.

Scholar. Yet then there remain the first seven kinds of Numbring.

Master. So there doth: howbeit if I shall speak exactly of the parts of Numbring, I must make but five of them: for Progression is a compound Operation of Addition, Multiplication and Division, and so is the Extraction of roots. But it is no harm to name them as kinds several, seeing they appear to have some several working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them as seven kinds distinct. But now I desire you to instruct me in the use of each of them.

Master. So I will, but it must be done in order: for you may not learn the last so soon as the first, but you must learn them in that order as I did rehearse them, if you will learn them speedily and well.

Scholar. Even as you please. Then to begin: Numeration is the first in order, what shall I do with it?

Master.

Master. First, you must know what the thing is, and then after learn the use of the same.

Numeration.

NUmeration *is that* Arithmetical *skill, whereby we may duely value, express, and reade any* Number *or* Summe *propounded: or else in apt* Figures *and* Places *set down any* Number *known or named.*

Scholar. Why? then methinks you put a difference between the Value and the Figures.

Master. Yea so do I. For the Value is one thing, and the Figures are another thing; and that cometh partly by the diversity of Figures, but chiefly in the places wherein they be set.

Scholar. Then must I know here 3 things, the Value, the Figure, and the Place.

Master. Even so. But yet adde Order to them as the fourth. And first mark, that there are but ten Figures that are used in Arithmetick; and of those ten, one doth signifie nothing, which is made like an o, and is privately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured. A Cypher

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you must mark, that every Figure hath two values: one alwaies certain, that it signifieth properly, which it hath of his form; and the other uncertain, which he taketh of his place.

place. A place is called the seat or room that a figure standeth in. And look how many figures are written in one summe, so many places hath that whole number. And that must be called the first place that is next to the right hand, and so reckoning by order towards the left hand, so that that place is last that is next to the left hand. As for example: If there stood before you six men in a row, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand; then he that were next him should be called your second, and so forth to the farthest from your right hand, which is the sixth and the last.

Scholar. I perceive you well: so might I reckon Letters or any other thing. As if I should write 8 Letters after this order, a, b, c, d, e, f, g, h: then must I say, h is the first, g the second, f the third, e the fourth, d the fifth, c the sixth, b the seventh, a the eighth.

Master.

Master. That is well done. And after the same sort use hereafter, that what I declare by one example do you express by another: and so shall I perceive whether you understand it or no. And so pass over nothing till you perceive it well, and be expert therein.

Schol. I pray you how many of these places be there in all?

Master. There is no certain number of them, but they are sometimes more, and sometimes fewer, according to the sum that is expressed. For so many as the figures are, so many are the places: and the last place is so called, not because it is the last of all other, but it is the last of that present summe, and it may be the middle place in another summe.

Scholar. Me seemeth I perceive this very well, as touching the order of reckoning of the places: but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diversity of places, which you called the value uncertain. Value uncertain.

Master. But first let me hear whether you know perfectly the certain value. Value certain.

Scholar. Yes, sir, as you wrote them so I marked them.

Master. How write you then five?

Scholar. By this figure 5.

Master. And how six?

Scholar. Thus, 6.

Master. Write these three numbers, each by it self, as I speak them, vii. iiii. iii.

Scholar. 7. 4. 3.

Master. How write you these four other, ii. i. ix. viii.

Scholar. Thus (I trow) 2. 1. 6. 8.

Master. Nay there you miss: look on mine example again.

Scholar. Sir, true it is, I was to blame, I take 6 for 9, but I will beware hereafter.

Master. Now then take heed, those certain values every figure representeth when it is alone written without other figures joyned to him; and also when it is in the first place, though many other do follow: as for example, this figure 9 is ix, standing now alone.

Scholar. How is he alone, and standeth in the middle of so many letters?

Master. The letters are none of his fellows: for if you were in France in the middle of a thousand French-men, if there were no English-man with you, you would reckon your self to be alone.

Scholar. So it is. Then 9 without more figures of Arithmetick betokeneth ix, whatsoever other letters be about it.

Master. Even so; and so doth it, if it be in the first place joyned with other, how many so ever do follow: as in this example, 3679, you see 9 in the first place, and it doth betoken nine as if it were alone.

Scholar. I perceive that; and doth not 7 that standeth in the second place (between 9, and 6 in the third place) betoken vii, and so 3 in the fourth place betoken three?

Master. Their figures be as you have said, but their values are not so: for as in the first place every figure betokeneth his own value certain onely, so

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in the second place every figure betokeneth his own value certain ten times: as in the example, 7 in the second place is seven times ten, and is lxx. And in the third place every figure betokeneth his own value an hundred times, so the 6 in that place betokeneth vi C: and in the fourth place every figure betokeneth his own value a M. times, as in the aforesaid number, 3 in the fourth place standeth for 3 M: and in the fifth place every figure standeth for his own value x M. times, and in the sixth place a C M. times, and in the seventh place a M M. times, and in the eighth place x M M: so that every place exceedeth the former ten times.

Scholar. As thus: if I make this number at all adventures, 91359684, here are eight places. In the first place is 4, and betokeneth but four: in the second place is 8, and betokeneth ten times 8, that is 80: in the third place is 6, and betokeneth six hundred: in the fourth place 9 is nine thousand, and 5 in the fifth place is x M times 5, that is, fifty M. So 3 in the sixth place is a C M times 3, that is, CCC M. Then 1 in the seventh place, one M M; and 9 in the eighth place, ten thousand thousand times 9, that is, xc M M. But now I cannot easily nor quickly read it in order. A general Rule.

Master. That shall you practise by this means. First, put a prick over the fourth figure, and so over the seventh, and (if you have so many) over the tenth, thirteenth, sixteenth, and so forth, still leaving two figures between each two pricks. And those two rooms between the pricks are called Ternaries. Ternaries.

Then begin at the last prick, and see how many

figures are between him and the end, which cannot pass them, reckoning himself for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands as your numbers have pricks.

After that come to the next three figures, and sound them as if they were apart from the rest, and adde to their value so many times thousands, as there are pricks between them and the first place of your whole number. And so do by every other three figures following, if you have more. As in example, 91359684, this was your number.

Put a prick over 9 in the fourth place, and over 1 in the seventh place, and then no more, (for your places come not to ten) as thus: 91359684.

Now go to the last prick over 1, and take it and the figure 9 that followeth it, and value them alone.

Scholar. 91, that is xci.

Master. So it is. Then adde for the number of your pricks twice M.

Scholar. That is, xci. thousand thousand.

Master. So it is. Then take the three other figures from one to the next prick, and value them.

Scholar. 359, that is CCC. lix.

Master. Now adde for the one prick, that is between them and the first place, M.

Scholar. CCC. lix. thousand.

Master. Then come to the other 3 figures that remain.

Scholar. 684. That is, vi. C. lxxxiiii.

Master. Now have you valued all. And at the end of the last number you shall adde nothing, because there remaineth no prick nor number after it:

it: yet prove it in another number, as thus, 2308640891 05340.

Scholar. 2308640891 05340. I have pricked them as you taught me, but I am in doubt whether I have done well or no, because of the Cyphers, for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a figure in setting of the pricks; and again, I know not wherefore they serve.

Master. That will I tell you now. Indeed they are of no value themselves, but they serve to make up the number of places, and to make the figure following them to be in a farther place, and therefore to signifie the more value: as in this example, 90, the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is, &c. So do two Cyphers thrust the figure following them into the third place, and so forth.

The use of Cyphers.

Scholar. Then I perceive in the example above, I have pricked well enough; for though that Cypher that is pricked signifies nothing, yet must he have the prick, because he came in the thirteenth place. Then will I prove to number that summe. First, there is 230, M, M, M, M, and then followeth 864, M, M, M. And what shall I now doe? There is a Cypher in the third place, and no figure after him, but they that I have reckoned.

Master. He did serve for them that you have already reckoned, to make them in a place farther then they should be, if he were away, and therefore now ye shall let him go. And so doe alwaies when he occupieth that place next before any

prick, which is the last of that Ternary; and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, M, M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must value them CCC. xl. onely.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity; for you have numbred or valued the summe most truly, and by the aid of the pricks each Denomination is distinct most plainly.

Trinity.

Scholar. What call you Denomination?

Denomination.

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty Pounds, Pounds is the Denomination. And likewise in saying 25 men, Men is the Denomination: and so of other. But in this place (that I spake of before) the last number of every Ternary is the Denomination of it. As for the first Ternary the Denomination is Unites; and of the second Ternary, the Denomination is thousands; and of the third Ternary, thousand thousands, or Millions; of the fourth, thousand thousand thousands, or thousand millions: and so forth.

Scholar. And what shall I call the value of the three figures that may be pronounced before the Denomination, as in saying, 203000000, that is, two hundred three millions? I perceive by your words that millions is the Denominator: but what shall I call CCiii, joyned before the millions?

Master. That is called the Numerator, or Valuer; and the whole summe that resulteth of them both is called the Summe, Value, or Number.

Numerator.
Summe or Value.

Scholar.

Scholar. Now is there any thing else to be learned in Numeration? or else have I learned it fully?

Master. I might shew you here who were the first Inventors of this Art, and the reason of all these things that I have taught you: but that I will reserve till ye have learned over all the practice of this Art, lest I should trouble you with over many things at the first.

But yet this you must mark, that there are three kinds of Numbers, one called Digits, another Articles, and the third mixt numbers. Three kinds of numbers.

A Digit is any number under ten, as these, 1. 2. 3. 4. 5. 6. 7. 8. 9. Digits.

And 10 with all other that may be divided into ten parts just, and nothing remain, are called Articles; such as are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000. &c. Articles.

And that number is called Mixt that containeth Articles, or at the least one Article, and a Digit; as 12. 16. 19. 21. 38. 107. 1005. and so forth. And for the more ease of understanding and remembrance, mark this: The Digit number is never written with more then one figure, but the Article and the Mixt number are ever written with more then one figure. And thus they differ, that the Article hath evermore this Cypher 0 in the first place, and the Mixt number hath ever there some Digit. Mixt.

Scholar. By these last words I perceive it much better then I did before: and now (I think) I will never miss to know those three asunder.

Master. If you remember now all that I have said, you have learned sufficiently this first kind of Arithmetick called Numeration. Howbeit I will exhort you now to remember both this that I have

ſe ma- eth ma- ry.

ſaid, and all that I ſhall ſay, and to exerciſe your ſelf in the practice of it: for rules without practice are but a light knowledge, and practice it is that maketh men perfect and prompt in all things.

And as you have learned to gather and expreſs the value of a ſumme propounded and ſet down before you, ſo muſt you practiſe to mark, note, and write down with apt figures and in due places, any number onely named or recited to you, or of your ſelf imagined: as for a proof. Now note you, or write down this ſumme, five thouſand two hundred fifty and ſeven?

Scholar. This troubleth me now, whether I ſhould begin at the firſt, or at the laſt. For reaſon (methinks) ſhould cauſe me to begin at the firſt; and yet if I write it as you ſpeak it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt; but the more eaſe for your hand is to begin with the laſt, that is to ſay, as I did ſpeak them: yet for the more ſurety a while you may begin at the firſt, repeating my words backwards thus, ſeven, fifty, two hundred, five thouſand; or elſe counting them all by their digit or value, as thus, ſeven, five, two, five; for that way is eaſieſt. But then muſt you look well whether there be any cypher in your ſumme, that he may be ſet in his place: as if the laſt valuer of your ſumme (as you ſpeak it) be above 9, then is there a cypher in the firſt place; and if it be an hundred, or above, then is there two cyphers, one in the firſt place and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch as cannot be ſet forth without many words; I think beſt here now

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at the end of Numeration, to adde a Table easie and ready for the first exercise of it.

Lo this is the Table.

The left hand or side.

The denominators of the place or value uncertain.	*X.M. of Millions.*	*M. of Millions.*	*C. of Millions.*	*X. of Millions.*	*Millions.*	*C. of Thousands.*	*X. of Thousands.*	*Thousands.*	*Hundreds.*	*Tens.*	*Unites.*
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seven.	7	7	7	7	7	7	7	7	7	7	[illegible]
Six.	6	6	6	6	6	6	6	6	6	6	6
Five.	5	5	5	5	5	5	5	5	5	5	5
Four.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cyph.	0	0	0	0	0	0	0	0	0	0	0
The order of places.	Eleventh.	Tenth.	Ninth.	Eighth.	Seventh.	Sixth.	Fifth.	Fourth.	Third.	Second.	First.

The right side or hand. The names of Digits, Values certain or Valuers.

This Table (as you may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertain on the left hand: so that by this Table you may learn both how to

expresse

expresse any number that you list, (if that it exceed not eleven places, that is to say, X C. thousand Millions) and so may you by help of it value all summes proposed under the said number.

For example, take the summe that I proposed before, which was five thousand two hundred fifty and seven. And if you will expresse it, take the first number (as I speak it,) which is five M, whose valuer or certain value is v, and his uncertain value or Denomination is M. First, you shall seek at the right hand of the valuer 5: then seek along under the title of Denomination toward the left hand till you find thousands, and under it, right at the foot of the Table, is the number of the place, that is the fourth, wherein you must write your Digit, or valuer, 5.

Afterward come to the second part of the number two hundred, whose valuer is 2, and his Denomination C. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand till you come under C, then look to the foot of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your Digit 2.

Then do so by your other two numbers that remain; and you shall finde 5 in the second place for your fifty, and 7 in the first place for your seven. And thus you may do with other numbers.

Scholar. Master, I thank you heartily. I perceive you seek to instruct me most plainly and briefly, and not to hide your knowledge with subtile words, as many do. For this rule is so plain, that I can desire it no plainer. And though it seem somewhat

what long, yet I perceive it to be a sure way.

Master. So it is; and though it be long, yet it is neither too long, neither too plain for young learners that lack practice: for this Table is in stead of a Teacher to them that lack one. But now I trust I have said enough of Numeration: which after you have well practised, then may you learn forth.

Scholar. Yet I pray you in one thing to tell me your judgment. Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men do agree, that the Chaldees, which first invented this Art, did set these Figures as they set all their letters; for they write backward, as you term it, and so do they reade. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not onely written from the right hand to the left, and so must be read, but also the right end of the Book is the beginning of it: whereas the Greeks, Latines, and all Nations of Europe, do write and reade from the left hand toward the right, and all their Books begin at the left side.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither liketh me well, because I see that the Chaldees and Hebrews do not so use their own Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that seeing in pronouncing of Numbers we keep the order of our own reading, from the left hand to the right; and again, we do ever name the greater numbers

before

prick, which is the last of that Ternary; and a Cypher in the last place doth nothing.

Scholar. Then shall I say but 89, M, M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my pricks spent, and yet remain 340, so that I must value them CCC. xl. onely.

Master. Now can you reckon after this sort: and remember that every such room, so parted, is called a Ternary, or Trinity; for you have numbred or valued the summe most truly, and by the aid of the pricks each denomination is distinct most plainly.

Trinity.

Scholar. What call you Denomination?

Denomination.

Master. It is the last value or name added to any summe. As when I say, an hundred two and twenty Pounds, Pounds is the Denomination. And likewise in saying 25 men, Men is the Denomination: and so of other. But in this place (that I spake of before) the last number of every Ternary is the Denomination of it. As for the first Ternary the Denomination is Unites; and of the second Ternary, the Denomination is thousands; and of the third Ternary, thousand thousands, or Millions, of the fourth, thousand thousand thousands, or thousand millions: and so forth.

Scholar. And what shall I call the value of the three figures that may be pronounced before the Denomination, as in saying, 203000000, that is, two hundred three millions? I perceive by your words that millions is the Denominator: but what shall I call CCiii, joyned before the millions?

Master. That is called the Numerator, or Valuer, and the whole summe that resulteth of them both is called the Summe, Value, or Number.

Numerator.

Summe or Value.

Scholar.

Scholar. Now is there any thing else to be learned in Numeration? or else have I learned it fully?

Master. I might shew you here who were the first Inventors of this Art, and the reason of all these things that I have taught you: but that I will reserve till ye have learned over all the practice of this Art, lest I should trouble you with over many things at the first.

But yet this you must mark, that there are three kinds of Numbers, one called Digits, another Articles, and the third mixt numbers. Three kinds of numbers.

A Digit is any number under ten, as these, 1. 2. 3. 4. 5. 6. 7. 8. 9. Digits.

And 10 with all other that may be divided into ten parts just, and nothing remain, are called Articles; such as are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000. &c. Articles.

And that number is called Mixt that containeth Articles, or at the least one Article, and a Digit; as 12. 16. 19. 21. 38. 107. 1005. and so forth. And for the more ease of understanding and remembrance, mark this: The Digit number is never written with more then one figure, but the Article and the Mixt number are ever written with more then one figure. And thus they differ, that the Article hath evermore this Cypher 0 in the first place, and the Mixt number hath ever there some Digit. Mixt.

Scholar. By these last words I perceive it much better then I did before: and now (I think) I will never miss to know those three asunder.

Master. If you remember now all that I have said, you have learned sufficiently this first kind of Arithmetick called Numeration. Howbeit I will exhort you now to remember both this that I have

ſe ma- th ma- ry.

ſaid, and all that I ſhall ſay, and to exerciſe your ſelf in the practice of it: for rules without practice are but a light knowledge, and practice it is that maketh men perfect and prompt in all things.

And as you have learned to gather and expreſs the value of a ſumme propounded and ſet down before you, ſo muſt you practiſe to mark, note, and write down with apt figures and in due places, any number onely named or recited to you, or of your ſelf imagined: as for a proof. Now note you, or write down this ſumme, five thouſand two hundred fifty and ſeven?

Scholar. This troubleth me now, whether I ſhould begin at the firſt, or at the laſt. For reaſon (methinks) ſhould cauſe me to begin at the firſt; and yet if I write it as you ſpeak it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt; but the more eaſe for your hand is to begin with the laſt, that is to ſay, as I did ſpeak them: yet for the more ſurety a while you may begin at the firſt, repeating my words backwards thus, ſeven, fifty, two hundred, five thouſand; or elſe counting them all by their digit or value, as thus, ſeven, five, two, five; for that way is eaſieſt. But then muſt you look well whether there be any cypher in your ſumme, that he may be ſet in his place: as if the laſt valuer of your ſumme (as you ſpeak it) be above 9, then is there a cypher in the firſt place; and if it be an hundred, or above, then is there two cyphers, one in the firſt place and another in the ſecond, and ſo forth.

But becauſe this thing is ſuch as cannot be ſet forth without many words; I think beſt here now

at

at the end of Numeration, to adde a Table easie and ready for the first exercise of it.

Lo this is the Table.

The right side or hand.

The names of Digits, Values certain or Valuers.

The denominators of the place or value uncertain.	Nine.	Eight.	Seven.	Six.	Five.	Four.	Three.	Two.	One.	Cyph.	The order of places.
Unites.	9	8	[illegible]	6	5	4	3	2	1	0	First.
Tens.	9	8	7	6	5	4	3	2	1	0	Second.
Hundreds.	9	8	7	6	5	4	3	2	1	0	Third.
Thousands.	9	8	7	6	5	4	3	2	1	0	Fourth.
X. of Thousands.	9	8	7	6	5	4	3	2	1	0	Fifth.
C. of Thousands.	9	8	7	6	5	4	3	2	1	0	Sixth.
Millions.	9	8	7	6	5	4	3	2	1	0	Seventh.
X. of Millions.	9	8	7	6	5	4	3	2	1	0	Eighth.
C. of Millions.	9	8	7	6	5	4	3	2	1	0	Ninth.
M. of Millions.	9	8	7	6	5	4	3	2	1	0	Tenth.
X.M. of Millions.	9	8	7	6	5	4	3	2	1	0	Eleventh.

The left hand or side.

This Table (as you may see) hath eleven Places, and in each of them are set all the Digits, whose certain value is written on the right hand of the Table, and the value uncertain on the left hand: so that by this Table you may learn both how to

expresse

expreſs any number that you liſt, (if that it exceed not eleven places, that is to ſay, ✠C. thouſand Millions) and ſo may you by help of it value all ſummes propoſed under the ſaid number.

For example, take the ſumme that I propoſed before, which was five thouſand two hundred fifty and ſeven. And if you will expreſs it, take the firſt number (as I ſpeak it,) which is five M, whoſe valuer or certain value is v, and his uncertain value or Denomination is M. Firſt, you ſhall ſeek at the right hand of the valuer 5: then ſeek along under the title of Denomination toward the left hand till you find thouſands, and under it, right at the foot of the Table, is the number of the place, that is the fourth, wherein you muſt write your Digit, or valuer, 5.

Afterward come to the ſecond part of the number two hundred, whoſe valuer is 2, and his Denomination C. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand till you come under C, then look to the foot of the Table, and there you ſhall ſee the number of the place, that is to ſay, the third, wherein you muſt ſet your Digit 2.

Then do ſo by your other two numbers that remain; and you ſhall finde 5 in the ſecond place for your fifty, and 7 in the firſt place for your ſeven. And thus you may do with other numbers.

Scholar. Maſter, I thank you heartily. I perceive you ſeek to inſtruct me moſt plainly and briefly, and not to hide your knowledge with ſubtile words, as many do. For this rule is ſo plain, that I can deſire it no plainer. And though it ſeem ſomewhat

what long, yet I perceive it to be a sure way.

Master. So it is; and though it be long, yet it is neither too long, neither too plain for young learners that lack practice: for this Table is in stead of a Teacher to them that lack one. But now I trust I have said enough of Numeration: which after you have well practised, then may you learn forth.

Scholar. Yet I pray you in one thing to tell me your judgment. Why do men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Master. In that thing all men do agree, that the Chaldees, which first invented this Art, did set these Figures as they set all their letters: for they write backward, as you term it, and so do they reade. And that may appear in all Hebrew, Chaldee and Arabick Books; for they be not onely written from the right hand to the left, and so must be read, but also the right end of the Book is the beginning of it: whereas the Greeks, Latines, and all Nations of Europe, do write and reade from the left hand toward the right, and all their Books begin at the left side.

Scholar. That reason hath satisfied me.

Master. It neither satisfieth me, neither liketh me well, because I see that the Chaldees and Hebrews do not so use their own Numbers, as at another time I will declare. But this plain reason may best satisfie you presently, that seeing in pronouncing of Numbers we keep the order of our own reading, from the left hand to the right; and again, we do ever name the greater numbers

before

before the smaller; it was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places, containing the greater numbers, to proceed toward the left hand.

Scholar. This reason is to me so plain, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied, hoping that practice shall make me fully ready and expert in it. And in the mean season I desire to learn the other kinds of Arithmetick.

Master. That is well said: but what should you next learn? can you tell?

Scholar. I remember you said that Addition was next.

Master. Even so, and what that is, must you first know.

Addition.

Addition is the gathering together and bringing of two numbers, or more, into one summe. As if I have 106 Books in the Latine tongue, and 136 in the Greek tongue, and would know how many they be in all, I must write these two numbers one over another, writing the greatest number highest, so that the first figure of the one be under the first figure of the other, and the second under the second, and so forth in order.

When you have so done, draw under them a right line, then will they stand thus.

$$\begin{array}{r} 160 \\ 136 \\ \hline \end{array}$$

Now begin at the first places toward the right hand alwaies, and put toge-

ther

ther the two firſt figures of theſe two numbers,
and look what cometh of them write 160
under them, right under the line. 136
As in ſaying 6 and 0 is 6, write 6 —
under 6, as thus: 6

And then go to the ſecond figures, and doe like-
wiſe: as ſaying 3 and 6 is 9, 160
write 9 under 6 and 3, as here you 136
ſee: —

And likewiſe doe you with the fi- 96
gures that be in the third place, ſaying 1 and 1 be 2;
write 2 under them, and then will 160
your whole ſumme appear thus: 136

So that now you ſee that 160 and —
136 do make in all 296. 296

Scholar. What? this is very eaſie to doe, methinks I can doe it even ſince.

There came through Cheapſide two droves of Cattell: in the firſt was 848 ſheep, and in the ſecond was 186 other Beaſts.

Thoſe two ſummes I muſt write as you 141
taught me, thus. Then if I put the two firſt 186
figures together, ſaying, 6 and 8, they make —
14. That muſt I write under 6 and 8 thus: 14

Maſter. Not ſo: and here you are twice deceived. Firſt, in going about to adde together two ſummes of ſundry things, which you ought not to doe except you ſeek onely the number of them, and care not for the things: For the ſumme that ſhould reſult to that Addition, ſhould be a ſumme neither of ſheep nor of other beaſts, but a confuſed ſumme of both. Howbeit ſometimes ye ſhall have ſummes of divers Denominations to be added, of which I will tell you

anon:

anon: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8, under 6 and 8, which is unpossible; for how can two figures of two places be written under one figure and one place?

Scholar. Truth it is, but yet I did so understand you.

Master. I said indeed, that you should write that under them that did result of them both together: which saying is alwaies true, if that summe do not exceed a Digit. But if it be a mixt number, then must you write the Digit of it under your figures, as you have said before: and if it be an Article, then write 0 under them: and in both sorts you shall keep the Article in your mind. And therefore when you have added your second figures, which occupy the place of tens, you shall put that one thereto which you kept in your mind; for though it were ten indeed, yet in that place it is but as one, because that every one of that place is ten, for that it is the place of tens. And in like manner, if you have in the second place so great a number that it amounteth above 9, then write the Digit, and reserve the Article in your mind, ever adding it to the next place following; and so of all other places, how many soever you have. And if you have a mixt number when you have added your last figures, then write the Digit under the last figures, and the Article in the next place beyond them: so shall your number resulting of Addition have one place more then the numbers which you shall adde together.

place.

Scholar. Now do I perceive you, and the reason of this is, (as I understand) because that no

one

one place can contain above 9, which is the greatest figure that is, and then all tens or Articles must be put to the next place following: for every place (as I may see) exceedeth the other place next before him by 10.

Now, if it please you, I will return to my example of Cattell. But I remember you said I might not adde summes of sundry things together, and that I may see by reason.

Master. Truth it is, if you seek the due summe of any thing, but if you onely seek a bare summe, and have no respect to the thing, then were it better to name the summe onely without any thing: as in saying 848, without naming sheep or any thing else, and likewise 186, naming nothing.

Now let me see how you can adde those two summes.

Scholar. I must first set them so that the two first figures stand one over another, and the other each one over his fellow of the same place; then shall I draw a line under them both. And so likewise of other figures, setting always the greatest number highest, thus as followeth.

Then must I adde 6 to 8, which make 14, that is a mixt number; therefore must I take the digit, which is 4, and write it under 6, and 8, keeping the Article 1 in my mind, thus:

$$\begin{array}{r} 848 \\ 186 \\ \hline 4 \end{array}$$

Next that, I do come to the second figures, adding them together, saying 8 and 4 make 12, to the which I put the one reserved in my minde, and that maketh 13, of which number I write the Digit 3 under 8 and 4, and keep the Article in my mind, thus:

$$\begin{array}{r} 848 \\ 186 \\ \hline 34 \end{array}$$

Then come I to the third figures, saying, 1

and

and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cypher under 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Master. What needeth that, seeing there follow no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last figures, I must write the Cypher under them, and the Article in a farther place after the Cypher, thus:

848
186
——
1034

Master. So now you see, that of 848 and 186 added together, there amounteth 1034.

Scholar. Now I think I am perfect in Addition.

Master. That will I prove by this example. There are two Armies of Souldiers: in the one are 106800, and in the other 9400. How many are there in both Armies say you?

Scholar. First, I set them one over another, beginning with the first number on the right hand, thus:

106800
9400

But the nether number will not match the other number.

Master. That forceth not.

Scholar. Then do I adde 0 to 0, and there amounteth 0; that must I write under the first place, thus:

106800
9400
——
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write under the second place, thus:

106800
9400
——
00

Then

Then I come to the third place, saying, 4 and 8 make 12, of which I write the Digit, and keep the Article 1 in my mind, thus:

	106800
	9400
	200

Then I adde 9 to 6, which make 15, to that I adde the Article 1 that was in my mind, and it is 16, I write 6 under 6 and 9, and keep 1 in mind, thus:

	106800
	9400
	6200

Mast. Why do you not write both figures, seeing you are come to the last couple of numbers?

Scholar. Nay, reason sheweth me, that I must adde that Article that is in my mind unto the next figure of the over summe, though there be no more in the nether summe.

Master. That is well considered: then do so.

Scholar. Then say I, 0 in the over summe, and 1 in my mind maketh 1: that write I under 0. Then followeth there yet one more in the over summe, which hath none to be added to it, for there is none in the nether summe, nor yet in my mind: therefore I think I must write that even as it is.

Master. Yea.

Scholar. Then doth my whole summe appear thus:

	106800
	9400
	116200

Master. If you mark this, you have learned perfectly the common Addition of all summes which are of one Denomination; so that ye observe this also, that in Addition you must have two numbers at the least; or else how can you say that you do adde? And ever let the greatest number be written highest, for that is the best way, though it be not necessary.

And forget not this, that (if you have many num-

bers to adde together) you ſhall have oftentimes an Article of a greater value then 10, ſometimes 20, ſometimes 30, ſometimes more, yea (peradventure) 100. Therefore as you did with the Article 10, ſo do with them, reſerving them in your mind, and adding to the number next following ſo many as their value or value certain is: that is to ſay, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and ſo forth of other like. So that if the Article be 100, then muſt you ſet down the 0, and keep 10 in mind, to be carried to the next row of Figures or place, if any ſuch happen to come. For your better underſtanding, take this example for all. I would adde theſe thirteen ſummes into one, which I ſet after this manner: then do I begin and gather the ſumme of the firſt row of Figures, which come to 107, (for I take 9 there ten times, and that is 90) then 9 and 8 is 17, that is in all 107, of which ſumme I write the 7 under the firſt row of Figures, and then for that 100 is ten tens, I keep ten in mind, which ten I muſt adde unto the next row of Figures, which are in the ſecond place:

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389

which ſecond row of Figures (when they are added together with that ten that I had in my mind) make in all 125, of which ſumme write the Digit 5 under the ſecond row; and then (for that 120 containeth twelve tens) I keep twelve in mind to be added to the third place or row of Figures; which being added together make in all 60: the Cypher 0 I ſet down under the row of figures in the third place.

And

And the Figure 6 I keep in mind to be added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I set down under the fourth place. And because it is my last work, I set down the 2 also that I have in my mind to the 9 in the fifth place. So these summes do make in all 29057.

¶ But (for your more ease in work) when you have an Addition of so many summes to be added together, you were best part that summe into two or three parts, and work them severally, and so put their Additions together: and this were the best thing you could doe when over-many summes fall to be added.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
382
29057

Scholar. This seemeth somewhat hard, by the reason of so many numbers together. Howbeit, I think (if I do often prove, even with the same example, either by working of it alone, or else by parting it, as you said even now) that I shall be able to do so shortly with any other summe.

Master. So shall you. For it is often practice that maketh a man quick and ripe in all things. But because as well in great summes as in small there may chance to be some errour, I will teach you how you shall prove whether you have done well or no.

Scholar. That were a great help and ease.

The proof of Addition.

Master. Begin first with the highest number, and then take all the other orderly, and adde them together, not having regard to their places, but as

 though

though they were all unites: and still (as your number encreaseth above 9) cast away 9. Then go forth, ever casting away 9 as often as it amounteth thereto: and so do till you have gone over all the numbers that you intended first to adde. And whatsoever remaineth after such Addition and casting away of 9, write it in some void place by the end of a line, for the better remembrance: and thus is the first place of your work proved. Then, secondly, put together the figures that result of the Addition under the line, still casting away 9 also; and then that that remaineth write at the other end of the line: and if those two figures be alike, then have you well done, but if they be unlike, then have you missed. As for example, in this present summe; The first figure of the over line is 9, let him goe, then 8 and 8 is 16, take away 9, there resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9, therefore overpass them both, and take the 5 to the 2 which did remain in the first row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that goe to the third line, whose two first numbers you may let pass, because they are nines: then take the two figures of 2, which (with the other two that remained in the second row) make 6. Then go to the fourth row, whose two first numbers let go, and take the 6 to the 6 that remaineth, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next maketh 6. And so goe through all the

other numbers, and you shall find that there remaineth 5, after you have cast away 9 as often as you can find it: therefore write 5 at the end of the line in a void place thus,

5————————

Then gather all the figures of the totall summe, which is under the lowest line, and cast away 9 as often as you can find it; as thus, 7 and 5 make 12, take away 9, there resteth 3; so that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5————————5

And then you see that these two figures be alike, whereby you may know that you have done well: and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First, in the highest line 8 and 6 make 14, then 9 taken away, there remains 5, to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I find first 4, which with 6 maketh 10, from that I take 9, and there resteth one; the next figure is 9, and therefore I let him alone, so find I 1 remaining, which I set at the end of a line, thus,

$$\begin{array}{r} 106800 \\ 9400 \\ \hline 116200 \end{array}$$

1————————

Then I come to the totall summe, and there I

find

And that all the Figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line thus,

1 ——————— 1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no. And now I will teach you how to adde summes of divers Denominations together; which thing can never be but when the one Denomination is such that it containeth the other certain times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination,) but after such a sort as I will now shew you, that is to say;

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then look that you set every Denomination by himself, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one be set under the other of the same Denomination. As for Example, if your Denominations be pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, methinks it needeth not to warn me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I should have been deceived in it.

Master. If you do say it is plain, I will speak no

no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together. Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see, with a line under them.

li.	s.	d.
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwaies be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination, then must I keep it in my mind till I come to that place: and under that first place must I note the residue, (if there remain any of the same Denomination) but if there remain none, then need I to write under it nothing. And this is all that you must mark in this Addition: for all other things are like to the manner of Addition before mentioned. *Therefore the chiefest point of this Addition is, to know the values of common Coins, and rated summs*; as how many shillings be in a pound, how many pence in a shilling; of which (and of other like things) I will instruct you hereafter in teaching of Reduction: but now I may not disturb your wit from the thing that we are about.

Therefore let us return to that former example which I proposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

bers to adde together) you shall have oftentimes an Article of a greater value then 10, sometimes 20, sometimes 30, sometimes more, yea (peradventure) 100. Therefore as you did with the Article 10, so do with them, reserving them in your mind, and adding to the number next following so many as their value or value certain is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so forth of other like. So that if the Article be 100, then must you set down the 0, and keep 10 in mind, to be carried to the next row of Figures or place, if any such happen to come. For your better understanding, take this example for all.

understanding, take this example for all.	4889
I would adde these thirteen summes into	4599
one, which I set after this manner: then	2290
do I begin and gather the summe of	3699
the first row of Figures, which come to	2299
107, (for I made 9 there ten times, and	4099
that is 90) then 9 and 8 is 17, that is	1099
in all 107, of which summe I write the	3298
7 under the first row of Figures, and then	299
for that 100 is ten tens, I keep ten in	699
mind, which ten I must adde unto the	499
next row of Figures, which are in the se-	899
cond place:	389

which second row of Figures (when they are added together with that ten that I had in my mind) make in all 125, of which summe write the Digit 5 under the second row; and then (for that 120 containeth twelve tens) I keep twelve in mind to be added to the third place or row of Figures; which being added together make in all 60: the Cypher 0 I set down under the row of figures in the third place.

And

And the Figure 6 I keep in mind to be added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I set down under the fourth place. And because it is my last work, I set down the 2 also that I have in my mind to the 9 in the fifth place. So these summes do make in all 29057.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
382
29057

¶ But (for your more ease in work) when you have an Addition of so many summes to be added together, you were best part that summe into two or three parts, and work them severally, and so put their Additions together: and this were the best thing you could doe when over-many summes fall to be added.

Scholar. This seemeth somewhat hard, by the reason of so many numbers together.

Howbeit, I think (if I do often prove, even with the same example, either by working of it alone, or else by parting it, as you said even now) that I shall be able to do so shortly with any other summe.

Master. So shall you. For it is often practice that maketh a man quick and ripe in all things. But because as well in great summes as in small there may chance to be some errour, I will teach you how you shall prove whether you have done well or no.

Scholar. That were a great help and ease.

Master. Begin first with the highest number, and then take all the other orderly, and adde them together, not having regard to their places, but as though — *The proof of Addition.*

though they were all unites: and still (as your number encreaseth above 9) cast away 9. Then go forth, ever casting away 9 as often as it amounteth thereto: and so do till you have gone over all the numbers that you intended first to adde. And whatsoever remaineth after such Addition and casting away of 9, write it in some void place by the end of a line, for the better remembrance: and thus is the first place of your work proved. Then, secondly, put together the figures that result of the Addition under the line, still casting away 9 also; and then that that remaineth write at the other end of the line: and if those two figures be alike, then have you well done, but if they be unlike, then have you missed. As for example, in this present summe; The first figure of the over line is 9, let him goe, then 8 and 8 is 16, take away 9: there resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9, therefore overpass them both, and take the 5 to the 2 which did remain in the first row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that goe to the third line, whose two first numbers you may let pass, because they are nines: then take the two figures of 2, which (with the other two that remained in the second row) make 6. Then go to the fourth row, whose two first numbers let go, and take the 6 to the 6 that remaineth, and that maketh 12: take away 9, and there resteth 3, which with the 3 that is next maketh 6. And so goe through all the

other numbers, and you shall find that there remaineth 5, after you have cast away 9 as often as you can find it: therefore write 5 at the end of the line in a void place thus,

5————————

Then gather all the figures of the totall summe, which is under the lowest line, and cast away 9 as often as you can find it; as thus, 7 and 5 make 12, take away 9, there resteth 3; to that if you adde the 2 that is last, (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus:

5————————5

And then you see that these two figures be alike, whereby you may know that you have done well: and so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First, in the highest line 8 and 6 make 14, then 9 taken away, there remains 5, to which I adde the 1 that followeth, and that maketh 6: then come I to the second line, where I find first 4, which with 6 maketh 10, from that I take 9, and there resteth one; the next figure is 9, and therefore I let him alone, so find I 1 remaining, which I set at the end of a line, thus,

106800
9400
116200

1————————

Then I come to the totall summe, and there I

find

And that all the Figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other end of the line thus,

1 ——————— 1

And because they be like, I know that I have well added.

Master. So you know now both how to adde two summes or more together, and also how to prove whether you have done well or no. And now I will teach you how to adde summes of divers Denominations together; which thing can never be but when the one Denomination is such that it containeth the other certain times. And yet you shall adde them to the other, not after this sort (as you did them that were of one Denomination,) but after such a sort as I will now shew you, that is to say;

Addition of numbers of divers Denominations.

If you have a summe of divers Denominations, then look that you set every Denomination by himself, with some note or figure of his Denomination, as they are wont to be written. Then write your other summes so under that first, that every one be set under the other of the same Denomination. As for Example, if your Denominations be pounds, shillings, and pence, write pounds under pounds, shillings under shillings, and pence under pence: and not shillings under pence, nor pence under pounds.

Scholar. Now that you have spoken it, methinks it needeth not to warn me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peradventure I should have been deceived in it.

Master. If you do say it is plain, I will speak

no

no more of it, but with an example make the matter to appear evidently.

First, one man oweth me 22 l. 6 s. 8 d. another oweth me 5 l. 16 s. 6 d. and another oweth me 4 l. 3 s. I would know what this is all together. Therefore must I first set down my great summe, and then the other, every one under his Denomination agreeing to the greatest summe, as here you see, with a line under them.

li.	s.	d.
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwaies be set next to the right) and adde them together: and if the summe will make 1 or 2 or 3 of the next Denomination, then must I keep it in my mind till I come to that place: and under that first place must I note the residue, (if there remain any of the same Denomination) but if there remain none, then need I to write under it nothing. And this is all that you must mark in this Addition: for all other things are like to the manner of Addition before mentioned. *Therefore the chiefest point of this Addition is, to know the values of common Coins, and rated summs*; as how many shillings be in a pound, how many pence in a shilling; of which (and of other like things) I will instruct you hereafter in teaching of Reduction: but now I may not disturb your wit from the thing that we are about.

Therefore let us return to that former example which I proposed of the Debtors: which summes when I had set orderly, they stood thus with a line under them.

Then, to adde them into one summe, I must begin at the right hand, where the smallest Denomination is, and adde them together, first saying 6 and 8 make 14. Now seeing these 14 are pence, which contain one shilling and 2 pence; the 2 pence I set down under the line of pence, and the one shilling I keep in my mind to carry to the next row, being the place of shillings.

li.	s.	d.
22	6	8
5	16	6
4	3	0

Then do I adde the shillings together, saying, 1 in my mind and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10 make 26, which is 1 pound 6 s. The 6 s. I set down under the place of shillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

li.	s.	d.
22	6	8
5	16	6
4	3	0
	6	2

Then come I to the pounds, adding them all together, saying, 1 that I kept and 4 make 5, and 5 make 10, and 2 make 12. The Figure or Digit 2 I set down right under that place or row of pounds where I gather them, and the Article 1 I keep to carry to the next place, saying, 1 in my mind and 2 is 3, which 3 I set down directly under the 2. And then appeareth my whole summe thus:

li.	s.	d.
22	6	8
5	16	6
4	3	0
32	6	2

And thus must you doe with any such like summes whatsoever, whether they be money, weight, or measure, by which (if you practise divers summes) you

you shall be well acquainted with the feat of Addition.

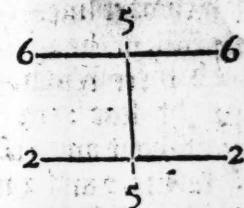
But now can you tell how to prove this Addition, or such other like of divers Denominations, and to try whether you have well done or no?

Scholar. I would I could.

Master. That shall you doe by this means: You must make a Cross which shall have as many lines as you have sundry Denominations in your Addition. As if you have but two Denominations then you may take it thus: that the over part and nether part may serve for one Denomination. — Proof of Addition of divers Denominations.

And if you have three Denominations, (as pounds, shillings and pence) then must you make three lines, thus: The upright line may serve for pounds, and the highest thwart line for shillings, and the lowest for pence. As for example, the summe which we last wrought.

li.	s.	d.
22	6	8
5	16	6
4	3	0



For the proof of which, because it containeth 3 Denominations, I must make a cross of 3 lines, as in the example before. Then I reckon first at the right hand the pence, 6 and 8 make 14; from which I take 12 for the next Denomination, that is to say, a shilling, and there resteth 2, which I must write

write at one end of the nether thwart line.

After that I gather the summe of the shillings, 3, 16, 6, which make 25; to whom I put 1 that I took of the pence, and that maketh 26; from those I take 20, the quality of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds, 4, 5, and 2, which make 11; to them I adde the one that came of shillings, and they make 12; from whence I cast 9, and there resteth 3: that three I joyn to the 2 in the next place, and they make 5, which 5 I set at the Cross also. And thus is my first part of my work proved.

That done, I come to the total summe under the line, and examine it, beginning at the pence, where I find but 2, and cannot take 9 from him; therefore I set him at the other end of the nether thwart line: then I come to the shillings, where I find onely 6, which (because it is less then nine) I set at the other end of the line of the shillings, that is, the overmost thwart line.

Last of all, of the 32 li. I take three times 9, which is 27, and there remaineth 5, which I write under the upright line: or else I may reckon them simply without any respect of their valuation or place, saying, 2 and 3 make 5, which, because it is less then nine, I set under the upright line as before. Then I consider every number, comparing it to the number that is against it: and because I find them to be every one like his match, I know that I have well done.

Scholar. This Cross I perceive doth serve for these

theſe 3 Denominations, pounds, ſhillings, pence: but what if I had l. s. d. ob. and q?

Maſter. Theſe lines, as I have ſaid, do ſerve for 3 Denominations, ſuch as they be, as here 3 do ſerve for pounds, ſhillings and pence; but if you have no pounds in your ſumme, then may they ſerve for ſhillings, pence and half penies; yea for d. ob. and q; or in weight for C, q. and l; or in meaſure for Ells, Quarters and Nails, if you have no greater Denomination: ſo that you remember that the upright line ſerveth for the greateſt Denomination, and the higheſt thwart line for the next, and the loweſt for the leaſt.

And ſo if you have four Denominations, you muſt make your croſs with ſo many lines: And if your ſumme be of more Denominations, make ſo many lines in your croſs. And thus will I make an end of Addition, ſaving that here (for the better underſtanding of this Rule) I have ſet you down certain examples both of money, weight, and meaſures, with their works and proofs.

Examples of Addition.

li.	s.	d.	li.	s.	d.
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
25	13	6	120	0	0
131	13	3	292	17	10

The

The Proofs.

	5	
4		4
3		3
	5	

	4	
8		8
1		1
	4	

C.	q.	li.
34	1	3
12	2	2
7	3	4
13	0	13
67	2	23

yards.	q.	nails.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3

	3	
2		2
4		4
	3	

	8	
1		1
3		3
	8	

Subtraction.

Schol. T*Hen have I learned the two first kinds of* Arithmetick *: now (as I remember) doth follow* Subtraction, *whose name (me thinks) doth sound contrary to* Addition.

Subtraction. Master. So it is indeed: for as Addition increaseth one gross summe, by bringing many into one; so, contrariwise, Subtraction diminisheth a gross summe, by withdrawing of others from it. *So that*

that Subtraction *or Rebating is nothing else but an* Art *to withdraw and abate one summe from another, that the Remainer may appear.*

Scholar. What do you call the Remainer?

Master. That you may perceive by the name.

Scholar. So methinks: but yet it is good to ask the truth of all such things, lest in trusting to mine own conjecture I be deceived.

Master. So it is the surest way. And, as I see cause, I will still declare things unto you so plainly, that you shall not need to doubt. Howbeit, if I do overpass it sometimes, (as the manner of men is to forget the small knowledge of them to whom they speak) then do you put me in remembrance your self, and that way is surest.

And as for this word that you last asked me, take you this description: The Remainer is a summe left after one Subtraction made, which declareth the excess or difference of the two other numbers: as if I would abate or subtract 14 out of 18, there should remain 4, which is called the Remainer, and is the difference between those two numbers 14 and 18. Remainer.

Scholar. I perceive then what Subtraction is: now resteth to know the order how to work it.

Master. That shall you doe by this means. First, you must consider, that if you should go about to rebate, you must have two sundry summes proposed: the first, which is your gross summe, (or summe total) and it must be set highest; and then the rebatement, (or summe to be withdrawn) which must be set under the first, (whether it be in one parcell or in many) and that in such sort, that the

the first figures be one just over another, and so the second and third, and all other following, as you did in Addition: then shall you draw under them a line, and so are your summes duely set to begin your working.

Then begin you at the right hand (as you did in Addition,) and withdraw the nether number out of the higher, and if there remain any thing, write that right under them beneath the line: and if there remain nothing, (by reason that the two Figures were equal) then write under them a Cypher or nought: And so doe you with all the other Figures, evermore abating the lower out of the higher, and write under them the Remainder still, till you come to the end: And so will there appear under the line what remaineth of your gross summe after you have deducted the other summe from it, as in this example.

I received of your Father 48 s, of which I have laid out for you 36 s. now would I know what doth remain. And therefore I set my number thus in order. First, I write the greatest summe, and under him the lesser, so that the Figures at the right side be even one under another, and so the other, thus:

$$\begin{array}{r} 48 \\ \underline{36} \end{array}$$

Then do I rebate 6 out of 8, and there resteth 2, which I write under them right beneath the line thus:

$$\begin{array}{r} 48 \\ \underline{36} \\ 2 \end{array}$$

Then I goe to the second Figures, and do rebate 3 out of 4, where there remaineth 1, which I write under them right: and then the whole summe and operation appeareth thus:

$$\begin{array}{r} 48 \\ \underline{36} \\ 12 \end{array}$$

Whereby

Whereby it appeareth, that if I withdraw 36 out of 48, there remaineth 12.

Scholar. Now I will prove in a greater ſumme, and I will ſubtract 2367924 out of 3468946. Thoſe ſummes I ſet in order thus:

3468946
2367924

Then do I begin at the right ſide, and deduct 4 out of 6, and there reſteth 2, which I write under them. Then goe I to the ſecond figures, and withdraw 2 out of 4, and there remaineth 2, which I ſet under them alſo. Then I take 9 out of 9, and there reſteth 0, which I write under them, (for you ſay, that if the Figures be equal, ſo that nothing doth remain, I muſt write the Cypher 0 under them.)

Maſter. It was well remembred: now go forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1, which I write under them alſo. Then in the fifth place I take 6 out of 6, and there reſteth 0, (for it I write under them the Cypher 0.) Then in the ſixth place 3 rebated from 4 there remaineth 1, which I write under them: and likewiſe in the ſeventh and laſt place, 2 taken from 3 there is left 1, which I write under them. So have I done my whole working, and my ſummes do appear thus. Whereby I ſee that (if I do rebate 2367924 out of 3468946) there remaineth 1101022.

3468946
2367924
1101022

Maſter. This is well done. And that you may be ſure to perceive fully the Art of Subtraction, let me ſee how you can ſubtract 52984732 out of 8250003456.

Scholar.

Scholar. First I set down the greatest summe, and after that I will write under it the lesser number, beginning at the right side; and then my figures will stand thus:

8250003456
52984732

Note. Then take I 2 from 6, and there resteth 4, which I write under them. Then do I withdraw 3 from 5, and there remains 2, which I write under them. Then take I 7 out of 4, but that I cannot, what shall I now do?

Master. Mark well what I shall tell you now, how you shall doe in this case, and in all other the like. If any figure of the nether summe be greater then the figure of the summe that is over him, (so that it cannot be taken out of the figure over him) then must you put 10 to the over figure, and then consider how much it is, and out of that whole summe withdraw the nether figure, and write the rest under them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example, where I should have taken 7 out of 4, and could not, I put ten to that 4, which maketh 14, from it I take away 7, and there resteth 7 also, which I write under them.

Master. So have you done well. But now must you mark another thing also; that (whensoever you do so put ten to any figure of the over number) you must adde one still to the figure or place that followeth next in the nether line: as in the example there followeth 4, to which you must put 1, and make him 5, and then go on as I have taught you.

8250003456
52984732
018724

Scholar.

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be; therefore I must put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written under them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I should take out of 0, and cannot; therefore I put thereto 10, and that maketh 10, from 10 I take 9, and there remains 1, which I write under them.

Thus do I adde 1 likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure above, but I cannot, for it is 0; therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written under them.

Then come I to the next figure, which is 2, and to him I doe adde 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doe take 3; so there remaineth 7 to be written under them: likewise doe I put 1 to 5, which make 6, that 6 I cannot take out of 5, therefore I adde 10 to that 5, and make it 15, from which I rebate 6, there remaineth 9, which I write under them. Now have I spent all the nether Figures, and what shall I do more?

8250003456
52984732
8197018724

Master. You should have added one to the next figure following, (if there had been any) because you added 10 to the last figure before of the over line: but being there is no figure following, you

must adde that one to the place following, and then deduct that one from the number above.

Scholar. Then shall I say, Because I borrowed 10 to the over 5, I must put 1 in the next place beneath, that is under 2; then must I subtract that 1 from 2, and there resteth 1, to be written under that in the ninth place. Now I have no more to subtract, for there is not any figure remaining beneath, neither yet any unite to be added, because I borrowed not 10 to the figure last before: and yet is there 8 remaining in the over line, which I think (by reason) should be set at the end of the Figures in the lowest row, which is under the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But, Sir, I beseech you, shall I always when any number so remaineth alone, as thus 8 did, write him under the line straight against his own place?

Master. Yea, what else? whether they be one or many: and this well remembered, you have sufficiently learned Subtraction: Howbeit, because of certain things that might deceive you, if you did not take good heed to your working, I will propose to you another example of many numbers to be subtracted, as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500, at another time 368, at another time 440, at another time 80, and at another time 64; now would I know how many do rest behind.

Therefore first I set down my gross summe

2869

2869 *Crowns received*, and underneath it I set all the parcells thus, and under them a double line.	2869
	500
	368
	440 *Delivered.*
	80
	64

Then first I begin at the first place, and gather together the sum of all those lines (save the overmost) in the first figures, & so I doe with all the figures of the second place, and so forth, as I did in Addition; save that I leave out the highest row of numbers, (as the line warneth me) that sum so gathered between the double line, is the sum delivered in all; which summe I doe afterwards subtract out of the highest row of numbers, and the remainder doe I set under the nethermost line: as for example.

I set the summes as before: then do I gather the first figures of all the places delivered together; where I find but 4 and 8, that maketh 12, (for these Cyphers encrease no sum in Addition, as you learned before:) of the 12 therefore doe I write the Digit 2 between the double line, and keep the Article in my mind, till I come to the second place, where I find 6, 8, 4, 6, that maketh 24, to them I put the Article in my mind, and it is 25, 5 of which I write under the second place, and keep the Digit 2 in my mind for the third place, where I find 4, 3, 5, that makes 12, to the which I adde the 2 in my mind, and it maketh 14, thereof I write the 4

2869 *Crowns received.*

500
368
440 *Delivered.*
80
64

1452 *Delivered in all.*

1417 *Rest behind.*

under the third place, and because there remain no more Figures to be added, I write the Digit in the fourth place, as you see in the Example: and so it appeareth I have delivered in all a thousand four hundred fifty two Crowns.

Then come I to the subtracting of this summe between the lines, for by Addition it is equall to the five parcels over it, therefore I proceed to subtract it from the overmost summe, saying, 2 from 9, remains 7 to be written under them beneath the lowest line. Then in the second place I take 5 from 6, and there resteth 1 to be written under them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2, remaineth 1. And thus I see that after those five sums are subtracted from 2869, the Remainer is 1417.

Scholar. This I perceive: but is there no shorter way and more speedy?

abridg- ent of former nner of btracti-

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest sum. But if in quantity those numbers added together exceed the highest sum, or upper number, then shall you (as before hath been taught) imagine to borrow 10, 20, or 30 moe, as need shall require, and put them to the upper number, to help to further the abatement, reserving or restoring the Articles that you borrowed to the next place again: and so still goe forward till you have ended your work: as for example. In the last summe proposed, I gather first in the first place 4 and 8, that maketh 12, which 12 I should deduct or take out of 9 in the upper number above the

line,

line, but I cannot; and therefore I adde unto 9 an Article of 10, and make the upper number 19, from whence I take 12, then there resteth 7: then for the Article 10, I adde to the next place of money delivered, saying, 1 that I bring and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25, which 25 I should take out of 6 in the upper number, but I cannot; therefore I adde 2 tens or 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26, resteth 1; then the tens which I borrowed, or have in mind, I adde to the next row or sum delivered, saying, 2 that I bring and 4 make 6, and 3 make 9, and 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18, resteth 4. Now because there are no more places to be added, the one that I borrowed, or have in mind, I rebate from 2 in the upper line, and there remaineth 1, which I set down in the remainer line: & so my sum appeareth (as before) to be 1417 Crowns.

Loe thus have you now a shorter way.

Scholar. I like both ways well, and I perceive both well: yet, as in one the working seemeth somewhat long, so in the other it leaveth very much (me seemeth) to remembrance, and therefore may cause errour quickly, except a man have a quick and an exercised remembrance. But yet for the sharpning of my wit, by your patience, (if you will give me leave) I will try what I can do in a like summe, to work it the shortest way: whereupon I would subtract out of 40301964 these three parcels. Therefore I set them first in due order: then I gather the parcels of the first place, which are 8, 2, 1, that is 11,

40301964	*Charge.*
20003428	
10002432	*Discharge.*
10101461	
43	

which I should take or deduct out of 4, which is over him, but I cannot; therefore I add an Article, or one ten to 4, which maketh 14; then 11 out of 14, there resteth 3 to be written under the first place between the two lines.

Then come I to the second place, saying, 1 that I borrowed to have in my mind and 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, and then 12 from 16, resteth 4, which I write under the second place between the two lines.

Then come I to the third place, saying, 1 that I borrowed or have in mind and 4 make 5, and 4 is 9, and 4 make 13, which I should take out of 9 that is over them, but I cannot: therefore I adde 10 to 9, which make 19, then 13 out of 19, rest 6.

Then come I to the fourth place, saying, 1 in mind and 1 is 2, and 2 is 4, and 3 make 7, which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that I come to the fifth place, where are only three Cyphers, which make nothing, unto which I adde 1 in mind; then should I take that (that is to say 1) from the Figure over them, which is also a Cypher: therefore I say thus, I cannot take 1 from 0, but 1 from 10, remaineth 9: so must I write 9 under them. Then in the sixth place I find but 1, and 1 in mind make 2, which I take out of 3 over him, and the remainder is 1: that must be written between the two lines in the sixth place. So I goe to the seventh place, where I find onely Cyphers, and in the gross

summe

summe over them a Cypher also: therefore must I write the remainer (which is nothing) with a Cypher also. Then in the eighth and last place I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is over them, there will nothing remain: and that must be noted with a Cypher between the two lines (as I have often said.) And so have I ended my work, and the figures stand as followeth.

Scholar. But, Sir, I remember you taught me that Cyphers should not come in the last place, for because they serve only to increase the value of other Figures which follow them, and serve not those figures that go before them: and now in my Example I have set two Cyphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not have set them here, but only because that I would make you plainly to perceive the Art of Subtraction. Therefore seeing that you doe now perceive it, whensoever you would write down a Cypher, look whether any other figures be yet behind; and if not, then let go the 0 also, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the use of Subtraction the plainer.

Therefore your figures must stand thus when the work is ended.

40301964	*Charge.*
20003428	
10002432	*Discha.*
10101461	
0164643	*Rest.*

Scholar. Sir, I do think, with that you taught me before, and by these two summes that you taught me

laſt alſo, that now I could ſubtract any ſumme.

Maſter. So may you, if you have marked what I have taught you. But becauſe this thing (as all other) muſt be learned ſurely by often practice, I will propound here two Examples to you: wherein if you often exerciſe your ſelf, you ſhall be ripe and perfect to ſubtract any other ſumme lightly; for in them is contained all the obſervances of whole numbers. And becauſe you ſhall perceive ſomewhat both how to doe it, and alſo whether it be well done when you have proved to doe it; therefore have I written under them both the Remainers.

30606.	*Lent.*	308964.	*Debt.*
10354		103145	
10249	*Paid.*	102597	*Paid.*
163		101024	
20766	*Paid in all.*	02198	*Reſt.*
9840	*Reſt to pay.*		

Scholar. Sir, I thank you: but I think I might the better doe it, if you did ſhew me the working of it.

Maſter. Yea, but you muſt prove your ſelf to do ſome things without my aid, or elſe you ſhal not be able to do any more then you are taught! And that were rather to learn by rote (as they call it) then by reaſon. And again, there is nothing in theſe examples, or any other of whole numbers, but

but I have taught you the rules of them already.

Scholar. Then I trust by practice to attain the use of it. And is this all that I shall learn of Subtraction?

Master. Yea, saving that (as you have seen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without some instructions be given of it) it might seem to a learner more difficult then indeed it is. Therefore I will briefly shew you the use of it only by an example or two.

A certain man owed to me 14 l. 12 s. 8 d. *of which he paid me at one time* 4 l. 6 s. 8 d. *at another time* 3 l. *at another* 2 l. 3 s. 4 d. *and last of all* 6 s. 8 d.

Now would I know what remaineth unpaid yet: therefore I set my sums thus, every one in their due place, as pounds under pounds, shillings under shillings, pence under pence.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray you why doe you write 2 l? for the common speech useth rather to say 40 s.

Master. We must here use the Denomination that is greatest in any summe, so that we may not write according as we use to speak, saying, 16 d, 18 d, or likewise 7 groats, 8 groats, 24 s, 40 s, 48 s, and such other: but we must write every Denomination that is in any summe by it self.

Note how the pen differeth from the common order of Counters.

Namely, shillings and pounds. So must we write for the last summes now named, 1 s. 4 d, 1 s. 6 d, 2 s. 4 d, 2 s. 8 d, 1 l. 4 s, 2 l. 8 s, and so forth of other like.

Scholar.

Scholar. So that we may not write in Arithmetick pence, when the summe amounteth to shillings, nor shillings, when the summe maketh pounds. Now (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which be paid me into one totall summe, directly to be set under them between the two lines, not medling with the 14 *l.* 12 *s.* 8 d. as the line warneth me: therefore must I begin with the smallest Denomination, saying, 8, 4, 8, is 20 pence, which maketh one shilling and 8 pence; the 8 d. I set down under the place of pence, and the one shilling I keep in mind to carry to the next Denomination of shillings. Then come I to the shillings, and say 1 that I bring or have in mind and 6 is 7, and 3 is 10, and 6 make 16,

li.	s.	d.	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	
4	16	0	*Rest.*

which, because it containeth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are 2, 3, 4, that is in all 9, that 9 doe I set down directly under the pounds. And so the totall or whole Addition of all the particulars paid amounteth to 9 l. 16 s. 8 d.

Now for the work of Subtraction, I must rebate that totall summe of Addition out of the highest number, that is to say, from the 14 l. 12 s. 8 d.

There-

Therefore to perform the work I say, 8 d. out of 8 d, remaineth or resteth nothing, therefore in the place of the rest or remain, right under the Denomination, I set down o. Then coming to the shillings, where I find 16, which should be taken out of 12, but I cannot; therefore I imagine to borrow 1 out of the next Denomination, that is, of the 14 l, and put that one pound so borrowed unto 12 s, that maketh 32 s.

Now 16 s. out of 32 s. resteth 16 s, which 16 s. I set down directly under the place of the rest.

Lastly, coming to the pounds, saying, one pound in mind that I borrowed and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remain appear to be 4 l. 16 s. o d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this work also?

Master. Yes, as in the last Example I will also shew you: for you may adde together the particular sums as they are set in order, beginning with the pence, saying, 8, 4, 8, make 20. d, which 20 d. you should take out of the 8 d. above the line, but you cannot: therefore shall you borrow 1 of the next Denomination, that is to say 1 of the shillings, and put it to the 8 d, that maketh 20 d.

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Now 20 out of 20 d. resteth o, which Cypher I set down

down directly under them. Then one shilling that I borrowed, or had in mind, and 6 make 7, and 3 make 10, and 6 make 16; the 16 out of 12 I cannot take, therefore of the next Denomination I do borrow one l. and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly I come to the pounds, saying, 1 *l.* in mind or that I borrowed and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appear as before to be 4 l. 16 s. 0 d.

Scholar. Then doe I perceive very well: and if there be no other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. We have done indeed with the Art of Subtraction, as touching the working.

Proof of Subtraction.

But yet before we go to Multiplication, I will instruct you how to examine your work, whether it be well done or not. For the performance whereof, if you mark what I said in Addition, you may easily perceive what is to be done for the proof of Subtraction, which is best made by the aid of Addition thus.

Draw under the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did subtract before together, and write that that amounteth under the lower line: and if the summe that cometh thereof be equall to the highest of the Subtraction, then is the Subtraction well wrought, or else not. As you may see for example in the summes set down before, and first in sums of one Denomination, whereof one was this.

Where

Where the Number 52984732 is ſubtracted from 8250003456, and the Remainer is 8197018724. How to prove whether it be	8250003456 52984732 8197018724

truly wrought or not, I adde the Remainer and the number ſubtracted together, beginning at the right hand; and firſt I ſay 4 and 2 is 6, which is ſet under the line.

Example in a ſum of one Denomination.

The number given	8250003456
The number to ſubtract	52984732
The Remainer	8197018724
The Proof	8250003456

Then again in the ſecond place I ſay, 2 and 3 is 5, which I write under; next that in the third place, 7 and 7 are 14, of which I write the Digit 4, and keep the Article 1 in my mind. Then in the fourth place 8 and 4 is 12, and 1 in my mind maketh 13, whereof I write down the Digit 3, and keep the Article 1 in my mind. Again in the fifth place, 1 and 8 is 9, and 1 in my mind is 10; whereof I ſet down 0, and keep the 1 in my mind. And ſo going on to the reſt, (as is taught in Addition) when I have made an end, I ſee that the loweſt line of numbers and the higheſt be alike: wherefore I know that I have well done.

So likewiſe the Proof is to be made in numbers of divers Denominations: as for Example, in our ſumme of that kind which in the firſt form of working ſtood thus, (all the particular numbers to be ſubtracted being drawn into one;)

Where,

Example in a sum of divers Denominations.

Where, in the title of pence, I find 8 & 0: the 8 I set down directly under in that of pence.

Then in the place of shillings I find 16 and 16, which make 32 shillings, wherein is contained 1 l. and 12 s: the 12 s. I set down directly under them in the due place of shillings, and one pound I keep.

Then coming to the pounds, I say, 1 that I keep and 4 is 5, and 9 is 14; which 14 in due order I set down directly under them as this Figure sheweth. And the whole summe is 14 l. 12 s. 8 d. agreeing with the upper number above. So I find the work is good, and the Subtraction well wrought.

	li.	s.	
	14	12	
	4	6	
	3	0	
	2	3	
		6	
Paid in all.	9	16	8
Rest.	4	16	0
Proof.	14	12	8

The same thing is to be done for the latter form of Subtraction, (where the particular summes are not gathered together into one gross.) For the Remainer and all the particular summs subtracted being added together, if the summe that cometh thereof be equall to the highest number above, then is the Subtraction well wrought, or else not.

Example of a proof in the latter form of Subtraction.

As for example also in the last sums which stood thus.

First, in the title of pence I adde, 8, 4, 8, that maketh 20 d. which containeth 1 shilling 8 pence.

The 8 I set down under the lowest line in the row or

li.	s.	d.
14	12	8
4	6	8
3	0	0
2	3	4
	6	8
4	16	0
14	12	8

title

title of pence, and that 1 shilling I keep to carry to the next Denomination or place of shillings.

Then returning to the shillings, saying, one in mind, or that I keep, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound 12 s: the 12 s. I set down under the title of shillings, and 1 pound I keep or have in mind to carry to the next Denomination or place of pounds. Then come I to the pounds, saying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14: then doe I write 14 under the pounds, and so have I ended the Addition: and I see that the lowest line is like unto the uppermost line in number, wherefore I know that I have well done.

And thus have I taught you the Art of Subtraction, and the means to prove whether it be well wrought or not. Therefore now will I make an end thereof, and will instruct you in Multiplication.

Multiplication.

Multiplication what it is.

Multiplication is an operation whereby two sums produce the third; which third sum so many times shall contain the first, as there are Unites in the second: And it serveth in stead of many Additions. As for Example.

When I would know how many are 30 times 48, if I should adde 48 thirty times, it would be a long work. Therefore was Multiplication devised, which shall doe that at once that Addition should do at many times.

Scholar. I perceive the commodity of it partly, but I shall not see the full profit of it till I know the whole use of it. Therefore, Sir, I beseech you, teach me the working of it.

Multiplication of Digits.

Master. So I judge it best: but because that great summes cannot be multiplied but by the Multiplication of Digits, therefore I think it best to shew you the way of multiplying them; as when I say, 9 times 8, or 8 times 9, &c. And as for the small Digits, under 5, it were but folly to teach any rule, seeing they are so easie that every child can do it: but for the Multiplication of the greater Digits, thus shall you do.

First, set your Digits one right over the other; then from the uppermost downwards, and from the nethermost upward, draw straight lines, so that they make a cross, commonly called Saint

ys

Andrews cross, as you see here. Then look how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference. As if I would know how many are 7 times 8, I must write those Digits thus.

Digit difference.

8

7

The difference.

Then doe I look how much 8 doth differ from 10, and I find it to be 2: that 2 doe I write at the right hand of 8, at the end of the line, thus.

Digit difference.

8 2

7

After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.

Digit difference.

8 2

7 3

Then doe I draw a line under them, as in Addition, thus.

Last of all, I multiply the two differences, saying 2 times 3 make 6; that must I ever set under the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other Digit, (not from his own) as the lines of the Cross warn me, and that is left must I write under the Digits. As in this example, If I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write under the Digit, and then there appeareth the multiplication of 7 times 8 to be 56.

Digit difference.

8 2

7 3

5 6

And so likewise of any other Digits, if they be above 5: for if they be under 5, then will their difference be

greater then themſelves, ſo that they cannot be taken out of them. And again, ſuch little ſums every child can multiply, as to ſay 2 times 3, or 4 times 5, and ſuch like.

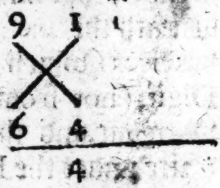
Scholar. Truth it is. And ſeeing meſeemeth that I underſtand the multiplying of the greater Digits, I will prove by an example how I can do it. I would know how many are 9 times 6.

Maſter. It is all one in value to ſay 9 times 6, or 6 times 9: but yet the order is beſt to put the leſs ſum firſt, ſaying, 6 times 9, and ſo of all other ſums.

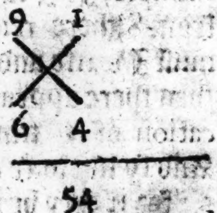
Scholar. Then would I know how many are 6 times 9: therefore I ſet the Digits thus, and make the croſs thus.

Then do I ſet their differences from 10 at the right ſide, the difference of 9, which is 1, againſt it, and the difference of 6, which is 4, againſt it alſo, as in this example.

And under them draw a line. Then do I multiply the differences together, ſaying, 1 time 4 maketh 4: that 4 do I write under them thus.



Then take I one of the differences from the other Digit, as, 1 from 6, or elſe 4 from 9, and each ways there reſteth 5, which I do write under the Digit thus. And ſo appeareth the multiplication of 6 times 9 to be 54. Thus I ſee the feat of this manner of multiplication of Digits.



Maſter.

Master. Now might you go streight to the multiplication of great numbers, save that both for your ease and surety in working I will draw you here a Table, whereby shall appear the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seek your first or last Digit in the greater figures; and from it go right forth towards the right hand, till you come under the number of your second Digit, which is in the highest row; and then the number that is in the meeting of the rows of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seek first 7 in the greater figures, and then go right forth toward the right hand till you come under 9 of the highest row, in which place where you so come under the

other Digit (as here for example you come under 9) is always contained the off-come or product which you seek; and that place we term to be in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle where the rows of little squares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater summes.

Scholar. This is very good and ready. And so may I find the multiplication of any Digits. But now how shall I doe in greater summes?

Master. *When you would multiply any* summe *by another, you shall mark that it is the meetest order to set the greatest* number *highest, which is the place of the* number *that must be multiplied; and likewise the lesser* number *under it, for that is the place of the*

Multiplier.

Multiplier *or* Multiplicator, *that is to say, the* number *by which the* Multiplication *is made, and is in* English *always put before this word Times, in such speaking as when I say* 20 *times* 70. *And the* number *that followeth this word Times is that which must be* Multiplied.

Times.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition: and under them must I draw a line. As for example; If I would multiply 264 by 29, I must set them thus.

264
29

Of which number thus set down to be multiplied may be formed a question, as thus: There are 29 men, and each man hath 264 Lambs. The question is, how many Lambs they have in all.

To

To the performance whereof, I must multiply every figure of the high row by every figure of the nether row: and that that amounteth I must set under the line, as thus.

First I do multiply 4 by 9, saying, 9 times 4, (or 4 times 9, which is all one) and that maketh 36, as the Table before of Digits doth declare: of that 36, I must write the 6, that is the Digit, under the 9, and the Article 3 I keep in mind to carry to the next place.

264
29
———
6

Then come I to the second figure of the higher row, which is 6, and say 9 times 6 make 54, and with the 3 in my mind make 57: the 7 I set down under the 2, and 5 I keep in mind.

264
26
———
79

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in mind maketh 23: wherefore because it is the last work of the Multiplier I set it down in order as you see.

264
29
———
2376

And so have I ended the first figure of the Multiplier. Wherefore I give it now a fine dash with my pen.

Then begin I with the next figure, and multiply it into all the higher figures, as thus.

264
29
———
2376

First, 2 times 4 make 8, that 8 do I write under the second place: for evermore the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set under the Multiplier of it, the other in their order toward the left hand.

Scholar. I understand you thus, that the Digit of the summe amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must be set under the first place; and that that amounteth of the same first figure by the second Multiplier, must be set under the second place: and so of the other, if there be more Multipliers.

Master. So mean I indeed: and if there amount but a Digit, then must it be set under the Multiplier.

And now to goe forth; I multiply by the same 2 the second figure of the higher row, which is 6, saying, two times 6 make 12; wherefore I write the Digit 2 under the third place, and the Article 1 I keep in mind.

264
29
2376
28

Then doe I multiply the last figure of the higher summe by that same 2, saying, 2 times 2 is 4, and with the 1 that I have in mind maketh 5, which I write under the fourth place, And so have I ended the whole Multiplication: wherefore I also give the 2 a dash with my pen, thus: and so I do ever as soon as I have dispatched any Digit by which I multiply: and the sums stand thus.

264
29
2376
528

Then must I draw a line under all those summes that mount of the multiplication, and must adde all them into one summe, as in the Example you may see.

264
29
2376
528
7656

Where

Where in the firſt place I find but 6, and therefore write I it under the line. Then in the ſecond place 8 and 7 make 15, whereof I write 5, and keep 1 in my mind, and ſo forth, as you learned in Addition. And ſo appeareth the whole ſumme to be 7656, which amounteth of the Multiplication of 264 by 29. And that is the juſt number of the Lambs that 29 men had.

Scholar. If there be no more to be obſerved in it, then can I doe it I ſuppoſe, as by this Example I ſhall prove.

¶ There is a piece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards ſquare there is in all this piece of ground: which numbers I ſet down with the greater above, and the leſſer under, as you ſee.

```
 1365
  236
 ----
```

Then doe I multiply 5 by 6, ſaying, 6 times 5 make 30, of which I write the Cypher in the firſt place, and the Article 3 I doe keep in mind to carry to the next place.

```
 1365
  236
 ----
    0
```

Then do I by the ſame 6 multiply the ſecond figure of the higher ſum, which is 6, ſaying, 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 under the ſecond place, and the Article 3 I keep in mind.

```
 1365
  236
 ----
   90
```

Then doe I multiply the third figure, which is 3, by the ſame 6, and that maketh 18, and 3 in my mind make 21. The 1 I ſet down, and keep 2 in mind.

```
 1365
  236
 ----
  190
```

Then come I to the last figure of the higher sum, and multiply it by 6, saying, 6 times 1 make 6, and 2 in my mind make 8: that 8 doe I write under the fourth place. And so have I ended the first Multiplier, and dash him slightly with my Pen.

1365
236
8190

Then begin I with the second Multiplier, and say first, 3 times 5 make 15, of which I set the 5 under the second place, because that the Multiplier is there, and the Article 1 I keep in mind.

Then come I to the second figure, that is 6, and multiply it by 3, which maketh 18, and with 1 in mind maketh 19: the 9 I set down under the third place, and 1 I keep in mind.

1365
236
8190
95

Then come I to the third figure, which is 3, and multiply it by 3, saying 3 times 3 make 9, and with 1 in mind make 10: the Cypher I set under the fourth place, and the Article 1 I keep in mind.

1365
236
8190
095

And then coming to the last figure 1, I multiply it by 3, and it maketh 3, and with the 1 in mind it maketh 4, which 4 I set in the fifth place: and then have I ended two of the Multipliers, and the summes stand as you may see in the latter end of the page going before, and then I give 3 his dash.

1365
236
1190
4095

Then

1365
236
8190
4095
0

Then come I to the third Multiplier, and multiply it into every figure of the higher summe; and first I say, 2 times 5 make 10, of which I set the Cypher under the Multiplier in the third place, and the Article 1 I keep in mind.

1365
236
8190
4095
2730

And so multiplying the second figure 6 by that same 2, there amounteth 12, and 1 in my mind maketh 13, whereof I write the Digit 3 under the fourth place, and the Article 1 I keep in mind.

Then do I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and the one in mind make 7, which 7 I set down under the fifth place, as appeareth by the example.

1365
236
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the sixth place, and then doth the summe stand thus.

1365
236
8190
4095
2730

And so have I ended the whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then under the line will appear, as you may see, the gross or total summe is 322140. Whereby I know there is so many yards square in that piece of ground.

1365
236
8190
4095
2730
322140

Master.

Master. This is well done.

Scholar. Then methinketh I could call it well done, when I know whether I had well done or no.

Master. It is to be proved by 9, as Addition was, but the surest proof is by Division; and therefore I will reserve that proof by Division, till you have learned the Art of Division. And anon I will shew you how it is commonly proved.

¶ *But first, for your farther instruction in this exercise of* Multiplication, *I will with one example try your cunning, and so make an end: And the* question *is this. I would know how many days it is since* the Nativity of our Lord and Saviour Jesus Christ, *unto this year 1630. Which to perform, you must* multiply *this present year 1630 by the days in one whole year, which are 365.*

Scholar. Now for that you have given me so much light into the question, you shall see I will handsomely finish the work; for according to your former instructions, I set them down with a line under them thus.

$$\begin{array}{r} 1630 \\ 365 \\ \hline \end{array}$$

Then say I, 5 times 0 is 0, which I set down under the first place, as here appeareth. Then say I, 5 times 3 make 15; the Digit 5 I set down in the second place under 3, and the Article 1 I keep in mind to be added to the next Multiplication. Then saying 5 times 6 make 30, and one in mind 31, the 1 I set down in the third place, and 3 I keep in mind. Then coming to the last Figure, I say once 5 is 5, and 3 in mind make 8; that 8 doe I set down under the fourth place. And thus have I ended my first Multiplier, and

and therefore I give it a dash with my pen.

Then come I to the second Multiplier, which is 6, and do likewise multiply it into the upper number, saying, 6 times 0 is 0, which I set down in the second place, right under his Multiplier. Then say I, 6 times 3 make 18; the 8 I set down under the third place, and 1 I keep in mind. Then say I, 6 times 6 make 36, and 1 I keep in mind make 37; the Digit 7 I set down in the fourth place, and 3 I keep in mind. Then say I, 6 times 1 is 6, or once 6 is 6, and 3 in mind make 9, which I set down next. And so have I ended two Multipliers, wherefore I dash the 6 with my pen.

```
 1630
  365
 ----
 8150
9780
```

Then I begin to multiply the third Multiplier into the over number, saying, 3 times 0 is 0; the 0 I set down in the third place right under his Multiplier. Then say I, 3 times 3 make 9, which I set down in order next: then say I, 3 times 6 is 18; the 8 I set down, and 1 I keep. Lastly, I say, once 3 is 3, and 1 I keep is 4, which I set down orderly next. And so have I ended the Multiplication, and my figures stand thus.

```
              or thus,
  1630          1630
   365           365
------        ------
  8150           815
 9780           978
4890            489
------        ------
594950        594950
```

Master. I commend you for your diligence, the work is very perfectly done; which parcels if you now adde together into one sum, it will be 594950, which

which is the gross or total summe of that Multiplication, and declareth the number of days since our Lord and Saviour his Incarnation, unto the end of 1630 *years, besides 407 days and twelve hours for leap-years.*

Scholar. This is marvellous, methinks, that such great matters may so easily be atchieved by this Art, which heretofore I ever thought had been impossible, as infinite sorts of people are of that mind.

Master. Truth it is, that knowledge hath no greater enemy then ignorance; for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to justifie.

Scholar. The manner of Multiplication I perceive, if there be no more in it.

Master. *Yes, there are other forms and helps for ease and shorter labour of the work of* Multiplication, *but I will remit them till you have a little tasted* Division, *where also the like help unto* Division *may be used; and so therefore, under one example for both, will I shew you both ease in* Multiplication, *and also in* Division.

But sith the other forms and workings do nothing differ from these works in effect, but only in setting of the numbers, I will overpass them till a more meet place and time. And now will I instruct you in Division, so that you think your self sufficiently to perceive what I have taught you.

Scholar. Yes, Sir, I thank you, but I doe not perceive how to examine my work, to try whether I have well done or no: therefore as you

you promised me ere-while, I pray you first shew me how I shall prove it.

Master. *That is commonly used by the proof of 9, as you learned before in* Addition, *saving that it differeth from that form in divers respects: As for example.*

First, must you make a cross after this manner.

Then must you examine your summe that should be multiplied, and look what remaineth after casting away of 9, that set you at the one side of the cross; then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the cross: then must you multiply those two numbers together, and look what amounteth thereof, if it be under 9, write it at the higher part of the cross; but if it be above 9, then take thence 9 as often as you can, and write the rest at the head of the cross. As in the example put forth of the piece of ground that contained 1365 yards in length, and 236 yards in breadth.

Proof of Multiplication.

Therefore first I cast away all the nines from the summe to be multiplied, saying, 5 and 6 make 11, cast away 9, resteth 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one side of the cross thus:

Then do I examine the Multiplier, which is 236; wherein when

the 9 is cast out, there remaineth 2; that 2 therefore I set at the other side of the cross.

2 6

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the cross. Then doe I examine the gross summe amounting of the Multiplication, which is 322140, where I find 9 once, and 3 remaining; that 3 I set at the foot of the cross, and then I see it to agree with the other 3 at the top of the cross, and so know I that I have done well: for if they two did differ, then were my work vain, and the Multiplication false.

3

2 6

3

A sure proof of Multiplication.

This is the common proof: but the most certain proof is by Division, of which I will anon instruct you.

Schol. Sir, what is the chief use of Multiplication?

Master. The use of it is greater then you can yet understand: howbeit, these plain commodities it hath, that if you would resolve any great and whole value into many small and less proportions, as if you would change pounds into shillings and pence, or any other greater or smaller parcels, by Multiplication ye shall do it speedily and easily. Also if you should need to adde one summe to it self, or to any other often times, you shall doe it by Multiplication much more speedily, readily, easily, and surely, then by often and sundry Additions. Take you these commodities grosly shewed for an answer at this time, and hereafter I will more abundantly make you to perceive the use of it.

Division.

Scholar.

WEll, Sir, then in Division I pray you to instruct me. But methinketh by the name of it, that it should be all one with Multiplication: for I call that Division, when any thing is parted into divers and many parts.

Master. You take it as it is taken commonly: howbeit, if you mark well, you shall perceive that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but, contrarywaies, it bringeth many parcells into few, but yet so, that these few taken together are equall in value to the other many: for by Division pence are turned into shillings, and shillings into pounds. As for example, of 120 shillings it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the Denominators, you shall see that they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they do differ, and the latter summe is the lesser; and so it is always in Division; howbeit yet in the working the summe is parted by another, and thereof doth it take the name.

Scholar.

Scholar. I think I shall better understand the reason of the name when I know the use of the work, therefore now would I gladly learn that.

Division what it is.

Master. *Division is a distributing of a greater summe by the unites of a lesser: Or, Division is an Arithmetical producing of a third number, in respect of two propounded numbers; which third number shall so often contain an unite, as the greater of the two propounded numbers can contain the lesser.* So that as Multiplication did seem to serve in stead of many Additions, so Division may seem to be in place of many Subtractions; because that third number briefly expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practice will more largely appear. Therefore (as you may perceive) unto Division are required three numbers: The first, which should be divided, and that must (generally) be the greater: and the second, by which the other must be divided, and that is (generally) the lesser, and is called the Divisor: and the third, which answereth to the question (How many times?) and therefore is called the Quotient.

A general rule for placing the figures.

The first must be first written, and the second so set under it, that the last figure of the lower number be right under the last of the higher, contrariwise to the work of other kinds of Arithmetick: for in them the two first figures were set ever meet one under the other; but in Division the last figures must be set meet, except it chance so that the last figure of the Divisor be greater then the last of the higher number, for then you shall set the last of the Divisor under the last save one of the higher

higher number, as for example:

If you should divide 365 (which are the summe of the days of a year) by 28, which are the days of a common Moneth, then should you set them thus.

365
28

But if you should divide those 365 days by 52, which is the number of weeks in one year, then should you set them thus.

365
52

Likewise if I would divide the same 365 by 4, which is the summe of the quarters of years, then must I set them thus.

365
4

Scholar. Sir, this doe I understand; but now how should I doe to divide the one by the other?

Master. You must begin with the last Figure next the left hand, and see how many times the first Figure of the Divisor may be taken out of the last Figure of the other Number, and that shall you note within a crooked line toward your right hand. As for example, I would divide 365 by 28, then set I those two summes thus.

365 (
28

And I look how many times I may find 2 (which is the last figure of the Divisor) in 3, (which is the last of the number to be divided;) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Quotient number.

Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, & deface or cancell the 3 and the 2: then will the figures stand thus.

1
~~3~~65 (1
~~2~~8

G Then

the 9 is cast out, there remaineth 2; that 2 therefore I set at the other side of the cross.

2 6

Then doe I multiply 6 by 2, and it maketh 12, from which 12 I withdraw 9, then resteth 3, which 3 doe I set at the head of the cross. Then doe I examine the gross summe amounting of the Multiplication, which is 322140, where I find 9 once, and 3 remaining; that 3 I set at the foot of the cross, and then I see it to agree with the other 3 at the top of the cross, and so know I that I have done well: for if they two did differ, then were my work vain, and the Multiplication false.

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higher number, as for example:

If you should divide 365 (which are the summe of the days of a year) by 28, which are the days of a common Moneth, then should you set them thus. 365 28

But if you should divide those 365 days by 52, which is the number of weeks in one year, then should you set them thus. 365 52

Likewise if I would divide the same 365 by 4, which is the summe of the quarters of years, then must I set them thus. 365 4

Scholar. Sir, this doe I understand; but now how should I doe to divide the one by the other?

Master. You must begin with the last Figure next the left hand, and see how many times the first Figure of the Divisor may be taken out of the last Figure of the other Number, and that shall you note within a crooked line toward your right hand. As for example, I would divide 365 by 28, then set I those two summes thus. 365 (28

And I look how many times I may find 2 (which is the last figure of the Divisor) in 3, (which is the last of the number to be divided;) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Quotient number.

Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 over 3, & deface or cancell the 3 and the 2: then will the figures stand thus.

1
365 (1
28

Then come I to the next figure of the Divisor, and take it likewise so many times out of the figures that be over it; and look what doth remain, that I must write over them, and cancell them, as in this example.

Therefore now do I take once 8 out of 16, and there remaineth 8, which I must set over the 6, and cancel or cross out the 16, and the 8 of the Divisor: and then will the figures stand thus. ~~1~~8
And so I have once wrought. ~~3~~65 (1
Scholar. So I perceive that you take ~~2~~8
the nether figure, not only out of the other that is right over him, but out of that with the other also that remaineth before, and are written toward the left hand.

Master. So must you doe: for you must so take the Divisor out of the over number, that there remain not over it so great a summe as it self is: for then were your work in vain.

But yet again here must you mark, that when you seek how many times the last figure of the Divisor may be found in the number over him, that you look also whether you may as often find all the figures following in those that are above them, (considering all the remainers, if there be any:) if not, take your Quotient less by one, and then prove again, and so till you find a meet Quotient: and by that meet Quotient must you always multiply your Divisor, and set the product under your Divisor, so that the first figure stand under the first figure of your Divisor, and the second under the second, and so forth; and then subtract that product from the number to be divided that standeth directly

rectly over it, as you have seen me doe.

When you have thus wrought once, then must you begin again, and write your Divisor anew, near-er toward the right hand by one place, place : as in this example, you shall set 2 under 8, and 8 under 5, thus.

$$\begin{array}{ll} \cancel{1}8 & \\ \cancel{3}\cancel{6}5 & (13 \\ \cancel{2}88 & \\ 2 & \end{array}$$

Then (as before) seek how many times you may take your Divisor out of the number over him now.

Scholar. That may I doe here 4 times.

Master. Truth it is that you may find 2 four times in 8 : but then mark whether you can find the figure following so many times in the o-ther that is over him. Can you find 8 four times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once less.

Scholar. That is three times.

Master. Well then, 3 times 2 make 6 : if I take 6 out of 8, there remaineth 2 ; which 2 with the 5 following make 25, in which summe I find 8 three times also, and therefore I take 3 as a true Quotient, and write it within the crooked line of the Quoti-ent before the 1, thus :

Mark how to consider this kinde of Remai-ner.

$$\begin{array}{ll} 2 & \\ \cancel{1}8 & \\ \cancel{3}\cancel{6}5 & (13 \\ \cancel{2}88 & \\ 2 & \end{array}$$

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2 ; therefore I cancell the 8, and write over it the 2 that doth remain, thus.

Then do I take 8 as many times out of 25, saying, 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1 : so then I cancell 25 and 8, and over the 5 set 1 thus.

Or you might (after you find 3 to be a fit Quo-

 tient)

tient) straightway have multiplied the whole divisor 28 by that at once, which giveth 84, which being set under 28, and duly subtracted from 85, of the number divided, giveth 1, the remainer of the whole division, as before you had it. Work which way you list, here you may see also the form.

2
181
385 (13
288
2

And now have I done with the dividing; for I cannot find my divisor 28 any more in the over summe.

Scholar. No; except you would part the 1 that remaineth into 28 parts.

Master. That is well said, and so must we doe in such cases when there remaineth any thing: but I will let that pass now, and will make you perfect in division of whole numbers, and will hereafter teach you particularly of broken numbers called Fractions. Now if you do perceive the order of division, then doe you divide this summe 136280 by 452.

Scholar. First I set down the number that should be divided; then do I set the divisor under the last figure of the over number. Then will it be thus.

136280
452

Master. Can you take the last of your divisor (which is 4) out of 1, which is the last of the over number?

Scholar. I had forgotten: because the last of the divisor cannot be taken out of the last of the over number, in so much as it is the greater, therefore must I set the divisor one place more forward toward the right hand, thus.

136280
452

And

And then muſt I look how often I may find the laſt figure of the diviſor (that is 4) in 13, which I may doe 3 times, therefore do I ſay, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then do I make at the right hand of my ſummes a crooked line, and write before it my quotient 3, and I cancell 13 and 4, and over the 3 I ſet the 1 that remaineth: and then the figures ſtand thus.

1
~~1~~~~3~~6280 (3
~~4~~52

Then I multiply the ſame quotient into every figure of the diviſor, and withdraw the ſumme that amounteth out of the numbers over them: as firſt I ſay, 3 times 5 make 15, which I take from 16, and there reſteth 1: I cancell therefore 16 and 5, and write over the 6 that 1 that remaineth, thus.

~~1~~1
~~1~~~~3~~~~6~~280 (3
~~4~~~~5~~2

Then doe I ſay likewiſe, 3 times 2 make 6, which I take out of 12, and there reſteth 6; therefore I cancell the 12 and the 2 over, and then I write the 6 that remaineth, thus.

~~1~~~~1~~6
~~1~~~~3~~~~6~~~~2~~80 (3
~~4~~~~5~~~~2~~

Then ſhould I ſet forward the diviſor into the next place toward the right hand thus.

~~1~~~~1~~6
~~1~~~~3~~~~6~~~~2~~80 (3
~~4~~~~5~~~~2~~2
45

Maſter. But you may ſee that over the 4 is no figure, therefore I muſt ſet the diviſor yet forwarder by another place.

And mark, whenſoever it chanceth ſo that you ſhould ſet forward the diviſor, and that it cannot ſtand here, becauſe there is no number over

the last place, or if there be any, it is lesser then the last figure of the divisor, then must you remove the divisor yet once again: and because that his first place of removing served not to subtract him so much as once, therefore you shall write in the Quotient a Cypher, and if you should by chance need to doe so oft times, for every time write a Cypher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my summes thus.

116
136280 (30
45222
45

And because I removed the divisor, so that I overskipped one place, I must write a Cypher in the Quotient, and then must I seek a new Quotient: as in this example I must say, How many times 4 is there in 6? and sith it can be but once, therefore doe I write 1 in the Quotient: and then say I, 1 time 4 taken out of 6, remaineth 2: I cancell the 6 and the 4, and write 2 over them, thus.

2
1163
136280 (301
45222
455
4

Then say I again, once 5 out of 28, remaineth 23: I let the 2 stand as it did, and over that 8 I set 3, cancelling the 8 and the 5 under it, thus.

2
1163
136280 (301
45222
455
4

Master. You might as well have said, once 5 out of 8, and so remaineth 3: but now go forward.

Scholar. Then, once 1 out of 0 cannot be. What shall I now do?

Master. Borrow of the next number that is behind,

hind, (for there is 230) and do as you learned in Subtraction in like case.

Scholar. Then must I borrow 1 out of the 3 coming behind next, and make that 0 to be 10; and then take I 2 out of 10, and there resteth 8: and because I borrowed one of the 3, I must cancel the 3, and write 2 over it: then doth the figure stand thus.

22
11638
136280 (301
45222
455
4

Master. Now have you done, and yet remaineth 228. and your quotient sheweth you, that if you divide 136280 by 452, you shall find your Divisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I divided 365 by 28) the quotient was 13, and 1 remained; whereby I knew that in a year (which containeth 365 days) there are 13 months, reckoning 28 days (or 4 weeks) just to a month, and 1 day more.

Scholar. Why then do we call a year but 12 months?

Master. Of that at a more convenient time will I fully instruct you: but now it is not convenient to intangle your mind with other things then do directly pertain to your matter. Therefore if you remember what you have heard, you have learned a short manner of Division, which I would have you often to practise, so that you may be perfect in it, and hereafter I will shew you certain other proper points touching it.

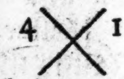
Scholar. Then I pray you tell me how I shall examine

examine and try my work, whether I have done it well or no, that though no man be by to tell me, yet I may perceive it my self.

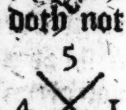
Proof of Division.

Master. Some men (yea and commonly most) doe try it by the rule of 9, as in all the other kinds, save that their order is; First, they cast away 9 as often as they can out of the Divisor, and that that remaineth they set at one side of a Cross: as in our first example the Divisor was 28, from which you may take 9 three times, and 1 remaineth, which they set by a Cross, thus.

Then they likewise examine the quotient, (which in our example is 13) and from thence they cast away 9 as oft as they can, and the remainder they set at the other side of the Cross; and then they multiply together those two remainers, and to it that amounteth they adde the remainer of the Division, if there were any; from that whole summe they withdraw 9 as oft as they can, and the rest they set at the head of the Cross: as in our example, the quotient is 13, from which take 9, and there remaineth onely 4, and therefore must you set 4 at the other side of the Cross, thus.



Then multiply 4 by 1, and it yieldeth but 4, thereto adde the remainer of the Division, (which was 1) and it will be 5; which summe doth not amount to 9, and therefore must be set wholly at the head of the Cross, as you see here.



And this number on the head of the Cross is the first proof, to which if you find another like in the

number

number that was divided, then you have done well.

Therefore now shall you likewise examine the whole summe that was divided, and take away 9 as often as you can, and that that remaineth set at the foot of the Cross: and if it be equall to that in the head of the Cross, then have you done well, else not.

As in our example, the whole sum was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of the Cross, thus.

And you shall see that they agree: therefore have you well done.

Now will I likewise examine our second example, where the Divisor was 452, which maketh 11: from thence I take 9, and the 2 that remaineth I set at the right side of the Cross, thus.

Then examine I the quotient, which was 301, where I find but only 4: that I set at the other side of the Cross, thus.

Then I multiply 4 by 2, and it maketh 8: to that I adde the remainer of the Division, (which was 228,) and it maketh 12, and they two make 20, wherein I find twice 9, and 2 remaining: that 2 must I set at the head of the Cross, thus.

Then I examine the whole number to be divided, which was 136280, where I find twice 9 and 2 remaining, which I set at the foot of the Cross, thus.

And because it doth agree with the

figure

figure at the head of the Cross, I know that the Division was well wrought.

The Proof of Division more certain by Multiplication.

Master. This is the common proof. Howbeit, the more certain working is by the contrary kind: as to prove Division by Multiplication, thus.

Multiply the quotient by the Divisor, and if the sum that amounteth be equal to the sum that should be divided, then have you well divided; else not.

Howbeit, this must you mark, that if there remained any thing after the Division, that must you adde to the summe that amounteth of the Multiplication. As in our first Example our quotient was 13, and the Divisor was 28: Now multiply the one by the other, and the summe will be 364: to that if you adde the 1 that remained after the Division, then will it be 365, which was the summe that should be divided: and therefore I know that I have well done.

Scholar. Now will I prove the same in the second example, whose Divisor was 452, and the quotient 301: these do I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280: which was the whole summe to be divided: and therefore I perceive that I have well done.

Master. This is the surest way to examine Division, by Multiplication: and contrariwise, the surest proof of Multiplication is by Division.

And therefore (according to my promise) now will I shew you how you may prove Multiplication by Division.

Proof of Multiplication by Division.

When you have ended Multiplication, and would know whether you have well done or not, see the

grofs

groſs ſumme that amounteth of the Multiplication overmoſt, and divide it by the Multiplier; and if the Quotient be the ſame number that ſhould be multiplied, then have you well wrought, elſe not: as in that example where we multiplied 264 by 29, the groſs ſumme was 7656.

Now if you will know whether that Multiplication be true, you ſhall divide that 7656 by the multiplier 29, and you ſhall perceive that the Quotient will be 264, and that is a token that you have well wrought.

Scholar. By your patience I will prove that, and firſt ſet down the groſs ſumme and the multiplier, not after the rule of Multiplication, but after the rule of Diviſion, for now that number is become the Diviſor that was before the Multiplier. I ſhould ſet them therefore thus,

7656
29

Then ſhall I ſeek how many times 2 in 7: that may be three times, and one remaineth: but then may not 9 be found ſo often in 16? therefore muſt I take a leſſer Quotient, that is to ſay 2: then ſay I, twice 2 maketh 4, which I take out of 7, and there remaineth 3: then do I cancell 7 and 2, and over 7 I write 3, and in the Quotient I ſet 2. So the figures ſtand thus.

3
~~7~~656 (2
~~2~~9

Then ſay I farther, two times 9 make 18, which I abate out of 36, and there reſteth 18: then cancell I 3, and over him ſet 1, and likewiſe I cancell 6 and 9, and over them I ſet 8: ſo that thus ſtand the Figures.

1
~~3~~8
~~76~~56 (2
~~29~~

Then I ſet forward the Diviſor by one place, and

and seek a new quotient, that is to say, how many times 2 are in 18? which I find to be 9 times: but then can I not find 9 so many times in 5, but therefore I take a lesser quotient, as to say 8: but yet that is too great; for if I take 8 times 2 out of 18, there remaineth but 2, and I cannot find 8 times 9 in 25: therefore yet I take a less quotient, that is 7, which is also too great; for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45: therefore yet I seek a lesser quotient, as to say 6; then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6: so I cancell 18, and the 2, and write 6 over 8, thus:

~~1~~6
~~3~~8
~~7~~~~6~~56 (26
~~2~~~~9~~9
~~2~~

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

1
~~1~~~~6~~
~~3~~81
~~7~~~~6~~~~5~~6 (26
~~2~~~~9~~~~9~~
~~2~~

Then must I set forth the Divisor again, and seek a new quotient, which will be 4: for though I may find 2 in 11 5 times, and 1 remain, yet I cannot find 9 so often in 6; therefore I set the figures thus:

1
~~3~~81
~~7~~~~6~~~~5~~6 (264
~~2~~~~9~~~~9~~9
~~2~~~~2~~

And the 4 in the quotient I multiply into the figures of the Divisor, saying, four times 2 makes 8, which I take out of 11, and there rests 3: therefore I cancell the 11, and the 2, and set 3 over the first place of 11, thus:

~~1~~
~~1~~~~6~~3
~~3~~~~8~~~~1~~
~~7~~~~6~~~~5~~~~6~~ (264
~~2~~~~9~~~~9~~~~9~~
~~2~~~~2~~

And then do I say forth, 4 times 9 maketh

9 maketh 36, which I take from 36, and there remaineth nothing: so that the quotient of this division also (where 7656 is divided by 29) is 264: which doth declare, that if 264 be multiplied by 29, the summe will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division also by Multiplication.

Master. Now have I ended the five common kinds of Arithmetick: For as touching Mediation, Duplation, Triplation, and such other, they are no several kinds of Arithmetick, but are contained under the other. For Mediation is contained under Division, and is nothing else but dividing by 2: and so are Duplation and Triplation contained under Multiplication; for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3: of which I will only propose an example, for the Rules you have heard already.

If you would mediate, or divide into 2, this sum 4531010, you shall set 2 for the Divisor, and work as you learned before, as thus. — An example of Mediation.

4531010 (
2

Then I find 2 in 4 two times, therefore my quotient must be 2: so I cancell 4 and 2, and remove the divisor forward thus, as the work requireth, and as before in Division hath been declared.

4531010 (2265505
2222222

Which mediation or division by 2 being finished, you shall have for your quotient 2265505, which is the half of 4531010, as you may try by Duplation; for double that quotient, or multiply it by 2, and the same number will amount.

I will

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certain easie forms both of Multiplication and of Division. And first of Multiplication.

Easie forms of Multiplication.

If you would therefore multiply any summe by 10, you shall need to doe no more but adde a Cypher before his first place; as for example, 36 multiplied by 10 make 360.

Likewise if you would multiply any sum by 100, put two Cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This do I well perceive, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now, look, as I have told you, that you both remember it, and also often practise it.

And now you have learned how to multiply easily by 10, 100, 1000: and in like manner may you do with any other of like sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 3000, and such like, where there is one Cypher in the first place, or many orderly in the first places, you shall take away those Cyphers, and multiply the summe only by the other figure or figures, (if they be many) and then at the beginning of the summe that amounteth, you shall set so many Cyphers as you took away.

Example of 2873, which I would multiply by 300. First, I omit the 2 Cyphers from the Multiplier,

plier, and I multiply the summe by three onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or took away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the Cyphers, I must only take away the Cyphers, and multiply by the other figures, as I learned before. As if I would multiply 93648 by 25000, I should take away the three Cyphers, and multiply the same by 25, and then at the beginning of that totall sum should I adde the 3 Cyphers again.

Master. Even so. But if it chance the number that should be multiplied, or both the summes, as well the number that should be multiplied, as the multiplier, to have Cyphers in their first places, evermore omit the Cyphers, and work by the rest. But remember to restore as many Cyphers to the amounting summe as you abated before. As in this example; 30200 shall be multiplied by 206; I shall only take away two Cyphers from the greater number, and then multiply 302 by 206, and afterward adde the two Cyphers again. But if I would multiply the same 30200 by 2060, I shall not only take away the two Cyphers from the number that should be multiplied, but also I may take away the one Cypher from the Multiplier, and then must I adde three Cyphers to the summe that amounteth: but take heed that you take away no Cypher that cometh after any signifying figure, as in the last example, you must not take away that in the fourth place of the higher number, neither

neither that in the third place of the multiplier. Howbeit yet thus you may doe: If one Cypher or more come in the last of your summes, you may multiply the other figures, and overskip them, but so that you give every figure his due place: as thus, I will multiply 3026 by 2004, therefore I set them thus. And thus do I multiply them. First, 4 times 6 make 24: I set 4 under the first place, and keep the two still in my mind. Then say I again, 4 times 2 make 8, and the 2 that is in my mind maketh 10: I set down the Cypher 0, and keep the Article 1 in my mind. Then 4 times 0 is 0, and the 1 in my mind maketh 1: I set down the figure 1, and say again, 4 times 3 is 12: I set down 2, and keeping the 1 still in my mind, (having no more places of the upper number to multiply it withall) I put it down next 2 in the fifth place.

```
 3026
 2004
 ----
12104
 52
```

But now when I come to the next place, (being a Cypher 0) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2, and multiply it into the 6 of the over number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the multiplication be set right under the multiplier, and the other orderly toward the left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as in this Example is shewn.

```
   3026
   2004
  -----
  12104
 6052
-------
6064104
```

Which

Which is indeed wrought so much the sooner and shorter by overskipping of the two Cyphers: whereas otherwise (if the same example were wrought at length) it would have had two workings more, as by the same example here also set down doth appear.

3026
2004
12104
0000
0000
6052
6064104

Scholar. Sir, I thank you, for I see great ease in this way of Multiplication: and if you can shew me such like in Division, you shall greatly further me.

Master. Yes, I will teach you some easie waies in Division also: and the first this. If you would divide any summe by 10, you shall onely with your pen make a square line between the first figure of your summe and the second, and then have you done: for the whole number that followeth the line standeth for the quotient, and the figure that is before the line is the remainer. As for example, 3648 divided by 10. Where 364 is the quotient, and betokeneth that so many times are 10 in 3648; and the 8 after the line is the remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet. — Ease forms of Division.

364 | 8

And so likewise if you would divide any summe by 100, with your pen you shall cut away the two first figures; and if you would divide by 1000, you must cut away the three first figures: and so of any other divisor, whose last figure is 1, and the other Cyphers; look how many Cyphers the divisor hath, and so many figures at the beginning shall

you cut away with the square line, and they stand alwaies for the remainer, because they are less then the divisor, and cannot be divided by it; and the other figures that are behind the line stand for the Quotient.

But now if your divisor have any other figure in his last place then 1, and in all his other places have Cyphers, look how many Cyphers there be, cut away so many of the first figures of the number that should be divided, and divide the rest that followeth the line by that figure that is in the last place, as if it were the whole divisor.

Example of 64284, which I would divide by 300: here I must cut away the two first figures, (for so many cyphers my divisor hath) and must divide the rest by 3, which is the figure in the last place of the divisor. First therefore I part away the two first figures, and the summe standeth thus:

642|84 (2
3|00

Then do I divide 642 by 3, and the Quotient is 214: for in 6 I find twice 3, and in 4 once, and one remaining, which 1 with the 2 next before doth make 12, wherein I find 3 four times. And this is a ready way to turn shillings into pounds: for sith one pound doth contain 20 shillings, I must divide the whole number of shillings by 20. Therefore easily do I see that my divisor hath one cypher; and therefore I cut away one figure from the beginning of the whole summe of shillings, and then do I mediate or divide by the two other figures or summe that followeth.

Scholar. I will put an example.

If you would divide 64287 shillings by 20, that

that is to say, if I would turn so many shillings into pounds, I must cut away the first figure, that is 7, and divide the rest, that is 6428, by 2, so shall the Quotient be 3214; whereby I know that 64287 shillings make 3214 pounds, and 7 shillings remaining.

Master. Now prove by Multiplication whether you have well done or no.

Scholar. The Quotient is 3214, which I do multiply by the divisor 2, and it doth amount to 6428.

Master. Hereby you may perceive not onely that you have well done, but also how by Division you may turn shillings easily into pounds, and contrariwise by Multiplication you may turn pounds into shillings.

But here shall you see amongst divers men divers forms of such division; but if you mark what I have told you, you shall perceive easily all the waies. For some men do not cut away so many of the first figures of the summe that they would divide, as there are Cyphers in the first place of the divisor; but they set all their Cyphers orderly under the first places of the number that they would divide, and then with the other figure, or figures, (if there be many) they divide the rest of their summe.

Example. If they would divide 725931 by 3400, they do set their summs thus:	725931 34 00

And then do they divide orderly till they come to the Cyphers: for there they stay and end their work, as in this example.

They seek how often 3 may be found in 7, which

is two times, and 1 remaining: therefore they set 2 in the quotient, and cancel 3 and 7, and over 7 they set the 1 that remaineth, thus:

1
~~7~~25931(2
~~3~~4 00

Then do I go forth, saying, two times 4 make 8, which they take out of 12, and there remaineth 4, thus:

~~1~~4
~~7~~~~2~~5931(2
~~3~~~~4~~ 00

Then remove they the divisor forward, and seek how often 3 may be found in 4, which is but once, and 1 remaineth: then set they 1 in the quotient, and cancel 3 and 4, and over them they set that 1, thus:

1
~~1~~~~4~~
~~7~~~~2~~~~5~~931(21
~~3~~~~4~~400
~~3~~

Then take they once 4 out of 15, and there resteth 11. Or else more easily, Take once 4 out of 5, and there resteth 1. So they cancel the 4 and 5, and set 1 over them, thus:

1
~~1~~~~4~~1
~~7~~~~2~~~~5~~921(21
~~3~~~~4~~~~4~~ 00
~~3~~

Then set they forth the divisor again, and seek how many times 3 are in 11, which they find three times, and 2 remaining: so they set 3 in the quotient, and cancel 11 and 3, and over them set 2, thus:

~~1~~2
~~1~~~~4~~~~1~~
~~7~~~~2~~~~5~~931(123
~~3~~~~4~~~~4~~400
~~3~~~~3~~

Then do they multiply 4 by 3, which maketh 12; that withdraw they out of 29, and there resteth 17; of which the 7 must be set over the 9, and the 1 over the 2 thus:

1
~~1~~~~2~~
~~1~~~~4~~~~1~~7
~~7~~~~2~~~~5~~~~9~~31(213
~~3~~~~4~~~~4~~400
~~3~~~~3~~

And now are the two Cyphers next ensuing, so that the divisor can no more be set forward, and therefore is the division ended, and the remainer is 1731. Now the quotient, which is 213, doth declare that if

you

you divide 725931 by 3400, you shall find it therein 213 times, and there remaineth 1731: so shall you find it, if you work as I taught you, by cutting away the two first figures, because of the two Cyphers. But this must you mark, (as you may perceive by this last example) that if there be left any other Remainer in the summe that was behind the square line, that the Remainer must be set to the latter end of the first Remainer, which was cut away with the square line: as if you would divide 725931 by 3400, after the form that I taught you, then would your summs appear thus: Note.

|1
~~12~~
~~141~~7
~~7259~~31 (213
~~3444~~
~~33~~

So that 17 which remaineth after the line must be set to the 31 (that was cut away with the line) in higher places, as you see here, where that 17 with the 31 do make 1731.

Scholar. Sir, is there no other form of Division in practice but this?

Master. Yes verily, there are other forms in practice; but because I love brevity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especial loving friend, Master Henry Bridges, wherein not any one figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus: 6)72

Then, if you please, you may write the divisor in a loose paper, that it may more easily without cancelling or defacing of the work be applied to, and removed from, the dividend at pleasure: then apply your divisor 6 to 7, the first figure of the Write the Divisor in a loose paper, to remove at pleasure.

 dividend,

dividend, and inquire how oft it may be had in 7; and seeing 6 is but once in 7, set 1 in the Quotient line, thus:

6)72(1

Then multiply the divisor 6 by the quotient 1, and set the Product 6 under 7, thus:

```
6)72(1
  6
```

Then draw a line under 6, and subduct 6 out of 7, setting the remainer 1 under 6, thus:

```
6)72(1
  6
 ---
  1
```

Then bring down the next figure of the dividend, and set it with the remainer 1 under the line, thus:

And bring the movable divisor 6 under the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12, set 2 in the quotient, thus:

```
6)72(1
  6
  12
```

```
6)72(12
  6
 ----
  12
  12
  0
```

And multiply 6 by that new quotient 2, setting the Product 12 under the other 12, and subducting it out of the upper number, there resteth nothing.

And since the unites of this Product do stand under the unites of the Dividend, the division is ended, otherwise you should proceed as before, bringing down the next figure, removing the divisor, dividing, multiplying, subducting, &c.

Scholar. This is very easie: but if there be greater numbers propounded, is the operation the same?

Master. If the numbers be never so great, the work is the same without any difference, as shall appear by this example.

Divide 7890 by 33. 33)7890(2

First set them thus: then bring the divisor under 78, and see how oft it is there found, which is twice, and therefore set 2 in the Quotient, by which multiply

may muster da rainy day

multiply the diviſor 33, and ſet the Product 66 under 78, and ſubduct it out of it thus:

$$
\begin{array}{l}
33)7890(239\frac{3}{33} \\
\quad 66 \\
\quad 129 \\
\quad 99 \\
\hline
\quad 300 \\
\quad 297 \\
\hline
\quad\ 3
\end{array}
$$

Then bring the next figure 9 down, and ſet it with the Remainer 12, it maketh 129, and removing the diviſor 33 thereto, enquire how often 33 is contained in 129, and I find it but thrice, (though at the firſt it made a ſhew of more;) therefore ſet 3 in the Quotient, and multiplying 33 by 3, ſet the Product under 129, ſubducting that product out of the number above, and proceed as before.

Then ſhall you find the Diviſor 9 times in the Remainer; therefore ſeeing 9 in the Quotient, multiply, and ſubduct as before, and at the laſt you ſhall find onely 3 remaining, which muſt be ſet above a line after the Quotient, and the Diviſor under, as above appeareth.

Scholar. Is there no more difficulty in the whole Rule?

Maſter. Not any, although your number be never ſo great, as before I have ſaid.

¶ And here will I make an end of Diviſion, (ſaying that I do requeſt you to exerciſe your ſelf well herein by many ſummes, till you have attained ſome expertneſs therein.)

For the reaſons and concluſions thereof are ſo many, and ſo available for all ſorts of men whatſoever, that if I ſhould ſpeak of the infinite uſes thereof, I ſhould rather lack words then matter. And therefore re-commending it to your judgment hereafter, upon your farther travel into the Art, I will here end this

this Treatise, representing unto you one example, or simple question of Division and Multiplication, in stead of many, which is this.

A question of shooting in Ordnance.

There are four brass Pieces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a Hold, and there is allowed by the Master Gunner 700 pounds of powder to be spent by these four Pieces in this assault. The question is twofold. The first, how many shot each Piece shall justly make about with this 700 pounds of powder. And secondly, how many pounds of powder ought justly to be allowed to each Piece for his true proportion.

Scholar. Why, Sir, you make me smile, to bear me in hand that these two demands may be simply resolved by Multiplication and Division.

Master. Truly that they may, and that you may by and by work your self with a little labour. First, adde together their quantities of powder, that is, 9 pounds, 5 pounds, 4 pounds, and 2 pounds, all which make 20: Divide the 700 pounds of powder by that 20, and your quotient giveth 35, as here appeareth: which sheweth for most certainty that they shall make just 35 shoots about.

```
  x
  ẋøø
  xøx(35
  x
```

Scholar. Sir, all this have I done, and I see it is so; but whether it be true or not, I cannot tell.

Master. To try the truth of the same, multiply the first piece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second piece which

spends

ſpends 5 pounds by 35, and you ſhall find 175 pounds his allowance. Then 4 by 35, and you ſhall find 140 pounds his allowance. Laſtly, multiply 2 by 35, and you ſhall find 70 pounds his allowance, All which four particular ſummes you ſhall adde together by Addition, as here appeareth, and it maketh juſt 700 pounds; and ſo is the queſtion truly abſolved.

315
175
140
70
700

Scholar. Truly, Sir, theſe excellent concluſions do wonderfully more and more make me in love with the Art.

Maſter. It is an Art, that the farther you travell, the more you thirſt to go on forward. Such a Fountain, that the more you draw, the more it ſprings. And to ſpeak abſolutely, in a word, (excepting the ſtudy of Divinity, which is the ſalvation of our Souls) there is no ſtudy in the world comparable to this for delight in wonderful and godly exerciſe: for the skill hereof is well known immediately to have flowed from the wiſedom of God into the heart of man, whom he hath created the chief image and inſtrument of his praiſe and glory.

Scholar. The deſire of knowledge doth greatly incourage me to be ſtudious herein: and therefore I pray you ceaſe not to inſtruct me farther in the uſe hereof.

Maſter. With a good will. And now therefore for the farther uſe of theſe two latter, that is, Multiplication and Diviſion, I will briefly ſhew you the feat of Reduction.

Reduction,

Reduction.

Reduction what it is.

REduction is by which all summes of gross Denomination may be turned into summes of more subtile Denomination: and contrariwise all summes of subtile Denomination may be brought to summes of grosser Denomination.

Scholar. What call you gross Denomination, and subtile Denomination?

Gross Denomination.

Master. That I call a gross Denomination, which doth contain under it many other subtiler or smaller: as a pound (in respect to shillings) is a gross Denomination; for it is greater then shillings, and containeth many of them: and shillings (in comparison to pounds) are a subtile Denomination, for because they are lesser then pounds, and many of them are contained in one of the other. And so likewise of other things: whatsoever thing is compared to other, if it be greater and containeth many of them, it is a gross Denomination; but if it be lesser, (so that many of them are in the other) then are they called the subtile Denominations: whereby you may perceive that one Denomination may be called a gross Denomination, and also a subtile, (that is to say, a great, and small) in divers comparisons. For shillings compared to pounds are a subtile or small Denomination; but compared to pence they are a gross or great Denomination.

Subtile Denomination.

Scholar. Now I understand the name, I pray you teach me the use.

Master. The use is easily learned, if you remember

member what you have learned before. For if you will reduce any summe of a gross Denomination into a summe of a smaller or subtiler Denomination, you must consider how many of that subtiler Denomination do make one of the grosser Denomination, and, by that number or numerator do ye multiply the summe. As if you would reduce 20 pounds into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400; whereby you may know that in 20 pounds are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360; whereby you may find that in 30 shillings are contained 360 pence. And thus may you reduce any gross Denomination into a more subtile by Multiplication, if you know how many of the lesser do make the greater: of which thing I will anon give you a brief Table for the most accustomed kinds of Money, Weights, Measures, and Time, and such like; whereby you may know how often each subtile Denomination is contained in the grosser, when you shall need it for the aforesaid kind of Reduction. And also the same shall serve you, if you would reduce any summe of a subtiler Denomination into a summe of a greater Denomination. For in such Reduction you must consider (as in the other form) how many of the smaller do make the greater; and by that number you must divide the other summe, and the Quotient will declare how many of the greater Denomination are comprehended in that summe.

To reduce gross Denominations to subtile.

As

As for example: If you would know how many shillings are contained in 3240 pence, consider that 12 pence do make 1 shilling: you must divide that 3240 by 12, and your Quotient will be 270, whereby you may know that so many shillings are in 3240 d. But if you would know farther how many pounds are in these 270 shillings, seeing that every pound containeth 20 shillings, divide that 270 by 20, and it will be 13, and 10 remaining: whereby you may know, that in 3240 pence (or 270 shillings) are 13 pounds and 10 shillings. For evermore the Remainer must be named by the name or Denomination of that summe that was divided, which in this place were shillings. And thus may you doe with any other kinds of Denominations.

¶ Wherefore, to the intent you may have certain light or knowledge in most common Coins, Weights and Measures, (which is the chief and principallest thing in Traffick to be known) I have in each Reduction, as they come in order, set down certain instructions incident thereunto. And first I have hereunto added this Table, wherein is comprehended, not onely our currant and common Coins, but also the most part of the usual Coins of Christendom, with their just weights and value currant in the Realm of *England*; intending, at the latter end of my Addition to this Book, to write of the ordinary Mony used in divers places, and their common values currant for Traffick, with the manner of their exchanges from place to place, &c.

A Ta-

A Table of the names, and now valuation, of the most usual Gold-coins throughout Christendom, with their several weights of Pence and Grains, and what they are worth of currant *Engl.* mony this year 1630.

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Great Sovereign.	10	0	23	0
Double Sov. K. H.	8	1	22	0
Doubl. Sov. of Q. E.	7	7	22	0
Royall.	4	22	16	6
Half Royall.	2	1 d.	8	3
Old Noble.	4	6	14	8
Half Noble.	2	3	7	4
Angel.	2	8	11	0
Half Angel.	1	16	5	6
Salute.	2	5	6	11 ob.
2 parts of Salute.	1	11	4	7
George Noble.	2	0	9	9 ob.
Half George Nob.	1	11	4	11 q.
First Crown K. H.	2	9	6	11 ob.
Base Crown K. H.	2	0	5	6
Sover. K. H. best.	2	14	11	8 ob. q.
Sovereign K. H.	4	0	11	0
Edward Sover.	3	15 d.	11	0
Elizabeth Sover.	3	15 d.	11	0
Elizabeth Crown.	1	9	5	6
Half Crown.	0	19	2	9
Unite.	0	12	22	0
Double Crown.	3	6	11	0
Britain Crown.	1	1	5	6
Thistle Crown.	1	7	4	4 ob. q.

Half

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Half Crown.	0	19 d.	2	9
Cross Dagg.	3	6 d.	11	0
Half Cross Dagger.	1	15	5	6
Rose Royal.	0	21	22	0
Spur-Royal.	4	10 d.	16	0
The Angel.	2	22 d.	11	0
Half Angel.	1	11 d.	5	6

All the several pieces of Gold heretofore mentioned are set down according to their valuation by the King's Maiestie's Proclamation for Gold, dated the 23 of November, 1611.

A Table of Foreign Gold Coin, according to their ancient valuation and several weight in Pence and Grains.

The names & titles of the Gold.	The weight in Pence.	Grains.	The value in Shil.	Pence.
Unicorn of Scot.	2	10	6	0
Scottish Crown.	2	5	6	0
French Noble.	4	16	13	4
All French Crowns	2	5	6	0
Flanders Riders.	2	6	6	6
Gelders Riders.	2	2	3	6
Philips Royal.	2	10	10	0
Philips Crown.	2	5	5	0
Golden Gilder.	2	2	4	8
New Ant. Gild.	2	2	5	0

Flanders

Flanders Noble.	4	10	12	0
Half Flan. Noble.	2	6	6	0
Flan. Angel olft.	3	6	3	0
Flan. Royallorke.	3	10	10	0
Carolus Gilden.	0	12	3	6
Flanders Royall.	2	6	5	0
Saron Gilden.	2	2	4	[illegible]
Flanders Crown.	2	5	6	[illegible]
Philips Gilden.	2	3	4	2
Half Phil. Gilden.	1	1	2	1
Golden Lion.	1	16	7	8
3 parts of gol. Lion.	0	21	2	5
½ parts of gol. Lion.	1	19	4	11
Davids Gilden.	2	2	4	9
Horne Gilden.	1	12	4	11
Old under Gilden.	2	3	4	10
Crusa. long Cross.	2	6	6	0
Crusa. short Gross.	2	6	6	2
Milreys.	4	20	13	4
Half Milreys.	2	10	6	8
Portague 1 ounce	2	16	68	0
Golden Castile.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Pistolet.	4	9	11	8
Single Pistolet.	2	4 d.	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Doub. duc. of Rome.	4	13	12	8

It is to be understood, (gentle Reader) that whereas in these Tables the weight is called by the name of a penny,

penny, it is not ment a penny of silver money, but a penny of Gold-smiths weight, which containeth 24 *Barly-corns. Concerning which see Troy-weight in* folio 133.

So if a man have not the weight wherewith to weigh any piece of gold, he may do it with Barly-corns, being dry, as it is said folio 133.

The Prices of Gold which the bringers in of Forein Gold shall receive at the Mint, according to the King's *Majestie's* Proclamation dated the 14 of *May Anno* 1612.

FOr an ounce of *French* Crowns being 22 Karacts fine————	3li. 6s.
For every ounce of *Spanish* Pistolets being 21 Karacts, 3 grains and a half fine ————————	3li. 6s.
For Ducats of *Spain* being 23 Karacts, 1 grain fine at least, the ounce—	3li. 8s. 8d.
For Milreas Crusado long cross, Crusado short cross, the ounce——	3li. 6s. 2d.
For *Hungary* Ducats being 23 Karacts, 1 grain fine at least, the ounce—	3li. 9s. 2d.
For the Checkeen of *Venice* being 23 Karacts, 1 grain fine at least, the ounce————————	3li. 10s.
For *Barbary* Gold being 23 Karacts and di. grain fine at the least, the ounce ————————	3li. 9s.

¶ *And if the said* Barbary *Gold be of less fineness, abatement to be made according to the rate.*

For

For Sultains being 23 Karects 1 Grain fine at the least, ounce———— } 3 li. 8 s. 8 d.
For all other Gold being 22 Karects fine, the ounce———————— } 3 li. 6 s.

¶ *And being finer, a greater price according to that rate; and being courser, a less, so that the bringer in supply the less fine with the more fine, in such sort, that in the totall it makes good the same rate of 22 Karects fine.*

The Price of Silver, which the bringers in of forein Silver shall receive at the Mint, according to the King's Majestie's aforesaid *Proclamation.*

FOr the ounce of *Spanish* Silver mony of *Sevill,*————— } 5 s.
For the ounce of *Mexico* mony—4 s. 10 d.
For Ingots of Silver, being 11 ounces 2 d. weight fine, according to the Standard of *England*, the ounce } 5 s.

¶ *And for other Silver of more fineness, a better price according to that rate; and for courser a less, so that the bringer in supply the less fine with the more fine, in such sort, that in the totall it makes good the said rate of 11 ounces 2 peny weight fine, according to the Standard of* England.

Of Silver Coyns currant in this Realm.

The *Edward* crown of 5 s.
The *Edward* half-crown of 2 s. 6. d.
The *Edward* shilling, half-shilling, and three pence.
Philip and *Marie*'s shilling, and half-shilling.
The *Mary* groat, and *Mary* two pence.
Queen *Elisabeth*'s shilling, 9 d, 6 d, 4 d, 3 d, 2 d, 1 d, three farthings, and half-peny.

HEre would I now express the values of sundry other Coins of divers Countries; but for three causes I now refrain. The first and chiefest is, because they are not currant by the Statutes of this Realm. Another cause is, by reason they are so uncertain, that they be never long at one rate. And again, they are so different in so many places, that it were matter enough for a great Book to speak sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in use; and first of France.

French Coyns. The most common Money are Deniers, Soulx, and Franks: 12 Deniers make 1 shilling, 20 Soulx make 1 Frank. So that you may see these three kinds are like in the rate to pence, shillings & pounds with us: but that this is the difference, that their Denier is but the ninth part of our peny, and of their Soulx (commonly called Souses) goe 9 to our shilling, and 9 of their Franks to an English pound of mony. So that three of their Franks make a Noble. And by those three you may practise how to reduce French mony into English mony, according as I have set forth here following.

2160	Deniers make	240 d. or 20 s.
3240		360 d. or 30 s.
8352		928 d. or 3 li. 17 s. 4 d.

2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their Coyns, I omit them till hereafter, that you have some understanding in broken numbers.

Flanders Coyns. But now as for the Coyns of Flanders, they be so changeable, that you must know them from time to time, else you cannot reduce them into our

mony

mony certainly: but yet that you may have an example of their mony to exercise you withall, you shall take those that be most common; as Stivers both single and double, Groats Flemish, Carolus and Gyldens. A Flemish Groat is a little above 3 farthings English. A single Stiver is 1 d. ob. q. half farthing. The double Stiver Carolus is 4 d. ob. half farthing. Then there is also the Carolus Gylden, which is worth 10 Stivers; and the Flemish Noble is worth 3 Carolus Gyldens and 12 Stivers.

So that if you would convert Flemish mony, or any other kind of mony whatsoever it be, justly into Sterling, you must reduce it first into the smallest part of English mony that is in that Coyn. As for example, If I would reduce 368 double Stivers into English mony, (considering that a double Stiver containeth 3 d. farthing) you shall first look how many farthings be in the double Stiver, and you shall find them 13; therefore multiply the summe of the Stivers by 13, and then have you their value in farthings, which is 4784. Now if you divide that by 4, then there will appear the number of pence: but better it were to divide it by 48, (for so many farthings are in one shilling) and then will the Quotient declare the summe of 4 li. 19 s. 8 d.

Likewise if you would reduce any summe of single Stivers into English mony, you must multiply the summe first by 13, and then have you reduced them into a certain summe, that is to wit, half farthings, which summe if you divide by 8, then will amount the summe of pence: or if you divide it by 96, the summe of shillings will appear.

But mark this in all Division: when you do reduce to bring one Denomination into another, if there be any remainer after the Division, that must be named by the Denomination of the gross summe that was divided. As for example, I would bring 254 farthings into pence, therefore I divide that 254 by 4, (for so many farthings make a peny) and the Quotient is 63, which is the summe of the pence, and then remaineth yet 2, which are farthings still, as one may prove by dividing. And this must be marked in all Division, namely, when it is done for Reduction.

Note well.

Danks Mony.

¶ Touching Danks Mony, they have their Soulx, whereof 20 is a Liver, which is 2 shillings sterling. They have also their Grash, whereof 80 make a Gilden, which is 4 shillings sterling. They have also Dollars, and their common or old Dollar is 35 Grash. New Dollars they have which be divers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Authour touching Danks mony.

Spanish mony.

Concerning Spanish Mony, whereof the most common are Cornadoes, Marveides, Marveide, 4 Marveides make a Ryall, and 11 Ryalls make one Ducat; so the Ducat containeth 374 Marveides, which is about 5 shillings 10 pence sterling. Therefore if you would convert 124 li. 5 s. sterling into Ducats, consider that pence is the last value or Denomination named in this question: therefore reduce 124 li. 5. s. into pence, and it maketh 29820 pence: which if you divide by pence that a Ducat is worth, (which is 70) you shall have for your Quotient 426 Ducats, your desire.

In

In Venice they have Bettes, Souldyes, Livers. 5 Bettes make an English peny, 60 Bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Liver of Venice, which is a pound sterling. Venice Money.

Thus much have I said of Money: Now will I shew you in the like sort the distinction of weights. Weights.

After a Statute made anno 11 *H*. 7. there ought to be but one sort of weight; as 24 Barley-corns dry, and taken out of the midst of the Ear, do make a peny-weight, 20 of these peny-weights make an ounce, 12 ounces a pound of Troy-weight, by which is weighed Bread, Gold, Silver, Pearl, Silk, and such like. Troy-weight. A peny-weight. An ounce. A pound Troy. But commonly there is used another weight called Haberdupoise, in which 16 Ounces make a pound. Haberdupoise weight. Therefore when you would reduce Ounces into pounds, you must consider whether your weight be Troy-weight or Haberdupoise: and if it be Troy-weight, you must divide your Ounces by 12, to bring them to pounds; but if it be Haberdupoise, you must divide them by 16. Now again, there be greater weights, which are called a hundred, half a hundred, and a quartern, and also a half quartern, &c. A hundred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they have not common weights made to their rate, as the other have. And again, these that I did name are not just in number as they seem by their name; for an hundred is not just 100, but is 112 pound. And so the half hundred is 56, the quartern 28, and the half quartern 14.

And theſe be the common weights uſed in moſt things that are ſold by weight.

Wooll weights. Todde. Stone.

Howbeit there are in ſome things other names: as in Wooll 28 pound is not called a quartern, but a Todde; and 14 pound is not named half a quartern, but a Stone; and the 7 pound half a Stone. Other names, becauſe they differ in many places, and agree in few, I let paſs.

Sack of Wooll.

But a Sack of a Wooll by the Statutes is limited to be 26 Stone.

Cheeſe weights.

¶ Now in Cheeſe, though it be ſold by the Hundred, and by the Stone in ſome places, yet the very weights of it are Cloves and Weyes. So that a Clove containeth 8 pound, and a Weye 32 Cloves, which is 256 pound, that is 12 ſcore and 16 pound; and ſo much weigheth the Weye of Suffolk Cheeſe: and the like is or ſhould be the Barrel of Suffolk Butter.

The Weye of Eſſex Cheeſe containeth ſix ſcore and ſixteen pound: and ſo much is alſo the Barrel of Eſſex Butter.

The Apothecaries weights.

Moreover this weight is uſed by the Apothecaries in their Phyſicall compoſition and mixture in Medicine, wherein the leaſt is a grain;

And	*20 Grains*	make	*A Scruple,*	thus charactered.	℈
	3 Scruples		*A Drachm or Dragm,*		ʒ
	8 Dragms		*An Ounce,*		℥
	16 Ounces		*A Pound,*		℔

Meaſures for Liquor. A Pint. Gallon. Pottle.

Now of weights are made other meaſures both for Grain and Liquor. For a Pound in Troy-weight maketh a Pint in meaſure, ſo that 8 pound or 8 pints do make a Gallon; half a gallon is named a Pottle,

and

and half a pottle is called a Quart, which containeth two pints. Now above a gallon the next measure is a Firkin; then the Tertian, or Kilderkin, or half a Barrell, and a Barrell. And by these measures are sold commonly Ale, Beer, Wine and Oyl, Butter and Soap, Salmon, Herrings and Eels. Quart. Firkin. Tertian: Kilderkin. Barrell.

But as these be unlike things, so the measures of their vessels do differ; for the measures of them all are as followeth.

Of Ale	*The firkin* / *The kilderkin* / *The barrel*	*containeth*	8 / 16 / 32	*Gallons.*	Ale measures.
Of Beer	*The firkin* / *The kilderkin* / *The barrel*	*containeth*	9 / 18 / 36	*Gallons.*	Beer measures.

Soap-measures, both Firkin, Kilderkin, and Barrell, should be equal to Ale-measure. Soap measures.

Moreover the Statutes do limit the weight of every of those three Vessels being empty;

A barrell / *Half a barrell* / *A firkin*	*to weigh empty*	26 / 13 / $6\frac{1}{2}$	*Pounds.*

Herrings also are sold by the same measures that Ale and Soap be sold by. Herrings.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eels have a greater measure.

Salmon and Eels	*The butt* / *The barrell* / *Half barrell* / *The firkin*	*holdeth*	84 / 42 / 21 / $10\frac{1}{2}$	*Gallons.*	Salmon and Eels.

Howbeit some Statutes did limit Eel-vessells equal with Herring-vessels.

Wine-measures. Now as for Wine-vessells, they are seldom smaller then Hogsheads, which are of 63 Gallons: Every Hogshead is two Barrels. Yet there are many other wine-vessels, but of them all see this Table, and mark the measures one by another.

Of Wine and Oyl	*The rundlet*	*holdeth*	$18\frac{1}{2}$	*Gallons.*
	The barrell		$31\frac{1}{2}$	
	The hogshead		63	
	The tertian		84	
	The pipe		126	
	The tun		252	

But you shall mark that there be other kinds of Tertian. Tertians: for there be Tertians (that is to say Thirds) of pipes, of hogsheads, and of barrels, as well of other things as of Wine.

Also Malmseys and Sack, &c. the half Tun is not called a Pipe, but rather a Butt. A Butt.

And thus much have I thought meet to tell you at this time,

Scholar. And is that alwaies true?

Master. I have told you how it should be; but how it is, I may not say: how they do differ daily from their just measure, that Gaugiers can tell you better then I. But I will let this pass now, and speak briefly of the other measure.

And as of Weights there did spring the liquid measures, (whereof I spake last) so of the same spring dry measures, as Pecks, Bushels, Quarters, and such like, whereby are measured Corn and like Grains, also Salt, Lime, Coals, and other like. And this is the order and quantity of them.

Dry measures.

A Peck. A Peck is the measure of two Gallons.

A bushell	containeth	*four pecks.*	A Bushell.
A quarter	containeth	*eight bushels.*	A Quarter.
A wey	containeth	*six quarters.*	A Wey.

These are the common names and measures, but in divers places there be divers sorts.

The Bushell in many places is two bushells, but then is that bushell there called a Strike: and in some places half a quarter is called a Cornock. But those diversities are too many to tell you briefly them all; and again, sith they are against the Law and Statutes, I count them unmeet to be used. Strike. Cornock.

But now remaineth yet another kind of measures, whereby men mete length, breadth and thickness; and those are an Inch, a Foot, and such other, whose names and quantities this Table sheweth. Measure to mete length, breadth & thickness.

3 *Grains of barley in length make an Inch.* An Inch.

12 *Inches*	*make*	*a foot.*	Foot.
3 *Foot*	*make*	*a yard.*	Yard.
3 *Foot and 9 Inches*	*make*	*an ell.*	Elle.
5 *Yards and a half*	*make*	*a pearch.*	Pearch.

1 *Pearch in breadth, 40 in length, do make a Rod of Land, which some call a Rood, some a Yard-land, some a Farthendele.* Farthendele.

2 *Farthendels*	*make*	*half an acre of ground.*	
4 *Farthendels*	*make*	*an acre.*	Acre.

¶ More, 40 *rods in length do make a furlong,* 8 *furlongs make an* English *mile, which containeth* 320 *Pearches.*

So that an English mile, grounded upon the Statute, is in length 1760 yards, 5280 foot, and 63360 inches.

Somewhat greater then the Italian mile of 1000 paces, and 5 foot to a pace,

Here

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures: but because it cannot be well done without the knowledge of Fractions, which as yet you have not learned, I will let them pass till another time, that I have taught you the knowledge of broken numbers.

The parts of time.

Scholar. But yet, Sir, of the parts of time I pray you tell me somewhat.

A Day. An Hour. Week. Month. Year.

Master. You know that a natural day hath 24 hours, and every hour hath 60 minutes. It needeth not to tell you that 7 days make a week, and 4 weeks make a common month, and 13 months make a year, lacking one day and certain hours and minutes: but of that I shall instruct you hereafter.

Here will I make an end of *Reduction* for this time; which though it be counted no kind several of *Arithmetick*, you see it is no less needfull to be known, or easier to be done, then any of the other.

Scholar. Marry, Sir, it seemeth unto me much harder then any other sort, for it requireth the knowledge of so many things. But now, Sir, when you see time, I am ready to learn forth: as much of Reduction as you have taught me I remember; but and if I do at any time forget, I shall have recourse to the Tables which you set forth for me.

Master. So doe you; for it will not be remembred without exercise. But inasmuch as you understand so much as we have intreated of, I will now instruct you in Progression.

Progression.

Progression.

Although untill this day the most part of Writers have defined Progression *as a compendious kind of* Addition, *yet truly it is not so: for* Progression *(as the very nature of the word doth inform any man) is a going forward and proceeding in numbers, and that regularly and orderly, whose place is aptly chosen to be very near, or rather next after, the exposition of the* four principal parts of Arithmetick; *for in it, after a most easie manner, are all the four former parts exercised and practised, and not onely* Addition, *as customably is done. Which custome hath been the cause why it hath so specially been named a kind of* Addition, *and defined to be a quick and brief* Addition *of divers summs, proceeding by some certain and reasonable order.* What Progression is.

You shall also understand there are infinite kinds of Progressions: but for you (as yet) two are sufficient to be exercised in; of which the one I call Arithmeticall, and the other Geometricall.

Arithmetical progression is a rehearsing or placing down of many *numbers*, *number* after *number*, in such sort that between every two next *numbers* rehearsed or placed down, the difference, diversity, or excess, be equal and alike. Arithmeticall Progression.

Scholar. Sir, I thank you for that you have both opened unto me what Progression is truly, and also why it is here placed.

But I pray you with an example make plain your definition.

Master.

Maſter. Examples cannot want, ſeeing all reaſonable creatures naturally uſe the order of one kind of Arithmeticall progreſſion, (which therefore is alſo named natural) whenſoever they diſtinctly do count or number any multitude by one, ſaying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and every one ſurmounting and exceeding his fellow next before by a like quantity, (which here is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſs, I ſet it down in this manner.

The common exceſs.

The progreſſion.

Scholar. This is moſt evident. And I think that I am able to tell you now of any progreſſion Arithmeticall propounded, what is that common exceſs or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholar. They exceed each other by 3: And that may I ſet down in ſuch evident order as you did your example of natural progreſſion, in this wiſe.

The common exceſs.

The progreſſion.

Maſter. And do you not alſo now perceive, that the whole Table of Multiplication may be made by the other of progreſſion Arithmeticall?

either

ther if you will begin at the first number of any of them on the left hand, and so proceed right overthwart; or any of the first number of the upper row, and go directly downward.

Scholar. I pray you let me consider the thing a little, and I will answer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this trial I perceive it now very well, for the common excess or difference between any two next is continually as much as the first number of every row, either from the left hand overthwart taken, or from any of the uppermost overthwart rows downward.

To know the totall summe of an Arithmeticall progression.

Master. Now then, if of any such progression you would speedily know the totall summe much quicklier then by common Rules of Addition, first, tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their summe down by it self: as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may see; therefore set down 7 in a place alone,

alone, then adde together the firſt number and the laſt, as in this Example; adde 2 to 14, and that maketh 16, take half of it, and multiply by the 7 which you noted for the number of the ſame places, and the ſumme that amounteth is the ſumme of all thoſe figures added together; as in this Example, 8 multiplied by 7 maketh 56, and that is the ſumme of all thoſe figures.

Scholar. That will I work by another example. I would know how much this ſumme is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places, and there are 9: that I note. Then I put the firſt number 5 and the laſt 29 together, and they make 34. I take half of it, that is 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of all the numbers.

Maſter. So ſhall you find it, if you try it.

Scholar. How ſhall I try it?

Maſter. By your common Addition; for if you adde all the parcels together, you ſhall ſee the ſame ſumme amount, if you did work well. And that manner of Addition trieth all kinds of ſumming any progreſſion.

Scholar. Then can I ſumme any progreſſion, if the number of the parcels be odde. But what if they be even, as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Maſter. When the number of the parcels is even, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the half of the number of the places do you multiply it: as in our example, the parcels are 8, that I note, then adding the firſt ſumme to the

laſt,

last, there amounteth 9, that do I multiply by half of the parcels, that is, by 4, and it maketh 36, which is the summe of the parcels.

But if you will take one Rule for these both, doe thus; Multiply the half of the one by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels is odde, so their half cannot be taken; and sometime it chanceth that the Addition of the first number and the last do bring forth an odde number, so that half of it cannot be taken: but they will never be both odde.

A general Rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Master. As accustomably it hath been taught, this hath been the chief and onely exercise in Progression used. But that you may perceive how divers waies and to how great profit so simple a thing (as this Arithmetical Progression is) may be considered and used, I will here propound you six Propositions, of which four of them were invented by a friend of mine, and never before this published; and the two first were never to my knowledge written of but by these men.

Scholar. This doth greatly incourage me to be attentive unto your words, seeing I shall not onely be instructed at your hands in the common known Rules of this excellent Art, but besides that so abundantly in other new Rules informed, as my very entrance shall seem to pass a great many mens farther study and longer continuance. Therefore, Sir, I beseech you let me know your six Propositions.

Master. These they are.

1 To know the *last number* without proceeding by continual *Addition* till you come unto it, so that the common *excess*, the first *number* and the number of the *places* be known.

2 The *first number* of the *Progression* and the *last* being known, with the common *excess*, to find the *number* of the *places*.

3 The *excess* being given, and the *first* or *last*, to know the quantity of any middle *number*, whose place is given from the *first* or *last*.

4 The total summe being given, and the *first* and *last*, to find out the *number* of the *places*.

5 The *totall summe* of any *Arithmetical Progression* being given, and the *first* and *last*, to find out the common *excess*.

6 The *totall summe* being given, and the mutual *excess*, with the *number* of the *places*, to give the *first* or *last number* of the same *Progression*.

Many moe considerations could I propound you in these *Arithmetical Progressions*; but these are sufficient to give you occasion to think, that Rules of knowledge and Arts are infinitely capable of enlargement.

Scholar. Happy were I if I did but well understand that which is already invented and written: And yet, in my simple fantasie, these things offer themselves (in manner) to be studied for about Progression; therefore I pray you to proceed to the Rules answering to these Propositions.

Master. I will orderly for every of these 6 Propositions give you Rules, and with every one an example, unless the plainness and easiness need no farther exemplifying.

For

For the Solution of the first, multiply the excess by a number less by 1 then the number *of the places, and the off-come adde to the* first number, *so you shall have the* last number, *which is sought for.* 1 Proposition.

As for example: If there were seven places in a progression Arithmeticall, whose continual increase or mutual excess were 4, and the first number were 5, and I would know what the last and seventh number is; I multiply 6, which is one less then 7, (the number of the places) by 4, thereof cometh 24, which I adde to 5, that maketh 29, and that is the last number, which I desire to know. And this you may straightway probe by continuall proceeding from 5 till the seventh place, increasing every one by 4, as thus:

5, 9, 13, 17, 21, 25, 29.

Loe here the last, being also the seventh, is 29.

Scholar. I perceive already one good property in this Rule, which in all works is to be desired, that is, it will ease one from great labour, if a progression were propounded of a hundred or two hundred places, or moe: And also it is very easy to work, and most necessary for the total summe finding, in a very long progression.

M. It is true; and therefore now let me see if you can answer me this question by this proposition.

A Merchant buieth 50 pounds of Spices, and agreeth to pay for the first pound 4 pence, for the second 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The question is, how much he must pay for the last pound, and then how much the 50 pound cometh to.

Scholar. According to the proposition, I multiply 49 (which is less by one then the number of the places) by the excess, which is 3 : to the product 147 I adde the first number, which is 4, it maketh 151 pence, the price of the last pound. Now I adde 4, the price of the first pound, to 151, the price of the last pound, it maketh 155, which I multiply by half the number of the places, which is 25, the Product, 3875 pence, is the totall sum or price of the 50 pounds of Spices, as appeareth.

49 places 1 less	151 last
3 excess	4 first
147	155
4 first	25 half places
151 the last	775
	310
	3875 totall summe, which amounteth to
	li s d
	16——2——11

Master. It is truly wrought.

Scholar. Then I intreat you to proceed to your second proposition.

Master. *The second* Rule *is this; From the last subtract the first, the* remainder *divide by the common excess, to the* Quotient *adde* 1, *and you have the* number *of the places which you would know, as in this* Progression.

6, 11, 16, 21, 26, 31.

If I know onely 6 and 31, and that they encrease by 5, then, according to the Rule, from 31 I

subtract

subtract 6, there remaineth 25, which 25 I divide by 5, (the common excess) the Quotient cometh forth 5, to which I adde 1, that maketh 6: and so many are the places, as you see.

Scholar. This Rule is so easie, that I were much to blame if I could not remember it.

Master. *The third* Proposition *may always thus be solved. Multiply the* excess *by a* number *less by one then the distance of the place is from the* first *or the* last number given: *the offcome adde to the* first, *if the* distance *be reckoned from the* first, *and the* first *also known; or subtract from the* last, *if the distance be from the* last counted, *and the* last given *also: and that which cometh forth, either in that* Addition *to the* first *or* Subtraction from *the* last, *is the* number sought. As for example, *I propound you this* Progression; 3 Proposition.

8	15	22	29	36	43	50	57

And for the apt considering the manner of this question, I will note over every place his distance from the first, and under every place his distance inclusively from the last, thus:

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the excess whereby this progression standeth be known to be 7, and the first number given be 8, I would know what number standeth under 4, that is to say, in the fourth place. I multiply 7 by 3, (which is less by 1 then the number of the place proposed) that yieldeth 21, to which I adde 8, the first number, so cometh 29; which I say to belong to the fourth place, as you see in the example. Or if in the

third place from the laſt you would know what number in this example ſhould ſtand, the laſt number being known to be 57, and the common exceſs 7, then by 2, which is leſs by 1 then the place propounded, I multiply 7, that giveth 14, which appertaineth to the third place incluſively reckoned from the laſt: and ſo my example giveth you.

Scholar. I perceive right good uſe of this rule: for if I had forgotten what the firſt number were, and remember ſtill but the laſt, the common exceſs, and the number of the places, then might I come by the knowledge of my firſt number again.

And methinketh that it differeth not much from the firſt propoſition, ſaving that which you make here a middle number, there was made the laſt: and alſo in this point it differeth, that in it the laſt was onely ſought, and no conſideration had in numbring the places from the laſt, as here I mark in your numbers noted under your Progreſſion.

Maſter. And think you not, the middle numbers of progreſſion ſtanding off a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſs, or abating from the laſt continually the common exceſs, as the very final Numbers in a ſhorter progreſſion would doe?

Scholar. Yes, ſir, that I think right well, and therefore I am glad of this new framed propoſition, and the manner of the working of it.

4 Propoſition. Maſter. *The Rule of the fourth is this: Adde the firſt and the laſt together, and by the offcome divide the total*

total summe; *double the* quotient, *and that will be the* number *of the places.*

Scholar. Then if in a progression whose summe were 207, and the first number 12, and the last 57, I adde 57 and 12 together, that maketh 69, and by it I divide 207, the quotient will be 3, which I double, and so I have 6: and so many must be the number of the places that this progression standeth on.

Master. Whether it be so or no, how will you trie?

Scholar. Half 6, which is 3, being multiplied by 69, must make 207, the total summe, if 6 be the number of the places: For so the whole work of your Rule in summing any Arithmetical progression did inform me. I will then multiply 69 by 3, thus:

It cometh forth justly.

69
3
207

Master. I must much herein commend your promptness both in memory and in well applying your Rule; although in manifest words it did contain no such matter.

Scholar. Sir, I pray you hear me frame one example or more.

Master. I am well pleased, so that ye be short, for you make me stay longer here then willingly I would have done: but I cannot perceive how I could have omitted any thing as yet, without your great lack thereof.

Scholar. If I had received 85 pounds of certain men, but of how many I have forgotten, yet I remember that the first gave me 7 pound, and the last 27 pound, and every payment after other did rise by

A question of money.

by a like summe; and the man for whom I received this money conditioned with me, that of every payment I should have twelve pence for my labour: now unless I can by Art find the truth of this case, I am like to lose the most part of my reward.

Master. I perceive you can handsomely frame an example which should concern your own gain: I pray you let me see how you would do justice in this point.

Scholar. I adde the first and the last together, that maketh 34; by which I divide 85, thus:

1	
27	17
85	(2
34	34

Why, how now, Sir? here is a remnant of 17, in which 34 cannot be had: so that now I am in the briers for doubling of my Quotient, and farewell then both my Justice, and a good lump of my gains.

Master. You are never the farther from the matter, though it fall into a Fraction. For you shall understand that the Fraction which of any such work proceedeth, is every half of one such as the unites of the quotient before are. And that you may try, if you double that which so remaineth, for then it will be equal to your divisor; as if ye double 17, (the remnant) it maketh 34, and your divisor also was 34, this noteth the remainder to be half of one.

Scholar. Now I am glad of this hard Example; for with it I have a general rule for the Fraction that may hap in this work. So that the quotient being 2 and a half, I double that, it maketh 5; therefore should my gain be 5 shillings. And to be sure (by your leave) I will try it; for I will multiply

half

half of 34 (which is the first and last number joyned together) by 5, thus.

	17
	5
	85

It is most true (I see) that I should lose nothing by the former working.

Master. *The fifth Proposition hath this Rule appertaining unto it: By the fourth rule find the* number *of the places, that being done, from the last subtract the first, and the residue divide by a* number *less by 1 then the* number *of the places, and the* quotient *will shew the excess which is sought for.* 5 Proposition.

An example hereof shall be this: If ye had disbursed 685 pounds to a certain number of men, you neither can tell how many they were, nor how much the ones money exceeded his next before, but you are sure that the excess was equal between every two next; and also you remember that the first had 19, and the last 118 pounds: how would you find the number of the name and the excess continually observed in the succession of their payments? Example.

Scholar. Your Rule doth plainly bid, first to find the number of the places, which I will do according to the fourth rule: I adde 19 and 118 together, thus:

118	
19	
137	

By this 137 divide 685 thus:

Seeing there is no Fraction, but a whole number, being 5, I double that; and then must the number of the places be 10. Now from the last I subtract the first, as 19 from 118, thus: and so remaineth 99.

~~13~~	
~~685~~	(5
~~137~~	

118	
19	
99	

This 99 I divide by a number less by one then the number of the places; and seeing the places were 10, I divide

99 by 9, thus: 9/9

The quotient is 11, and so was the 99 (11
excess, if I have followed your rule right.

Master. You have wrought every part of this question both well in order, and truly in the practice of your Rules.

Scholar. I will then set it down also formably, so that the number of the places, the excess, and the total summe may straight appear, as your first example stood.

The common excess	11	11	11	11	11	11	11	11	11	11
The progression.	19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from the first to the last the common excess is 11, I perceive most evidently: but whether the total sum be 685, I have not yet proved, which I will now doe. I adde 19 and 118 together, that maketh 137; I multiply that by half the number of the places thus: 137

All things agree most exactly, so that I am 5
perfect enough in these Rules, if I forget 685
them not again.

Master. Use maketh all things perfect.

6 Proposition.

Your sixth Rule is this: By the number of the places divide the total summe, double the Quotient, and that will be the first and last joyned in one summe. Then by a number less by 1 then the number of the places multiply the excess, that off-come subtract from the first doubled Quotient, and the half of the residue is the first number. The last number you may diversly find out, as by the first of our 6 Rules, *or by subtracting this first number from the* summe *which here contained both the first and last jointly, or thirdly, by continual adding the excess.*

Scholar.

Scholar. I pray you make this somewhat more plain with an example.

Master. If every month in the year (counting them now as 13) you gained clearly 40 shillings more then you did the month next going before, and at the years end you find the whole gain 5720 shillings, but you remember not how much either the gain of the first month or the last was, by this Rule it may be tried out. Example of gain.

Scholar. So that here you seem to apply the 13 months to thirteen places, the 40 shillings every one more then the other next before it to be the common excess, and 5720 shillings to be the total summe.

Master. It is true: by 13 then I divide 5720 in this manner:

1
1 5
5 7 2 0 (440
1 3 3 3
1 1

I double this Quotient, so have I 880 for the first and the last summe joyned together; by 12, which is less by one then the number of the places, I multiply 40, (the common excess) so cometh 480.

40
12
80
4
480

This 480 I subtract from 880, so remaineth 400; half whereof is the first number which we desired to know, that is 200.

And as for the last number, I can give you it three ways. As by the first of my six rules, I multiply the excess by a number less by 1 then the number of the places, as 40 by 12, that giveth 480, which I adde to the first, being 200, so shall the last be 680.

The same summe cometh forth if ye subtract 200 from 880.

And thirdly, if I begin at 200, and so proceed,

increasing

encreasing by 40, I shall at the thirteenth place have 680, as thus:

200 240 280 320 360 400 440
480 520 560 600 640 680

Scholar. I thank you most heartily for these six Rules. Now, if it be your pleasure, I would hear and learn somewhat of progression Geometricall.

Master. There are yet very many Rules and Propositions which fall into this Arithmeticall Progression.

And for the use and practice of them, I will propose unto you certain pleasant and necessary Questions of Arithmetical progression, and to the performance of their workings, such necessary rules and documents as are requisite for the better understanding of them, or any such like.

A question of Velvet. *A certain* Mercer *sold 20 yards of Velvet to be paid in* 12 *weeks by* Arithmetical proportion, *that is to wit, to receive the first week* 6 shillings, *the second week* 12 shillings, *the third week* 18, *and so forth, encreasing the number of weeks by* 6 shillings, *till the twelfth and last week were expired.* The question is, how many pounds he had for 20 yards of Velvet.

6
12
18
24
30
36
42
48
54
60
66
72

To the performance of this question, and such other the like, I set forth the 12 payments in such sort, as for example here appeareth.

Then touching the adding together of these summs,

ſumms, without the aid of Addition, according to the rules I taught you in progreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the progreſſion, which is 72, and the firſt number together, make 78, and multiplying 78 by half the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 li. 8 s. And ſo much hath the Mercer for his 20 yards of Velvet, which is nigh about 23 ſhillings 5 pence a yard.

Scholar. I underſtand this work very well: but is there any proof for the juſtifying hereof, as you have of other works?

Maſter. The work of it ſelf being ſo perfectly wrought, that in your proceeding and going forward from number to number each number exceedeth his fellow by an equal or like quantity, is all that is demanded for juſtifying of the ſame: yet notwithſtanding, because your requeſt is reaſonable, I will propoſe an example for the proof hereof.

A certain man is bound to pay for 20 yards of Velvet the ſumme *of* 23 pounds 8 ſhillings, *and it is to be paid weekly in* 12 *weeks or terms by* Arithmeticall progreſſion. The queſtion is therefore, to know with what number the ſame progreſſion is to be begun and continued in ſuch equal proportion Arithmeticall, that in 12 weeks the ſame may be juſtly accompliſhed. The proof of the laſt queſtion.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings all into ſhillings, which maketh 468 ſhillings. A general rule.

Then adde 1 unto 12, the number of the terms,

it

it maketh 13; which 13 you shall multiply by half the number of the terms, which is 6, it maketh 78: then divide 468 by 78, and you shall find 6 in the quotient, which is the true number that shall begin and continue the said Progression: That is to say, the first week 6 shillings, the second 12 shillings, and the third week 6 shillings more, which is 18 shillings, and so every week as they rise, 6 shillings more then the week before; as is manifest in the question aforesaid.

A question of a Farm.

A Farm is to be sold to be paid by the weeks in a year; the first week to pay 4 shillings, the second week 8 shillings, the third week 12 shillings, and so forth, increasing each number by 4, till the number of 52 (which are the number of weeks in a year) are expired. The question is, what the price of the Farm cometh to.

Scholar. I doubt not but by that you have already taught me to end this question very well; wherefore I set forth the Progression with his excess 52 times.

Master. Nay, stay a while. And here for your farther ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in any other the like) If a question were proposed of 100 or 200 places, or moe, and that this question, or any other the like, cannot be ended, unless you may know absolutely what the last number of the Progression of the 52 place is, (or ought to be) I will give you a general rule how to know the last number of any Progression Arithmetical, as well as if you had ordinarily proceeded by continuall Addition, till you had come to the last work; which is this.

Mul-

Multiply the excess by a number less by one then the number of the places, and thereto put the first number of the Progression, and you shall have your desire. A general rule.

Scholar. This Rule is well worth the noting: for if I understand you aright, I consider that my excess is 4, which I multiply by 51, which is one less then the number of the places, and it maketh 204, whereunto I adde the first number of the Progression, which is 4, and then it is 208, which you say is or should be the last number of the Progression.

Master. This is a most approved truth, if there were never so many places.

Scholar. This Rule is so easie, that I were much to blame if I do not remember it. For by the benefit hereof I have such an ease and light into this excellent Art, that my first entrance doth seem to pass a great many mens farther study and longer continuance.

Master. Many moe Considerations could I propound you in these *Arithmetical Progressions*; but these are sufficient for a taste, to give you occasion to think that *Rules* of Knowledge and *Arts* are infinitely capable of enlargement.

Scholar. Happy were I if I did but well understand that which is already invented and written. But these things, in my simple fantasie, offer themselves to be greatly beneficial unto the aid of Progression. Therefore now I will goe forward with your question.

Now considering that the 52 and last place is 208, I adde thereunto the first number of the Progression,

gression, which is 4, it maketh 212, which I multiply by half the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the total summe or addition of this progression, which maketh 275 pound 12 shillings, as appeareth here by my Tables.

Master. I like well your labour, and commend you for your diligence: I will here propose one example more, and therewithall for this time will end progression Arithmeticall.

A question of Holland

A certain man bought 20 Ells *of* Holland, *to be paid in* 17 *weeks, or terms, by* progression Arithmeticall, *and the first week to pay* 1 shilling 8 pence, *the second week* 3 shillings 4 pence, *the third week* 5 shillings, *the fourth week* 6 shillings 8 pence, *and so forth, each week succeeding,* 20 pence *more then the week before.* The question is, what the summe of his 20 Ells cometh to.

Scholar. Because here is mention made both of shillings and pence, I fear there is some harder matter contained herein then in the other before: therefore I pray you work it your self, and I will diligently mark your labour.

Master. There is no more to be done in this then in the other before: but because your request is so reasonable, be attentive unto me.

First, by the general Rules, I seek to find out the last Number of the 17 place, what this progression ought to be. Therefore here in my Tables multiplying the excess 20 by 16, which is one less then the number of the terms or places, it cometh to be 320; and thereunto adding the first number of the progression, which is 20 pence,

pence, all is 340 pence, or 28 shillings 4 pence, for so much ought the last number of the payments to be.

Then finally, to know what the whole 17 places amount unto, I adde the first number of the progression and the last together, which make 360. Now because 17 is an odde number, whose half cannot be taken, I take the half of 360, which is 180, and multiplying 180 by 17, cometh to 3060 pence, which maketh, as you see by Division, 12 pound 15 shillings. And so much is the buier to pay for his 20 Ells of Holland. Which 3060 pence if you divide by 20, the number of Ells that was bought, you shall find 12 shillings 9 pence; and so much payed he for an Ell one with another.

The Proof.

A certain man doth owe 12 pound 15 shillings, *to be paid in* 17 weeks, *or* terms, *by* Arithmetical progression. *The* question *is, to know with what* number *he shall begin and continue the* progression *in such equal proportion, as the same may be truly paid and satisfied in* 17 weeks. A question of Debt.

The Answer.

First, I reduce 12 pounds 15 shillings all into pence, which, as you see here in my Tables, make 3060 pence; that I let stand by a while.

Then I adde 1 to 17, the number of the places or terms, which maketh 18, which I should multiply by half the number of the weeks or terms,

which

which is 8 ½, which 8 ½ multiplied by 18 cannot well be done, unless you were acquainted with Fractions or broken numbers; therefore you shall let that pass, and multiply 17 by the half of 18, which is 9; (for that is all one with the multiplication of 8½) and the multiplication of 9 into 17 maketh as you see 153, with which number you shall divide the 3060 pence beforesaid, and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression withall: and so each week succeeding to rise 20 pence more then the week before, and thereby in 17 weeks shall 12 pounds 15 shillings be paid, as before was sufficiently declared. Thus much for Progression Arithmeticall.

Scholar. Certainly, Sir, I know not how to render you condign thanks for these benefits shewed me, which methinketh are so easie, delightfull and pleasant, that I count my self happy to be in your company.

Master. I am glad you delight so well herein, which is an Art of wonderful dexterity to all sorts of men, of what degree or profession soever they be. And now will I proceed to Progression Geometricai, wherein I will be more brief, both because I have been so long in this part of Arithmetical progression, and also for that it would require the knowledge of Roots and surd numbers, (whereof ye have learned nothing) if I should frame the like propositions in them as I have done in these. Therefore I will onely teach you to practice about it, and so end the considerations and works of these progressions.

Progression

Progression.

PRogression Geometrical *is when the numbers increase by a like proportion, that is, if the second number contain the first 2,3, or 4 times, and so forth, then the third containeth the second so many times also, and so the fourth the third, and the fifth the fourth: wherefore I set these 3 examples.* Progression Geometrical.

	3	6	12	24	48
	1	3	9	27	81
	2	10		50	250

Here in the first Example you see that every number containeth the other (that goeth next before him) two times, and in the second example three times, and in the third example five times. Now if you will know how to find easily the summe of any such number, doe thus: Consider by what numbers they be multiplied, whether by 2,3,4,5, or any other; and by the same number multiply the last summe in the Progression.

Scholar. I pray you work it by this example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continually multiply by 4.

Master. Then must I multiply the last summe (which is 2048) by 4 also, and it will be 8192. Now must I abate from this summe the first number of the progression, which here is 2, then resteth 8190, which summe I must divide by 1 less then was the number that I multiplied by. Seeing then I multiply by 4, I must divide by 3; so dividing 8190 by 3, the quotient will be 2730, which is the summe of all the progression. And now to prove whether you can doe the same, I give you these numbers to adde by this Rule, 3, 15, 75, 375, 1875, 9375, 46875. To find the total summe in any Geometrical Progression.

Scholar. I cannot well tell by what number this progression doth increase.

Master. In any such doubt doe thus: Divide the second number by the first, and the Quotient will shew you the number that engendreth the progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now work as I taught.

Scholar. The last number is 46875, which I multiply by 5, and it yieldeth 234375; from which I abate the last number of the progression, that is 3, and there resteth 234372, which I divide by 4, for that is one less then 5, and the Quotient is 58593; which is the whole summe of the progression.

Master. If you remember well this, you have learned the Art of progression both Arithmetical and also Geometrical: which you may prove either by subtracting of each number alone from the summe, and so will there nothing remain; or else by adding together of all the parcels, for so will the same summe amount.

A question of Satten.

A Mercer hath 12 yards of Satten, which he valueth at 16 shillings the yard, and selleth the same 12 yards to another man to be paid as followeth; that is to wit, for the first yard to have one shilling, for the second yard two shillings, for the third yard four shillings, for the fourth yard eight shillings, &c. doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargain of it, the buier or seller.

First, you may set down 12, the number of the yards, as you see here in this Example; and against each number the number of shillings due to be paid,

as

as the order of Duplation or Multiplication by two teacheth.

Then resort I to the adding up or summing of this Progression, where I consider that the increase of this summe proceeded by the Multiplication of 2, and therefore after I have drawn a line under the 12, I work and multiply the last summe by 2 also, and it yieldeth 4096; from whence I abate the first number of the progression, which is 1, and then resteth 4095: which I should divide by one less then I did multiply by, but seeing it is 1, I need not to divide it, for 1 (as I have said before) doth neither multiply nor divide; therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction amounteth to 304 pounds 15 shillings: and so much hath the Mercer for his twelve yards of Satten, which is 17 pound, 1 shilling 3 pence a yard. But I think you will buy none so dear.

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	

Scholar. No, Sir, by the grace of God, this year.

Master. *Then what say you to this* question: *If I sold unto you an horse having 4 shoes, and in every shoe 6 nails, with this condition, that you shall pay for the first nail one* ob. *for the second nail two* ob. *for the third nail four* ob. *and so forth, doubling untill the end of all the nails:* Now I ask you how much would the price of the horse come unto?

A question of an Horse.

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	

Scholar. First, to know the number of the nails, I must multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nails every one in order from 1 to 24, and against each number of the nails the summe of half-pence duly, as the order of Duplation or Multiplication by 2 teacheth, and as in the next figure following appeareth.

Then do I resort to the Rule of summing up the Progression, where I consider that the increase of this summe proceedeth by the multiplication of 2, as the last Example did; and therefore multiply the last summe by 2 also, and it yieldeth 16777216; from which I abate the first number, which is 1, and then resteth 16777215; which I should divide by one less then I did multiply, but seeing that it is 1, I need not to divide it, for 1 (as you have before said) doth neither multiply nor divide: therefore I do take the number 16777216 for the whole summe of the half-pence, which by Reduction I find to be 699050 shillings and 7 pence half-peny, that is 34952 pounds 10 shillings 7 pence ob.

Master. That is well done: but I think you will buy no horse of the price.

Scholar.

Scholar. No, Sir, if I be wise.

Master. Well then, answer me to this Question.

A Lord delivered to a Bricklayer a certain number of loads of Brick, whereof he willed him to make twelve walls, of such sort, that the first wall should receive two thirdels of the whole number, and the second two thirdels of that which was left, and so every other two thirdels of that that remained: and so did the Bricklayer: and when the 12 walls were made, there remained one load of Brick.

Now I ask you, how many load went to each wall? and how many load was in the whole?

Scholar. Why, Sir, it is impossible for me to tell.

Master. Nay, it is very easie, if you mark it well. Mark well that I said, that every wall should receive two thirdels of the summe that was left. Now take away two thirdels from any summe, and you must needs grant that that which remaineth is one thirdel of the summe last before. Example of 9, from which if you take 2 thirdels, there will remain 3, which is 1 thirdel of 9; likewise from three bate two thirdels, and there remains 1.

Scholar. This is true; and now I perceive the least wall had but two load of brick.

Master. And by the same reason may you know how many load every wall had, according as this figure following doth shew, and likewise what the whole summe of bricks was: for if you make 12 summes, multiplying by 3 still from the last remainer, as you may see here on the left side of the Table, there will appear all the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the summe of

 the

the loads which were delivered to the Bricklayer.

The remainer after every wall.			*Loads due to each wall.*
	1	12	2
	3	11	6
	9	10	18
	27	9	54
	81	8	162
	243	7	486
	729	6	1458
	2187	5	4374
	6561	4	13122
	19683	3	39366
	59049	2	118098
	177147	1	354294

Summe of the Loads delivered, 531440.

Again, if you double every Remainer, as you may see at the right side of this Table, those numbers will shew the summe of loads that went to each wall, whereby you may perceive that each wall was three times so great as the next less.

Scholar. Lo now it appeareth easie enough. Now surely I see that Arithmetick is a right excellent Art.

Master. You will say so when you know more of the use of it: For this is nothing in comparison to other points that may be wrought by it.

Scholar. Then I beseech you cease not to instruct me farther in this wonderful cunning.

The

The Golden Rule, or Rule of Proportion direct, called *The Rule of Three.*

Maſter.

The Rule of Proportion.

BY *order of the Science (as men have taught it) there ſhould follow next the* Extraction of Roots of Number; *which becauſe it is ſomewhat hard for you yet, I will let it paſs for a while, and will teach you the feat of* the Rule of Proportion, *which for his excellency is called* the Golden Rule. Whoſe uſe is, by three numbers known to find out any other unknown, which you deſire to know, as thus.

The Golden Rule.

If you pay for your Board for three months ſixteen ſhillings, *how much ſhall you pay for* eight months?

Queſtion of Boarding.

To know this and all ſuch like queſtions, you ſhall conſider which two of your numbers be of one Denomination, and ſet thoſe two one over the other, ſo that the undermoſt be it that the queſtion is of: as in my queſtion, 3 and 8 be both of one Denomination, for they both be months; and becauſe 8 is the number that the queſtion is asked of, I ſet the one over the other, and 8 undermoſt, thus, with ſuch a crooked draught of lines.

3
8

Then do I ſet the other number, which is 16, againſt 3 at the right ſide of the line, thus:

3 16
8

Note.

And now to know my queſtion, this muſt I doe: I muſt multiply the lowermoſt on the left ſide by that on the right ſide, and the ſumme that amounteth

I muſt divide by the higheſt on the left ſide: or, in plainer words, thus; I ſhall multiply the number of which the queſtion is asked (which is called the third number) by the number of another Denomination (which is called the ſecond,) and the ſumme that amounteth muſt I divide by the ſumme of like Denomination, (which is called the firſt.) Then for the knowledge of this queſtion, I multiply 8 into 16, and there amounteth 128, which I divide by 3, and it yieldeth 42 ſhillings, and 2 ſhillings remaineth, which I turn into pence, and they be 24 pence, of which the third part is 8 pence: ſo the third part of 128 ſhillings is 42 ſhillings 8 pence, which ſumme I write at the right hand of the figure againſt 8, thus.

The third number. The ſecond number. The firſt number.

3 ⁄ 16 ſhillings.
8 ⁄ 42 ſhil. 8 pence.

Hereby I know that if three months boarding do come to 16 ſhillings, that 8 months boarding will come to 42 ſhillings 8 pence; and likewiſe of any other like queſtion.

But here muſt you mark, that the firſt number and the third be of one Denomination, and alſo the ſecond and the fourth, for which you ſeek; or elſe be of ſuch Denominations, that you in working may bring them into one. As if a man ſhould ask me this queſtion:

Queſtion of expence.

Twelve weeks journeying coſt me 14 French Crowns at 6 ſhillings the piece; how many pounds is that in one year? Here you ſee no two numbers of one Denomination, but yet in working you may turn them into like Denomination; as thus: turn the one year into 52 weeks, and the fourth ſumme will be French Crowns, by the order of the working. Then to know this queſtion, multiply the third ſumme 52

by

by the second 14, and the summe will be 728: that divide by your first number 12, and the Quotient will be 60 Crowns, and 8 Crowns remaining; which if you turn into shillings, they will be 48 shillings, which if you divide by your first number 12, the Quotient will be 4, which signifieth 4 shillings: put those 60 French Crowns, which make 18 pounds, with the 4 shillings, for the summe that answereth to the question, and it is the just expences of a year; and the work will be thus.

W	C
12	14
52	$60\frac{2}{3}$

And take this evermore for a general rule touching this whole Art, that the doubtful or unknown number that you would be resolved of shall alwaies be set in the third place. Note also, the first number and the third must ever be of one nature and Denomination, or else must in working be brought to like Denomination; and then of necessity must the other number be in the second place. A general Rule.

Remember also, that the place of the first number is highest on the left side, and the place of the second right against it on the right side, the place of the third number is under the first; as by those examples you have seen.

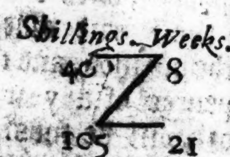
Scholar. This I trust I can doe.

Master. But and if the question be asked thus, *In 8 weeks I spend 40 shillings; how long will 105 shillings serve me?* Here you see that 8 weeks answers himself, and saith 40 shillings.

But how long time 105 shillings will serve you know not. Therefore you shall set 105 in the third place, according as I told you even now. And the first

first place must alwaies be of the same nature or denomination that the third is of, which here is 40: then must 8 needs be that other. Now multiply 105 by 8, and it will be 840; which if you divide by 40, it will yeeld 21, which is the fourth number, and sheweth how many weeks 105 shillings will serve, if you spend 40 shillings in 8 weeks.

The figure of this question is this: as if you should say, if 40 shillings serve for 8 weeks, 105 will serve for 21 weeks.



Other diversities there be of working by this Rule, but I had rather that you would learn this one well, then at the beginning to trouble your mind with many forms of working, sith this way can doe as much as all the other, and hereafter you shall learn the other more conveniently.

Note. ¶ And for your farther aid and instruction, to make you better acquainted with this *Golden Rule*, I have here proposed six Questions and their Answers, which I think most convenient and meet to prefer to the desirous to perfect understanding. The first four are all branches of one Question sprung out of the best tree (for a young learner to tast of) that groweth in this *Ground of Arts*; for that no manner of Question in the Rule of Three whatsoever it be can be proposed, but it must be comprehended under the reason or style of one of these four.

The

The Questions.

If 15 *Ells of Cloth cost* 7 pound 10 shillings, *what come* 27 *Ells to at that rate?* Answer. 13 pounds 10 shillings.

If 27 *Ells cost* 13 pound 10 shillings, *what are* 15 *Ells worth?* Answer. 7 pound 10 shillings.

If 27 *Ells cost* 13 pound 10 shillings, *how many Ells shall I have for* 7 pound 10 shillings? Answer. 15 *Ells.*

If I sell 15 *Ells for* 7 pound 10 shillings, *how many Ells are to be delivered for* 13 pound [illegible]ings? Answer. 27 *Ells.*

If 8 *pound of any thing cost* 16 shillings 6 pence, *what mony is to be received for* 49 pound? Answer. 5 pound 1 shilling 0¾.

If 4 *pound of any thing cost* 7 pence, *what mony will* 8765 *pound of that commodity cost?* Answer. 63 pound 18 shillings 2 d ¾.

Of all which questions I omit the work, of purpose that you may whet your wit thereby at convenient leisure, to climb each branch, and gather the fruit of them; and do mind now, before we make an end of this Rule, to give you some instruction of the backer Rule of Three, whose order is quite contrary to this that you have learned.

Scholar. I thank you heartily for these Questions, which I will (God willing) practice at convenient times: I pray you proceed therefore to the Backer or Reverse Rule.

The

The Golden Rule, or Rule of Proportion Backward, or Reverse.

Master.

Note this well.

IN the former, evermore look how much the third number is greater then the first, so much the fourth number is greater then the second: and contrariwise, look how much the first summe is greater then the third, (if it do chance so) so much is the second summe greater then the fourth.

The Backer or Reverse Rule of Three.

But in this Rule there is a contrary order, as this; That the greater the third summe is above the first, the lesser the fourth summe is beneath the second: and this Rule therefore you may call the *Backer* or *Reverse Rule*. As in example.

A Question of Cloth.

If I have bought 30 yards of Cloth of two yards breadth, and would have Canvas of three yards broad to line it withall, how many yards shall I need?

Scholar. Why, there is none so broad.

Master. I do not care for that, I do put this Example onely for yo[illegible]standing: [illegible] if I should put the [illegible] other measures, it would be harder to understand. But now to the matter: If you would know this question, set your numbers as you did before: but you shall multiply now the first number by the second, and that ariseth thereof you shall divide by the third. Which thing if you doe here, I mean if you multiply 30 by 2, it will

will be 60: which summe if you divide by 3, there will appear 20. Whereby I know, that if 30 yards of cloth of two yards broad should be lined with Canvas of three yards broad, 20 yards of Canvas would suffice, as this figure sheweth.

Breadth. Length.
2 30
3 20

And now because you found fault with my Example, how say you, perceive you this?

Scholar. Yes, Sir, I suppose.

Master. *Then answer me to this question: How many Ells of Canvas of Ell-breadth will serve to line twenty yards of Say of three quarters broad?*

Scholar. In good faith, Sir, I cannot tell, for I know not how to bring the summes to like Denominations.

Master. Then will I tell you: Sith there is mention here of quarters, and again every one of the measures both Ells and Yards may be parted into quarters, part them so both in the breadth and length, and then put forth the question by quarters.

Scholar. Then shall I say thus; How many quarters of Canvas of 5 quarters broad will line 80 quarters of 3 quarters broad?

Master. Now answer to the question.

Scholar. First, [illegible] them down in [illegible] thus: for 5 is joyned with the question, and is therefore the third number, then is 3 the number of the same Denomination, I mean because they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I divide by 5, and it yieldeth 48.

Breadth. Length.
3 80
5

Then

Then say I, that 48 quarters of 5 quarters broad will suffice to line 80 quarters of three quarters broad.

Master. Turn the quarters again into Ells and yards.

Schol. Then I say, that 9 Ells and three quarters of a yard of ell-broad will serve to line 20 yards of three quarters broad, as this figure sheweth.

Breadth.	Length.
3	80
5	48

Master. *Now what say you to this question? I lent my friend* 400 pound *for 7 months; how much mony ought he to lend me again for 12 months to recompense my courtesie shewed him? Can you answer to this?*

Scholar. Yes, Sir, I suppose, for I will set down my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I divide by 12, and it yieldeth 233 pound, and there is 4 pound remaining of my Division, what shall I doe therewith?

Months.	Pounds.
7	400
12	

Master. Turn the same 4 pound into shillings, and then divide it by 12, as you did before.

Scholar. Well, Sir, [illegible] be done: so have I 6 shillings for my Quotient, and yet remaineth 8 shillings upon my Division.

Master. You must also reduce that 8 shillings into pence, which maketh 96, and divide that also by your Divisor.

Scholar. So have I done, and I find 8 pence for my quotient, and nothing is left.

Master.

Master. This must you alwaies doe when any thing remaineth upon your Division, whether it be money, weight, measure, or any kind of thing whatsoever. This Rule is so profitable for all estates of man, that for this Rule onely (if there were no more but it) all men were bound highly to esteem Arithmetick.

By this Rule may a Captain in war work many things, as Master Diggs in his Stratioticos doth declare. Onely now in this my simple addition, for a tast and incouragement, I will inlarge the Author with a question or two more, wishing you and every my Countreymen or Gentlemen whatsoever, that by nature be any thing given to Military affairs, to be familiar and acquainted with this Excellent Art, the which he shall find not onely at the Sea, but also in the Camp and Field-service, abundantly to aid him, either in Fortification, paying of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Instruments whatsoever. As for example:

If it should chance a Captain which hath 40000 souldiers to be inclosed with his enemy, that he could have no fresh purveyance of victuals, and that the victuals he had would serve that Army but onely three months, how many men should he dismiss to make the victuals to suffice the residue 8 months? Question of an Army.

Scholar. As you taught me, I set the numbers thus, saying, If three months suffice 40000, to how many will 8 suffice?

Months,	*Men.*
3	40000
8	

To know this, I multiply the first number 3 into the

the ſecond 40000, and it yieldeth 120000; which ſumme I divide by 8, and there will be in the quotient 15000; which if I do ſubtract from 40000, the remainer will declare that he muſt diſmiſs 25000, as this figure ſheweth.

Months. *Men.*
3 40000
8 15000

A queſtion of a Fort.

Maſter. *Now anſwer me to this queſtion: If 136 maſons in a month be able to build a Fort to preſerve the ſouldiers from the Enemy, and ſuch expedition requireth that I would have the ſame finiſhed in eight days, how many workmen ſay you is there to be appointed?*

Scholar. As you taught me, I ſet the numbers thus, ſaying, If 28 days require 136 Maſons, what number of men by the like proportion will 8 days require?

28 136
8

To know this, I multiply the firſt number 28 into 136, and it yieldeth me 3808, which I divide by 8, and my quotient is 476: which is the juſt number of Maſons that ſhall ſupply this work. And now methinks theſe queſtions are very eaſie.

Maſter. Truly if you take delectation herein, you ſhall find this Art not onely eaſie, but wonderfull pleaſant and profitable. Now therefore one queſtion more I will propoſe, and ſo leave off this Rule in whole numbers, untill we come to the uſe of it in broken numbers: for had you the underſtanding of broken numbers preſently, not onely in this Rule but in all other, the queſtion that in the ſight or appearance ſeemeth to be 100 times harder to reſolve, may thereby be wrought as ſoon or ſooner then this.

Scholar.

Scholar. Your words do greatly incourage me to be studious to attain whole numbers; but might I once attain to be a Practitioner in broken numbers, I should think my self happy.

Master. *What say you then to this question? If 48 Joyners in two days make 200 light horsemen staves, (esteeming they work but 12 hours a day) and such need requireth that 384 Joyners are set to the finishing of those 200 staves; in what time, say you, will they make them up?*

Scholar. I see here that I must turn my 2 days into hours: And so doing, I set my numbers thus;

48 Z 24
384

Saying, if 48 men are 24 hours, 384 men will make an end quickly. For it is grounded upon an old Proverb, *Many hands make light work.*

I multiply 48 into 24, and it amounteth to 1152, which I divide by 384, and my quotient is 3 hours, which is my desire.

I take this for a Note worthy the marking, either in the Rule of Three forward or backward, When the two numbers are multiplied together, the product is of the same nature and determination that the second number is of. Note.

The double Rule of Proportion direct.

Master.

The double Rule.

WEll, sith you perceive now the use of this Rule, I will shew other which insue of the same, and first the *Double Rule*, which is so called, because there is in it double working, by which thing onely it differeth from this.

Scholar. Then by an example I shall understand it well enough.

Question of carriage.

Master. *So shall you, and let this be the example: If the carriage of 100 weight (that is 112 pound) 30 miles do cost 12 pence, how much will the carriage of 500 weight cost being carried 100 miles?*

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus: If C weight cost 12 pence, how much will five hundred weight cost? Set your figures thus:

C weight.	*Pence.*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you divide by one, the Quotient will be still 60. That is the price of 500 weight for 30 miles.

Then begin the second work, saying, If 30 miles cost 60 pence, how much will 100 miles cost? Set your figures thus:

Miles.	*Pence.*
30	60
100	

Then multiply 100 by 60, whereof amounteth 6000, which being divided by 30 will yield 200 pence. Then you must say, that so many pence shall cost the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceive it also.

¶ *Master.* These, and such other like questions of the double Rule of Three, are to be answered much sooner at one onely working by the Rule of proportion composed of five numbers, which anon I will shew you; and then when you have the use thereof, you may use it which way you think good.

Scholar. Sir, I thank you much for your courtesie. And I long now till this Rule be ended, that I may see how I may behave my self with that new Rule of five numbers; for that I have ever since you taught me hitherto in the Golden Rule, both forward and backward, wrought but with three numbers onely.

Master. But yet awhile we will go on forward with this Rule of Three; therefore answer to this question.

Thirty bushels of wheat sowed yielded in one year 360, how many will 80 bushels yield in 7 years? I mean sowing every year of those seven still fourscore bushels. Question of Sowing.

Scholar. First I say, that if 30 bushels will yield 360 in one year, then 80 bushels will yield 960 in one year. Then for the second work, I say, If one year yield 960, then 7 years will yield 6720; as these figures do shew.

Seed.	Encrease.	Year.	Encrease.
30	360	1	960
80	960	7	6720

A question of Corn.

But now, Sir, if I set forth 30 bushels of Corn to another man for 7 years, agreeing so, that he shall sow every year the whole increase of the Corn, and I at the end of these 7 years to have the half of the whole increase: I would know how many bushels will there amount to my part, supposing the increase to be after the rate of the last question, for 30 bushels in one year to yield 360.

Master. In such a question you must have so many several workings as there be years. As for Example, in the first year thirty bushels yield 360: Then to know the yielding of the second year, I must say, If thirty yield 360, how many yield 360? Work by your Rule, and you shall find 4320. Then say for the third year, If thirty yield 360, how many will 4320 yield? You shall have 51840. And so every year multiplying the whole increase by 360, and dividing it by 30, the increase of the next year will amount as these 7 figures following do orderly declare: where I have set 7 letters for the 7 years, of which the first is set without Art, because that is the increase which you do presuppose; and the last number of each other doth shew the increase of that year that it standeth for, which the letters do declare. So that the increase of the seventh year is 1074954240 bushels. How many quarters that is, and also how many weyes, you may by Reduction soon find.

a

30 —— 360

b

30 Z 360
360 Z 4320

c

30 Z 360
4320 Z 51840

d

30 Z 360
51840 Z 622080

e

30 Z 360
622080 Z 7464960

f

30 Z 360
7464960 Z 89579520

g

30 Z 360
89579520 Z 1074954240

Now with one question more I will prove you. If *six Mowers do mow 45 Acres in 5 days, how many mowers will mow 300 Acres in 6 days?* Question of Mowing.

Scholar. If 45 Acres require 6 Mowers, then 300 Acres require 40. Now again, if 5 days require 40 mowers, then 6 days need but 33 mowers.

Master. Why do you not make mention of the 2 that remaineth in the last Division? for the last part of the question is wrought by the Backer Rule, where the first number 5 is multiplied into the second, that is 40, whereof amounteth 200, which if you divide by the third number 6, the Quotient will be 33, as you said: but then will there remain 2, which cannot well be divided into 6 parts. Howbeit you may understand by the 6 part of 2 the third part of one mans work, which you must put to

to the 33; or else you must say that 33 Workmen will end all the 300 Acres in 6 days save 2 mens work for one day, or two days work for one man. But such broken numbers called Fractions you shall hereafter better perceive, when I shall wholly instruct you of them.

Master. *Yet one question more of field matters I will propose, and so I will make an end of this double Rule of Three.*

Scholar. With all my heart, Sir, I thank you, and I will dispatch it as soon as I can, because I would fain see the order of the next Rule of 5 numbers.

Question of Entrenchings.

Master. *If a Captain over a band of men did set 300 Pioneers awork, which in eight hours did cast a Trench of 200 Rods: I demand how many Labourers will be able with a like trench in three hours to intrench a camp of 3400 Rods.*

Scholar. I think I am now in the Backhouse ditch, for I know not well which way to go about it. And besides that, truly I think I shall never come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and service into the favour of your Prince, for the avail of your Countrey.

Example for Navigation: Sir *Francis Drake*, a man greatly honoured for his knowledge, was not the tallest man, and yet hath made as great an adventure for the honour of his Prince and Countrey as ever Englishman did.

Scholar. Sir, I thank you for your good incouragement. My minde, though I be little, is

is as desirous of knowledge as any other: I have pondred now a little of it, and thus I set forth the work.

Rod. Men.
200 300
3400

Saying, If 200 Rod require 300 men, whall shall 3400 rods require? I multiply 3400 by 300, and it yieldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then must I say for the second work, If in 8 hours 5100 men be able to discharge it, how many shall perform the same in 3 hours? Now if I would work by the Golden Rule of Proportion forwards, I should finde a less number of men; because 3 hours is less then 8 hours: but because reason teacheth me, that the lesser the time is wherein the trench must be made, the more Labourers I ought to have, thereupon I use now the Backer Rule, as in example. And I have in my quotient 13600. So many Pioneers must I have to intrench the Camp in 3 hours.

Master. You have answered the question very artificially; and truly I commend you for your diligence and apt understanding. And now, according to my promise, I will (in whole numbers) give you a little tast of the Rule of Proportion compounded in 5 numbers.

The Rule of Proportion, composed of 5 Numbers.

The first part of the Rule of Proportion compound, direct.

THe Rule of Proportion composed is distinct for most needfull questions into several parts or workings: And there belongeth unto it alwaies five numbers, whereof in this Rule, being the first part, the second number and the fifth are alwaies of one nature and like denomination, which Rule is to be wrought thus. You must multiply the first number by the second, and that shall be your Divisor: Then again multiply the other 3 numbers the one by the other, and their product shall be your Dividend.

And now, according to my promise, we will first work the question of Weight and Carriage, which I delivered you in the double Rule of Three, to be absolved by this Rule, which was this.

If the carriage of 1 C. weight 30 miles cost 12 pence, what will the carriage of 5 C. weight stand me in being carried 100 miles?

C. weight.	Miles.	Pence.	C. weight.	Miles.
1	30	12	5	100

Now mark well how these five numbers stand. Then multiply the first number by the second, as 30 by 1, which maketh but 30; that number keep for your divisor. Then multiply the other 3 numbers the one into the other, that is to wit, 12 by 5, which maketh 60. Lastly, 60 by 100, which, as you

you see here in our Tables, ariseth to 6000; which 6000 you shall divide by the Product of the two first numbers, which here is 30. And you see there is found 200 pence, which is the duty that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agreeth with the conclusion of the double Rule of Three.

Scholar. Sir, I thank you, it is even so.

Master. *Yet note this in a generality in this Rule; Look what nature of denemination your middle number is of, (which here are pence) and of the like denomination or nature is alwaies your quotient.* No

Scholar. Well now and if it please you, by your patience, I will see how I can end the question next following, of 30 bushels of wheat sowed, which in one year yieldeth 360; how many then will 80 bushels yield in 7 years following, every year of those 7 still 80 bushels? And according to your reasons I set my numbers thus.

Bush. Year. Bush. Bush. Year.

30——1——360——80——7

80
28800
7
201600

There I multiply 30 by 1, and it maketh 30 my Divisor; then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600, which I divide by 30: and my Quotient is 6720 bushels, my desire: for so much also it came to at two workings by the Rule of Three.

Master,

Master. Yet one question more I will propound unto you, and so leave this Rule, till it please God hereafter that I may make you work it in broken numbers.

Question of Interest. *What comes the interest of* 258 pound *for* 5 *months to, after the rate of* 8 pound *taken in the* 100 pound *for* 12 *months?*

Scholar. Sir, this is yet within the compass of some reasonable sciance. Therefore to minister equitie in this case, I will see how I can work the same, which I set down thus, praying you, if I have not done well, to shew me mine errour.

li. *moneths*. li. li. *mon.*
100—12-8-258-5.

Master. Proceed, you have done very well.

Scholar. Then I doubt not by the grace of God but to end it. I multiply 100 by 12, it yieldeth 1200, and the three other numbers multiplied together produce 10320, which I divide by 1200: and my quotient is eight pounds. Then, according as you have taught me heretofore, I turn the 720 pound that I left into shillings; and dividing it by the first number, my quotient is 12 shillings. So I answer, the loan of 258 pounds for 5 months, after the rate of 8 pound in the 100 pound for a year, comes to 8 pound 12 shillings.

Master. You say true, I commend your diligence. Now behold the manner of the second part of this Rule.

The Backer Rule, or the second part of the Rule of Proportion compound.

Master.

IN the second part of this Rule of Proportion composed, the third number is like unto the first. And the Rule is to be wrought thus: You shall now, contrary to the last Rule, multiply the third number and the fourth together; and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first; and that is the number that shall be divided. For example, I propound this question for a proof of my last question of Interest:

A Merchant hath received 8 pound 12 shillings *for interest of certain money for 5 months term, which he received after the rate of eight* pound *in the* 100 *for a year. The question is now, how much money was delivered to raise this interest?*

The proof of the last question.

Behold therefore the manner how the question is set forth.	li.	*months.*	li.	*months.*	li.	s.
	100	12	8	5	8	12

Scholar. Sir, I perceive it very well: and according to the doctrine which you prescribed for the working thereof, if it please you, now it is set down, I think I can follow the work.

Master.

Master. Nay, stay a while, and before you work, mark well how I deliver a reason for the perfect understanding of this Rule, which is thus: *If 8 pound in 12 months do yield me 100 pound, to take 8 pound 12 shillings for five months must needs yield a great deal more.*

Note.

So upon the knowledge that I have in this Art, the first part of this Rule is answerable to the Rule of Three forward; and this latter part accordeth to the rule of Three backward.

Scholar. Sir, I yield you most hearty thanks for these your last instructions, they have given me great light into these two Rules, whereby I may the better by deliberation conceive how to use them hereafter when occasion shall require.

Note.

Master. You say well: goe to now, if you will, and try your cunning in the question. But this note take with you by the way; Inasmuch as here is mention made of shillings, turn all your money as you work into shillings, for your more ease in working.

Scholar. If it please you to behold me a little, I will quickly end it: for I have but my first, my second, and my last number to be multiplied together for my Dividend; and my third into my fourth for my Divisor.

li. *Months.* *Months.* li. s.

100——12 8——5 8——12
20 20 20

2000 160 172
12 5

4000 800
2000

24000
172

48000 4128000 (
168000 8 00
24000
——4128000

Which 4128000 I divide by 800, and my Quotient is 5160 shillings, which in pounds yieldeth 258, my desire.

Master. I will here for this time in whole numbers end this rule, and I will instruct you in the Rules of fellowship. You may at your convenient leisure for your exercise work the same by the Rule of Three at twice. And for your aid and incouragement therein, I set down here a profer how to apply it.

	A			B
Months.	li.	s.	*pounds.*	li.
5	8——	12		100
12	$412\frac{4}{5}$		$412\frac{4}{5}$	258 li.

The

The Rule of Fellowship.

The Rule of Fellowship without time.

But now will I shew you of *the Rule of Fellowship* or *Company*, which hath sundry operations according to the divers number of the Company. This Rule is sometime without difference of time, and sometimes there is in it difference of time. First I will speak of that without difference of time, of which let this be an example.

A question of Company.

Four Merchants of one Company made a bank of money diversly; for the first laid in 30 pound, *the second* 50 pound, *the third* 60 pound, *and the fourth* 100 pound: *which stock they occupied so long, till it was increased to* 3000 pound. *Now I demand of you what should each man receive at the parting of this money.*

Scholar. I perceive that this Rule is like the other, but yet there is a difference which I perceive not.

Master. Then will I shew it to you. First, by Addition you shall bring all the particular summes of the Merchants into one summe, which shall be the first summe in your working by the Golden Rule, and the whole summe of the gains by that stock shall be the second summe. Now for the third summe, you shall set the portion of each man one after another, and then work by the Golden Rule, and the fourth summe will shew you each mans gains. As in Example.

30
50
60
100
240

The

The parcels of the four Merchants make in one summe 240 pounds: set that in the first place, the gains in the second, and the first mans portion of stock in the third place, thus;

240 Z 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appear 375 pounds, thus:

And that is the gains for the first man.

240 Z 3000
30 375

Now for the second man, set the 50 pound that he brought in the third place, and work as before; and his part will be 625 pound, as this figure sheweth.

240 Z 3000
50 625

Likewise for the third man, set his money, which was 60 pounds, and his part of gains will be 750 pounds, as here appeareth.

240 Z 3000
60 750

And so for the fourth man; if you set his summe, which is 100 pound, his gains will be 1250 pounds, as the work will declare.

240 Z 3000
100 1250

Scholar. This I perceive: but is there any way to examine whether I have well done or no?

Master. For the trial hereof, adde together all their four portions; and if their addition make the whole summe of their gains, then is the work well done.

Note this common proof.

Scholar. That will I trie by and by. The four parcels are these, which added together make 3000,

3000, which is the iust summe of money that they gained; whereby I know the work is well done.

Master. Well, now another example will I put to you, not of gains, but of loss: for one reason serveth for both.

A Question of Loss. *If three Merchants in one Ship, and of one fellowship, had bought Merchandise, so that the first had laid out 200 pound, the second 300 pound, the third 500 pound, and it chanced by tempest that they did cast overboard into the Sea Merchandise of the value of 100 pound, how much should each man bear in this loss?*

Scholar. If I shall doe in this as you did in the other question, then must I ioyn their 3 portions together, 200, 300, 500, which make 1000. Then say I, If 1000 lose 100, then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appear plain.

1000 Z 100
200 Z 20

1000 Z 100
300 Z 30

1000 Z 100
500 Z 50

Master. Well, sith now you have done these, I will propound a question of more importance, which shall make you not onely the abler to understand this Rule, but also it will greatly aid you in the next Rule of fellowship with time, if such need be that your money be of divers Denominations.

For

For this may not be forgotten in all such Questions: If the number be of divers kinds, you must by reduction bring it into one kinde, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of divers kinds, as some years, some months, weeks, and days, you shall make all months, weeks or days, according as the least name of time in the question is. As for example:

First in diversity of mony. *Three Companions bought 2000 sheep, and paid for them 241 pound 13 shillings 4 pence, of which summe one payd 101 pound 10 shillings, the second 82 pound 17 shillings 10 pence, and the third paid 57 pound 5 shillings 6 pence. How many sheep must each of them have?* Answer. *The first shall have 840, the second 686, and the third 474. And that must you work thus:* A question of Sheep.

First, considering that your mony is of divers Denominations, you shall (by Reduction) bring it all into the smallest Denomination which is in it, that is to say pence; and so will the Total summe be 58000 pence. Solution.

Now if you turn each mans mony into pence also, the first mans summe will be 24360 pence, the second mans mony will be 19894 pence, and the third mans mony will be 13746 pence.

Now to know how many sheep every man shall have, let the whole summe of mony, that is, 58000 pence, be set in the first place, and in the second place set the number of sheep, and then orderly in the third place set each mans mony; and then multiplying the third and the second summes together, and dividing that that amounteth by the first,

there will appear the number of sheep that each man ought to have; as these 3 figures do shew.

a

58000 Z 2000
24360 Z 840

b

58000 Z 2000
19894 Z 686

c

58000 Z 2000
13746 Z 474

Scholar. Why do you set the mony in the first place, seeing in the question you say 2000 sheep cost 58000 pence, and not thus, 58000 bought 2000 sheep?

Master. You remember I taught you at the beginning of the Golden Rule, that the first and the third numbers must be of one name, and of like things; and evermore the number that the question is asked of must be set in the third place.

Now is the question plainly this, If four men bought 2000 sheep for 58000 pence, how many sheep shall each man have?

But seeing in this question there ought more respect to be had to the summe of mony then to the summe of the persons, (for in the summe of mony is their proportion toward the sheep, and not in the number of persons,)

If 58000 pence bought 2000 sheep, how many did 24360 buy? Again, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceive it reasonable, and so shall I use it in all questions.

Note. Master. Even so. But for easiness of the work, mark this: Whensoever the first and second numbers

have

have Cyphers in the first places, you may both in the Multiplication and in the Division leave out those cyphers, so that you leave out like many out of both summes: as in this question the first number 58000 hath 3 cyphers, and so hath the second, that is 2000; therefore cast away there cyphers, and so will the first number be 58, and the second 2. Set them in their places, and work according to the Rule, and you shall perceive that will be all one, saving that this is the shorter and easier way, as these three figures do shew.

a

58 Z 2
24360 Z 840

b

58 Z 2
19894 Z 686

c

58 Z 2
13746 Z 474

And this you see is both easier, and also the more certain way to know the answer to this question.

Scholar. Truth it is as you say. But, Sir, meseemeth I might ask a farther question here, not onely how many sheep each man should have, but also what every sheep cost.

Master. That question doth not onely belong to this Rule, but may also be discussed by Division, especially if the questions number be one onely, as thus: Divide the total summe 58000 pence by 2000, (or 58 by 2) omitting the cyphers, and the Quotient will be 29 pence, that is 2 shillings 5 pence. Howbeit by this Rule you may doe it, and best when the number of the question doth exceed 1: as if I should

should ask this question, 2000 sheep cost 58000 pence, how much do 20 cost? Then shall I set my figures as before.

2000 58000
20

And doing after the Rule, there will amount 580 pence, that is, 2 pound 8 shillings 4 pence, the price of one score. But if you will use that easie way that I did teach you now, you may change the first and second numbers thus:

2 58
20

Thus do you perceive the use of the Rule without Time.

Scholar. All this I understand very well: I pray you now instruct me in the Rule of Fellowship with Time.

The Rule of Fellowship *with Time.*

Master.

The Rule of Fellowship with Time.

TO the intent you may as well perceive the same Rule with diversity of Time, I propose this example:

Four Merchants made a common stock, which at the years end was increased to 35145 pound. *Now to know what shall be each mans portion of gain, you must know each mans stock, and time of continuance.*

The first man of these four laid in 669 l. which he

he did take from the stock again at the end of 10 months; the second man laid in 810 pound, for 8 months; the third man laid in 900 pound, for 7 months; and the fourth laid in 1040, for 12 months. Question of a Bank.

This question shall you examine as you did the other before; saving that whereas in the third place of the figure you did set each mans summe alone, here you shall set the same being multiplied by the number of their time: and likewise in the first place of the figure you shall set the number which amounteth of their whole summes so multiplied by their time, and added into one whole summe, as thus: Note. A general Rule.

The first mans summe is 669 pound, which I multiply by 10, (that was the number of his time) and it maketh 6690. The second mans summe 810 pound, multiplied by 8, (which was his time) maketh 6480. The third mans summe 900 pound, multiplied by 7, (for that was his time) yieldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The four summes thus multiplied by their time must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all four, which is 31950; and the gain must be in the second place, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did 6690 get? Answer, 7359 pounds, as by this figure appeareth.

	a	
31950	Z	35145
6690		7359

Likewise the second man had to his part 7128 pounds, the third must have 6930 pounds, and the fourth man shall have for his part 13728 pounds: as these figures do partly declare.

b

31950 Z 35145
6480 Z 7128

c

31950 Z 35145
6300 Z 6930

d

31950 Z 35145
12480 Z 13728

Another Proof.

Scholar. This I like very well: but what Proof is there of this work?

Master. The same that I taught you for the other: howbeit there is used both for this work and the other also this manner of Proof, to adde all the portions together, and it must agree to the whole summe, then seemeth your work well done. But this is no sure Proof.

Schol. Yet will I prove in this example: The 4 parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceive the work is well done.

7359
7128
6930
13728
35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then do I understand this work also very well. But what have I now to learn?

Master. There are many other excellent parts behind, of which I will not as now make mention, because that without the knowledge of Fractions they cannot be duly taught, and much less understood. Therefore will I propose to you two or three questions

questions more, (that thereby you may better perceive the use of this Rule and all other the like) and so make an end for this time.

¶ *Three Partners by some ill adventure sustained the loss of* 160 pound, *whereof the first laid into the common stock* 200 pound, *for ten Months, the second laid in* 350 pound, *and the third* 100 pound; *but for how long the two latter is unknown: But breaking off their Partnership, the first found himself a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is, for how long time was the mony of the two latter in company?* A question of Loss.

For the solution hereof, and of such other like, you must also multiply the first mans 200 pounds that he put into the stock by his time of continuance, which was ten months, and it maketh 2000: wherefore now I affirm, If his money that lost 80 pound multiplied by his time make 2000, what will his money make that lost 56 pound, and his that lost 24 pound? which two numbers I commit to the trial of the Rule of Three at two workings, thus:

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80	2000	80	2000
56	1400	24	600

To conclude, If you now divide 1400, the second mans portion, by 350, which was his stock that he laid into company, you shall find in your quotient 4 months: and for so long time did the second man put his mony into the common stock.

Lastly, if you divide the third mans new laying in, which was 600, by 100, which was his stock

 that

that he put into the company, the quotient declareth his time of continuance, which was six months. And thus is the question resolved.

Scholar. Sir, I have attentively beheld your working, and the more we travell herein, the more methinks I am in love with this excellent Art.

Master. Then what say you to this Question?

A question of Canons. *There are in a Cathedral Church 20 Canons and 30 Vicars, those may spend by year 2600 pound, but every Canon must have to his part 5 times so much as every Vicar hath: how much is every mans portion say you?*

Scholar. I pray you make the answer your self also, so shall I perceive best the means to answer to such other like.

Master. In this Question you must doe as in those beforesaid that have diversity of time, for here is diversity of portions. Therefore shall you multiply the number of the persons by their difference of portions, (as you did in the other by time.) Then must you multiply the 20 (which is the number of Canons) by 5, (for that is the number of their portion) so will it be 100. Then 30 (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30. Put these two summs together, and they make 130. Then say thus, If 130 spend 2600 pounds, what may 100 spend? The Rule sheweth 2000 pounds.

Again for Vicars, if 130 spend 2600 pound, what may 30 spend? Answer, 600 pound, as these figures shew.

130	Z	600
100		2000

130	Z	2600
30		600

But

But if every Canon should have so often times 4 pound as the Vicar should have 3 pound, then should I multiply 20 by 4, (that were 80) and 30 by 3, (that were 90) and then both were 170. Then should the figures be set as followeth.

	li.	s.	d.		li.	s.	d.
170	2600			170	2600		
80	1223,	—10,	7	90	1376,	—9,	5

But this sort is too hard for you, by reason of the fractions, therefore I will let it rest to that place.

And by this Rule you see what the 20 Canons may spend; which summe if you divide by 20, you shall see each Canons proportion: and so of the Vicars, if you divide their summes by 30, the quotient will declare every Vicars proportion.

The Second Dialogue,

The Accounting by Counters.

Master.

N*Ow that you have learned* Arithmetick *with the Pen, you shall see the same Art in* Counters: *which feat doth not onely serve for them that cannot write and read, but also for them that can doe both, but have not at some time their pen or tables ready with them.*

This sort is in two forms commonly; the one by lines, and the other without lines. In that

that

that hath lines, the lines do stand for the order of places; and in that that hath no lines, there must be set in their stead so many Counters as shall need for each line one, and they shall supply the stead of lines.

Scholar. By examples I should better perceive your meaning.

Master. For example of the lines, loe here you see six lines, which stand for six places, so that the nethermost stands for the first place, and the next above is for the second, and so upward till you come to the highest, which is the sixth line, and standeth for the sixth place.

——100000——————
——10000——————
*——1000——————
——100——————
——10——————
——1——————

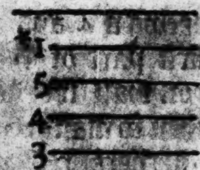
Numeration by Counters.

Now what is the value of every place or line, you may perceive by the figures which I have set on them, which is according as you learned before in Numeration of figures by the Pen: for the first place is the place of unites or ones, and every counter set in that line betokeneth but one; and the second line is the place of 10, for every counter there standeth for 10; the third line the place of hundreds, the fourth of thousands, and so forth.

Scholar. Sir, I do perceive that the same order is here of lines as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

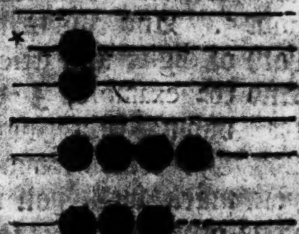
Master.

Master. If you do understand it, then how will you set 1543?



Scholar. Thus, as I suppose.

Master. You have set the places truly, but your figures be not meet for this use: for the meetest figure in this behalf is the figure of a counter round, as you see here, where I have expressed that same summe.

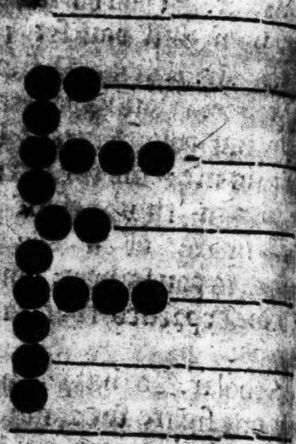


Scholar. So that you have not one figure for 2, nor 3, nor 4, and so forth; but as many digits as you have, so many counters you set in the lowest line, and for every 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 between two lines.

Master. You shall remember this, that whensoever you need to set down 5, 50, or 500, or 5000, or set forth any number whose numerator is 5, you shall set one counter for it in the next place above the line that it hath his denomination of. As in this example of that 500, because the numerator is 5, it must be set in a void space; and because the denomination is a hundred, I know that the place is the void place next above hundreds, that is to say, above the third line.

And farther you shall mark, that in all working by this sort, if you shall set down any summe

between 4 and 10, for the first part of that number you shall set down 5, and then so many counters more as there rest numbers above 5. And this is true both of digits and articles. And for example, I will set down this summe 29796, which summe if you mark well, you need none other examples for to learn the Numeration of this form.



But this shall you mark, that as you did in other kinds of Arithmetick set a prick in the places of thousands, in this work you shall set a Starre, as you see before.

Scholar. Then I perceive Numeration. But, I pray you, how shall I doe in this Art to adde two summes or more together?

Addition.

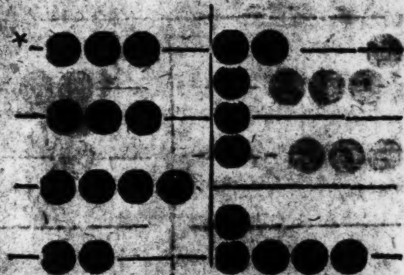
Master.

THe easiest way in this is to adde but two summs at once together: Howbeit you may adde more, as I will tell you anon.

Therefore when you will adde two summes, you shall first set down one of them, it forceth not which, and then by it

draw

draw a line cross the other lines: and afterward set down the other sum, so that the line may be betwæn them. As if you would adde 2659 to 8342, you set your sums as you sæ here.



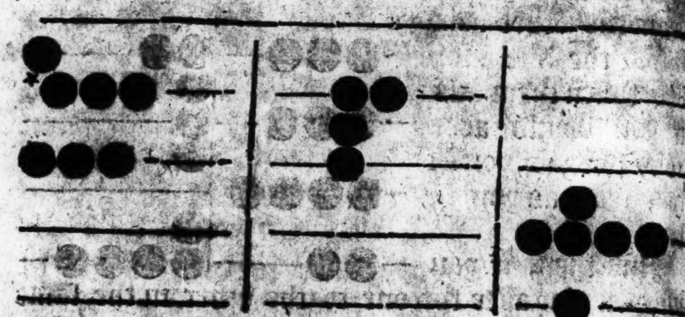
Addition of two summes.

And then, if you list, you may adde the one to the other in the same place; or else you may adde them both together in a new place: which way, because it is most plain, I will shew you first.

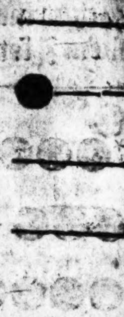
Therefore will I begin at the Unites, which in the first summe is but 2, and in the second summe 5, that maketh 11. Those do I take up, and for them I set 11 in the new room, thus:

Then do I take up all the Articles under a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90. Or you may say better, that in the first summe there are four Articles of 10, and the second summe 5, which maketh 9: but then take heed that you set them in their right lines: sæ here.

Where



Where I have taken away 40 from the firſt ſumme, and 50 from the ſecond, and in their ſtead I have ſet 90 in the third room; which I have ſet plainly, that you might well perceive: howbeit ſeeing that 90, with the 10 that was in the third room already, doth make 100, I might better for thoſe 6 Counters ſet one in the third line, thus:

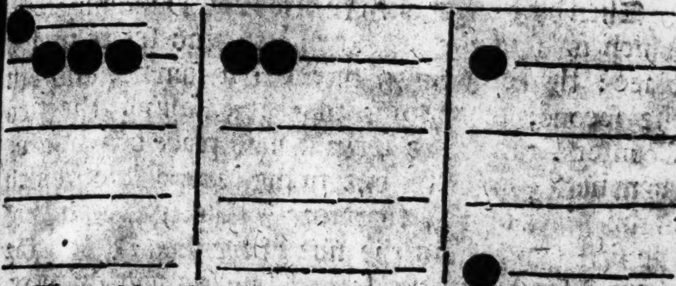


For it is all in one ſumme, as you may ſee: but it is beſt never to ſet five counters in any line, for that may be done with one counter in a higher place.

Scholar. I judge that good reaſon, for many are unneedful where one will ſerve.

Master. Well, then will I adde forth of hundreds. I find 3 in the firſt ſumme, and 6 in the ſecond, which maketh 6000; them do I take up, and ſet in the third room, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one counter in the fourth line for them all, as you ſee here.

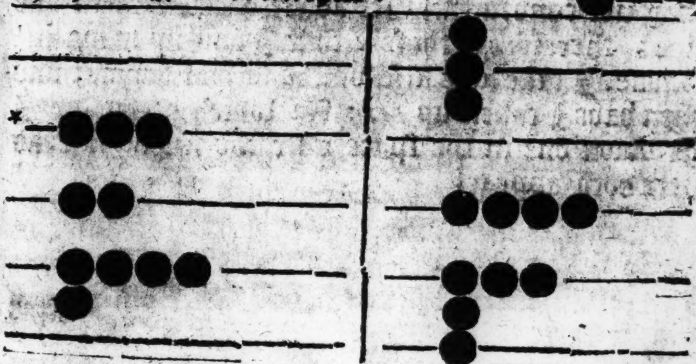
Then



Then adde I the thousands together, which in the first summe are 8000, and in the second 2000, that maketh 10000; them do I take up for those two places, and for them I set one counter in the fifth line: and then it appeareth, as you see, to be 11001, for so many doth amount of the Addition of 8342 to 2659. To adde summes together.

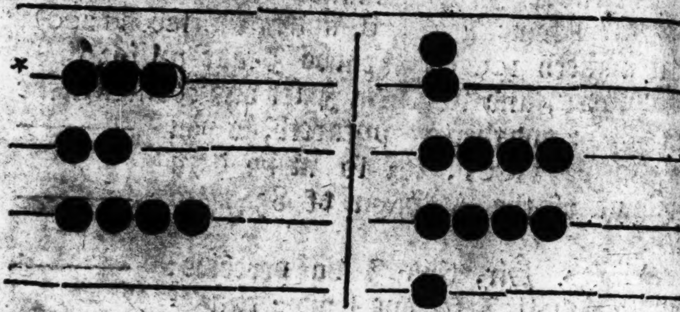
Scholar. Sir, this I do perceive: but how shall I set one summe to another, not changing them to a third place?

Master. Mark well how I doe it. I will adde together 65436 and 3245, which first I set down thus:

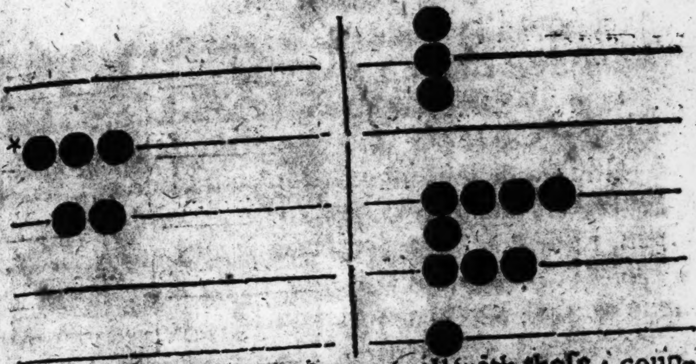


Then

Then do I begin with the smallest Denomination, which is 1 in the second summe, and set it in his place: then do I find 5 in the first summe, and 5 in the second, which put together, saving the two Counters, cannot be set in a void place of 5, but for them both I must set one in the second line, which is the place of 10; therefore I take up the five of the first summe and the five of the second, and for them I set one in the second line, as you see here.

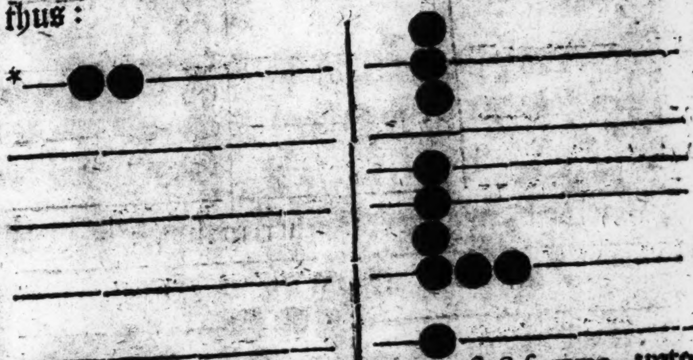


Then do I likewise take the 4 Counters of the first summe and second line, (which make 40) and adde them to the 4 Counters of the same line in the second summe, and it maketh 80: but, as I said, I may not conveniently set above 4 counters in one line; therefore to those 4 that I took up in the first summe, I take one also of the second summe, and then have I taken up 50: for which 5 counters I set down one in the space over the second line, as here doth appear.



And then is there 80 as well with those 4 counters, as if you had set down the other 4 also.

Now do I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600: therefore I take up the two counters in the first summe, and three of them in the second summe, and for them 5 I set 1 in the space above, thus:

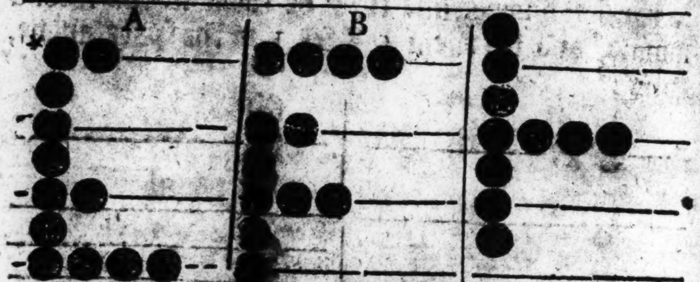


Then take I the 3000 in the first summe, unto which there are none in the second summe agreeing; therefore I do onely remove those three counters from the first summe into the second, as here doth appear.

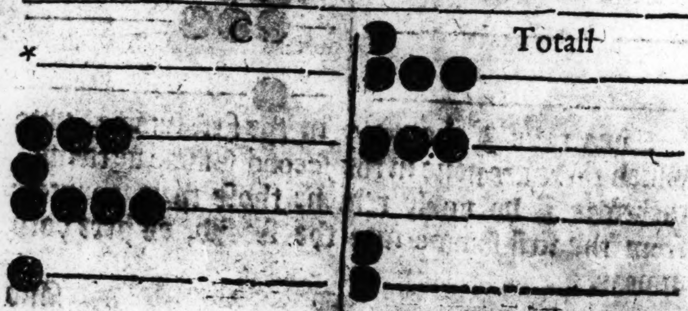
And you see the whole summe that amounteth of that Addition of 65436 with 3245 to be 68681.

And if you have marked those two examples well, you need no farther instruction in Addition of 2 onely sums: but if you have more then two sums to add, you may add them thus:

First add two of them, and then add the third and fourth, or more, if there be so many. As if I would add 2679 with 4286 and 1391, first I add the two first summs thus:



And then I add the third thereto thus:



And

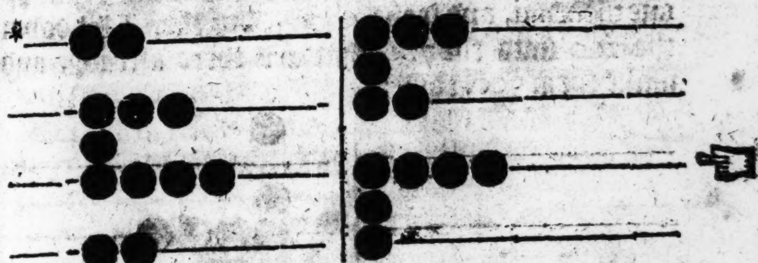
And so of more, if you have them.

Scholar. Now I think it best that you pass forth to Subtraction, except there be any way to examine this manner of Addition, then I think that were good to be known next.

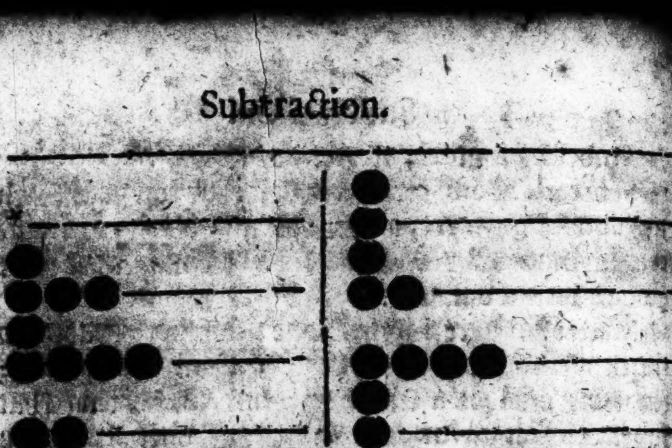
Master. There is the same proof here that is in the other Addition by the Pen, I mean Subtraction; for that onely is a sure way: but considering that Subtraction must be first known, I will first teach you the Art of Subtraction, and that by this Example.

Subtraction.

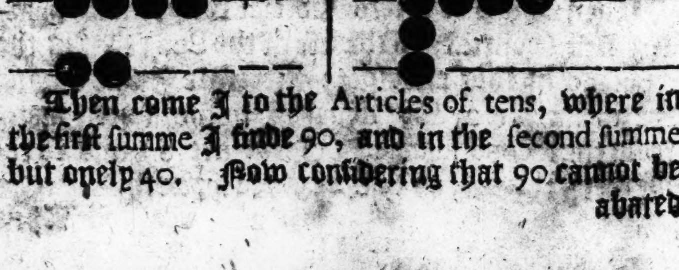
I Would subtract 2892 out of 8746. These summs must I set down as I did in Addition: but here it is best to set the lesser number first, thus:



Then shall I begin to subtract the greatest numbers first, (contrary to the use of the Pen) that is the thousand in this example: therefore I find among the thousands 2, for which I withdraw so many from the second summe, (where are 8) and so remaineth there 6, as this example sheweth.

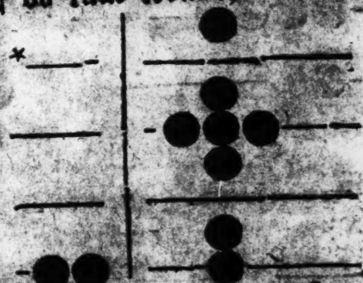


Then doe I likewise with the hundreds, of which in the first summe I find 8, and in the second summe but 7, out of which I cannot take 2, therefore this must I doe; I must look how much my sum differeth from 10, which I find here to be 2, then must I abate for my summe of 800 one thousand, and set down the excess of hundreds, that is to say 2, for so much as 1000 is more then I should take up: therefore from the first summe I take that 800, and from the second summe (which is 6000) I take up one thousand, and leave 5000; but then I set down the 200 unto the 700 that are there already, and make them 900, thus:



Then come I to the Articles of tens, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be abated

abated from 40, I look how much that 90 doth differ from the next summe above it, that is 100; or else (which is all to one effect) I look how much 9 doth differ from 10, and I find it to be 1: then, in the stead of that 90, I do take from the second summe 100: but considering that is 10 too much, I set down 1 in the next line beneath for it, as you see here.



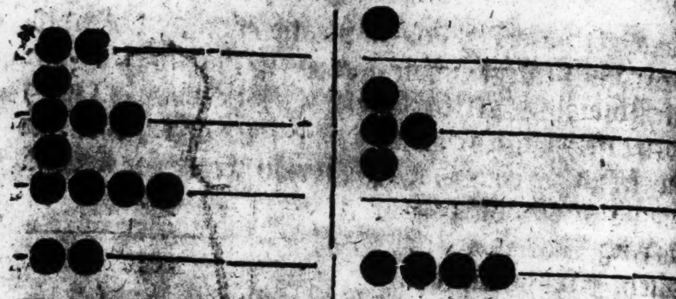
Saving that here I have set 1 Counter in the space instead of 5 in the next line. And thus I have subtracted all save 2, which I must abate from 6 in the second summe, and there will remain 4 thus:

So that if I subtract 2892 from 8746, the remainer will be 5854.

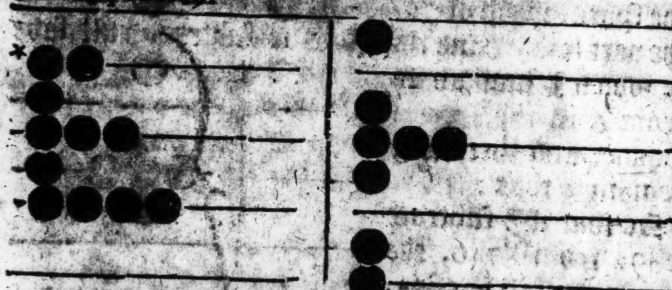
And that this is truly wrought, you may prove by Addition: for if you adde to this remainer the same summe that you did subtract, then will the former summe 8746 amount again.

A proof of Subtraction.

Scholar. That will I prove: and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus:



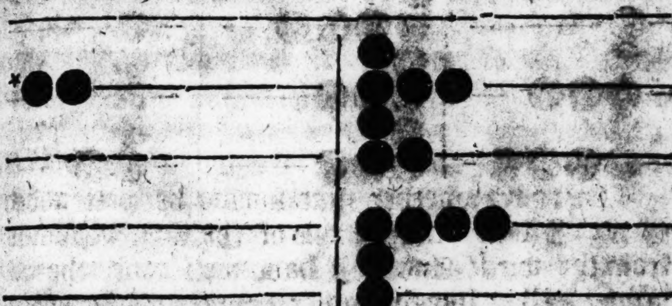
Then do I add the first 2 to 4, which maketh 6: so take I up 5 of those Counters, and in their stead I set 1 in the space, and one in the lowest line, as here appeareth.



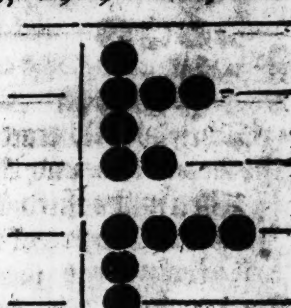
Then do I add the 90 next above to the 50, and it maketh 140. Therefore I take up those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line, thus:

Then

Then do I come to the hundreds, of which I find 8 in the first summe, and 8 in the second, that maketh 1600: therefore I take up those 8 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and in the third line, as you may see here.



Then is there left in the first summe but onely 2000, and in the second 5000, which is 7000, which I shall take up from thence, and set in the same line in the second summe to the one that is there already; and there will the whole summe appear, as you may well see, to be 8746, which was the first gross summe: and therefore I do perceive that I had well subtracted before.



And thus may you see how Subtraction may be tried by Addition.

Scholar. I perceive the same order here with Counters that I learned before in Figures.

Master. Then let me see how you can trie Addition by Subtraction.

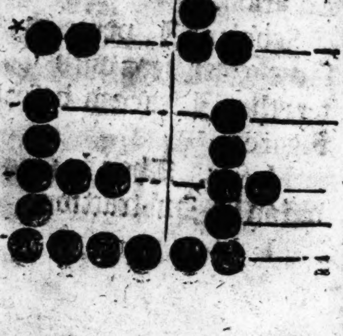
Scholar. First I will set forth this example of Addition, where I have added 2189 to 4988: and the whole summe appeareth to be 7177.

Proof of Addition by Subtraction.

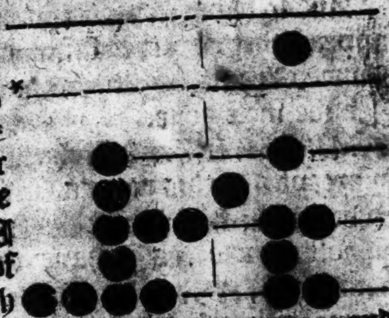
Now to try whether that summe be well added or no, I will subtract one of the first two summs from the third. And if I have well done, the remainer will be like that other summe. As for example, I will subtract the first summe from the third, which I set thus in order.

Then do I subtract 2000 of the first summe from the second sum, and then remains there 5000, thus:

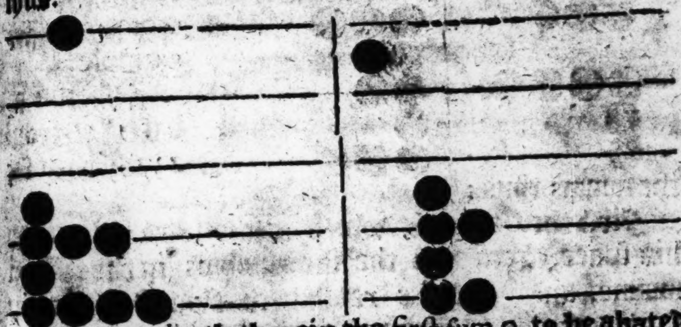
Then in the third line I subtract the 100 of the first from the second summe, where is onely 100 also; and then in the third line resteth nothing, as you may see in this example following.



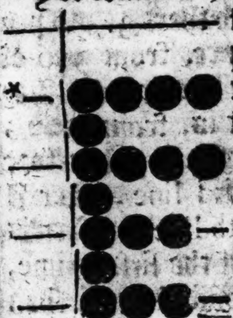
Then in the second ne with his space er him I find 80, * which I should sub- from the other m: then seeing there re but onely 70, I must take it out of some higher sum, which



is here onely 5000; therefore I take up 5000: and seeing that is too much by 4920, I set down so many in the second room, which with the 70 being there already do make 4990: and then the summs do stand thus.



Yet remaineth therein the first sum 9 to be abated from the second sum, where in that place of unites doth appear onely 7: then must I abate a higher sum, that is to say 10; but seeing that 10 is more then 9 (which I should abate) by 1, therefore shall I take up one Counter from the second, and set down the same in the first line, or lowermost line, as you see here.



And

And ſo have I ended this work, and the ſumme appeareth to be the ſame which was the ſecond ſumme of mine Addition; and therefore I perceive I have well done.

Another way of Addition.

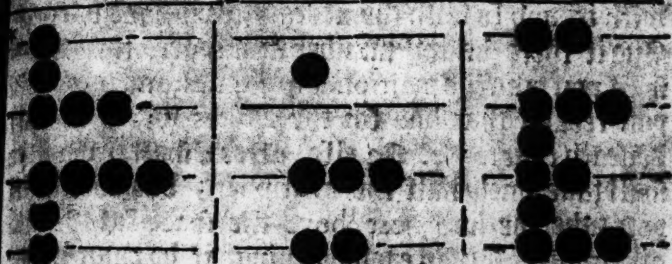
Maſter. To ſtand longer about this it is but folly: except that this you may alſo underſtand, that many do begin to ſubtract with Counters, not at the higheſt ſumme, as I have taught you, but at the nethermoſt, as they do uſe to adde; and when the ſumme to be abated in any line appeareth greater then the other, then do they borrow out of the next higher room: as for example,

I ſhould abate 1846 from 2378; they ſet the ſumms thus:

Firſt they take 6, which is the lower line, and his ſpace, from 8 in the ſame rooms in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, and his ſpace, from 300 of the ſecond ſumme cannot be; therefore do they abate it from a higher room, that is, from 1000; and becauſe 1000 is too much by 200, therefore muſt I ſet down 200 in the third line, after I have taken up 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then

do

all the Figures stand in order thus, 532.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that way best, so some like the other: therefore you now knowing both, may use which you list.

Multiplication.

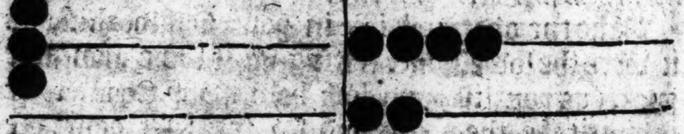
BUT *now touching Multiplication; You shall set your* numbers *into two* rooms, *(as you did in those other kinds) but so that the Multiplier be set in the first* room: *then shall you begin with the highest numbers of the second* room, *and multiply them first after this sort.*

Take the overmost line in your first working as it were the lowest line, setting on it some moveable mark, (as you list) and look how many Counters be in him, take them up, and for them set down the whole multiplier so many times as you took up Counters, reckoning (I say) that line for the Unites. And when you have done with the highest number, then

then come to the next line beneath, and doe so even with it, and so with the next, till you have done all. And if there be any number in a space, then for it shall you take the multiplier five times; and then must you reckon that line for the Unites which is next beneath that space. Or else, after a shorter way, ye shall take onely half the multiplier; but then shall you take the line next above the space for the line of Unites. But in each working, if by chance your multiplier be an odde number, so that you cannot take the half of it justly, then must you take the greater half, and set down that, as if that it were the just half: and farther, you shall set one Counter in the space between that line which you reckon for the line of Unites, or else onely remove forward the same that is to be multiplied.

Scholar. If you set forth an example hereof, I think I shall perceive you.

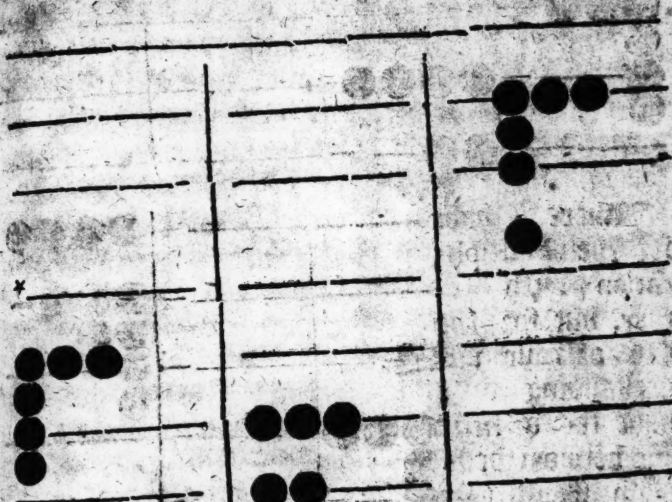
Master. Take this example: I would multiply 1542 by 365; therefore I set my numbers thus:



Then first I begin at the 1000 in the highest room, as if it were the first place, and I take it up, setting down for it so often (that is once) the

Multi-

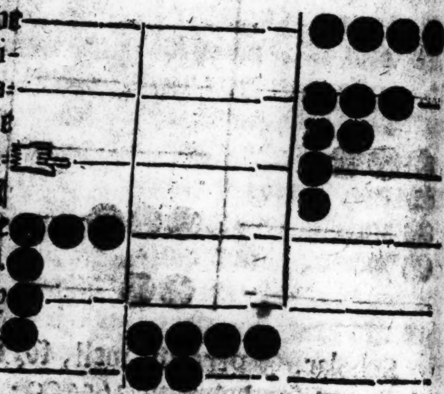
Multiplier, which is 365, thus as you ſee here: where, for the one Counter taken up from the fourth line, I have ſet down other ſix, which make the ſumme of the Multiplier, reckoning the fourth line as if it were the firſt; which thing I have marked by the Star ſet at the beginning.



Scholar. I perceive well, for indeed this ſumme that you ſet down is 365000; for ſo much doth amount of 1000 multiplied by 365.

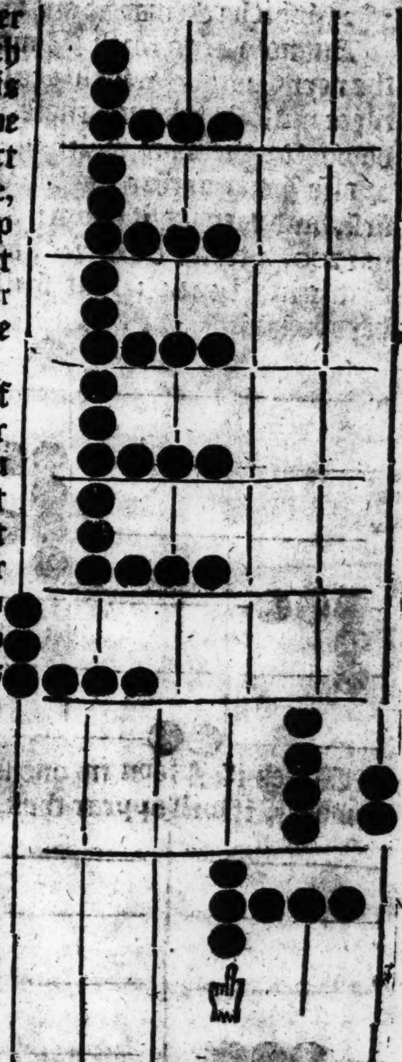
Maſter. Well then goe forth: In the next ſpace I find one Counter, which I remove forward, but take it not up, but (as in ſuch a caſe I muſt) ſet down the greater half of my Multiplier, (ſeeing it is an odde number) which is 182; and here do I ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſee.

Where I have set the Multiplication with other, but for the ease of your understanding I have set a little line between them. Now should they both in one summe stand thus:



Now-

Howbeit another form to multiply such Counters in space is this: first to remove the finger to the next line beneath the space, and then to take up the Counter, and to set down the Multiplier five times; as here you see.



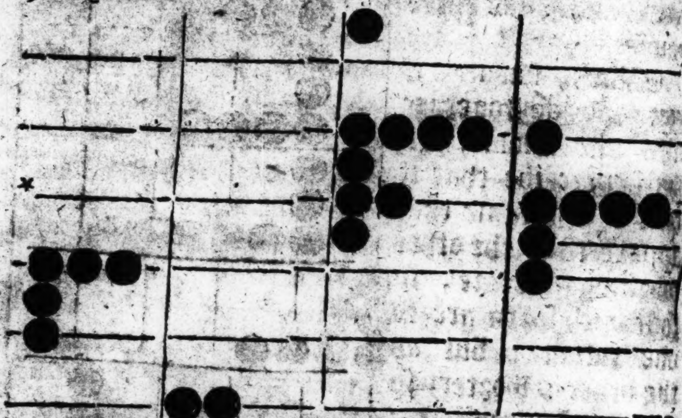
Another form of Multiplication.

Which summs if you do adde together into one summe, you shall perceive that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter, so this is plainer to reason for such as have had small exercise in this Art.

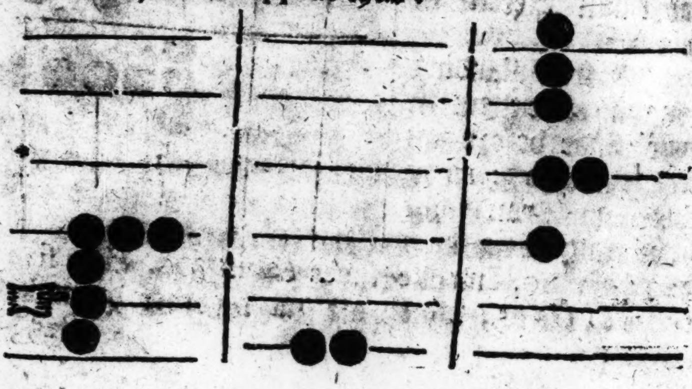
Notwithstanding you may adde them in your minde before you set them down: as in this example you might have said five times 300 is 1500, and five times 60 is 300, also five times five is 25, which all put together do make 1825;

1825; which you may at one time set down, if you list.

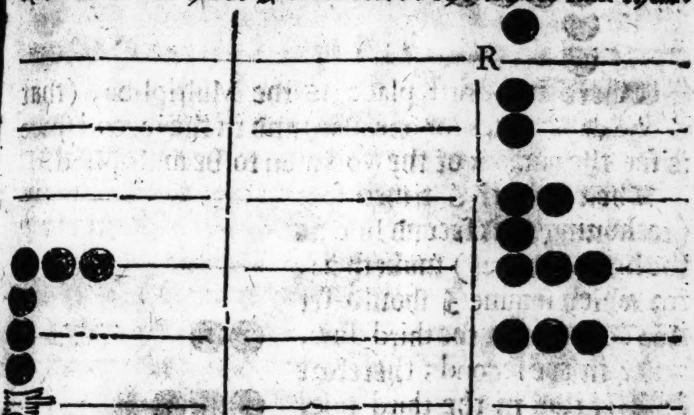
But now to go forth; I must remove the hand to the next Counters which are in the second line, and there must I take up those four Counters, setting down for them my multiplier four times severally, or else I may gather the whole summe in my mind first, and then set it down: as to say, 4 times 300 is 1200, 4 times 60 is 240, and 4 times 5 make 20, that is in all 1460: that shall I set down also, as here you see.



Which if I joyn in one summe with the former numbers, it will appear thus:



Then, to end this Multiplication, I remove the finger to the lowest line, where are onely 2; them do I take up, and in their ſtead do I ſet down twice 365, that is 730; for which I ſet one in the ſpace above the 3. line for 500, and 2 more in the 3. line with that one that is there already, & the rest in their order: and ſo have I well ended the whole ſum thus:

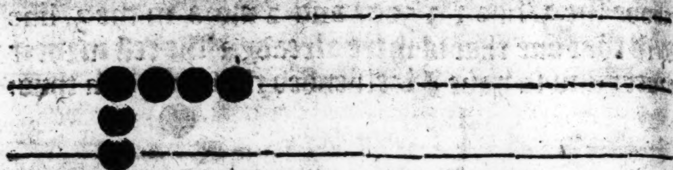


Whereby you ſee that 1542 (which is the number of years ſince Chriſt his Incarnation) being multiplied by 365 (which is the number of the days in 1 year) doth amount to 562830, which declareth the number of daies ſince Chriſt's Incarnation unto the end of 1542 years, beſides 385 days and 12 hours for leap-years.

Scholar. Now will I prove by another example; as this: 40 Labourers (after 6 pence the day for each man) have wrought 28 daies: I would know what their wages doth amount unto. Example of Wages.

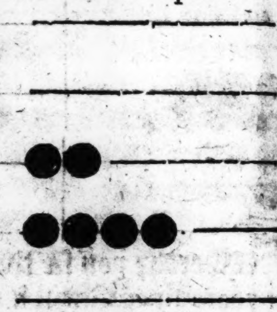
In this caſe muſt I work doubly; firſt, I muſt multiply the number of the Labourers by the wages of a man for one day, ſo will the charge of every day amount.

Then, secondly, shall I multiply the charge of one day by the whole number of days, and so will the whole summe appear. First therefore I shall set the summes thus:



Where in the first place is the Multiplier, (that is one days wages for one man) and in the second space is set the number of the workmen to be multiplied.

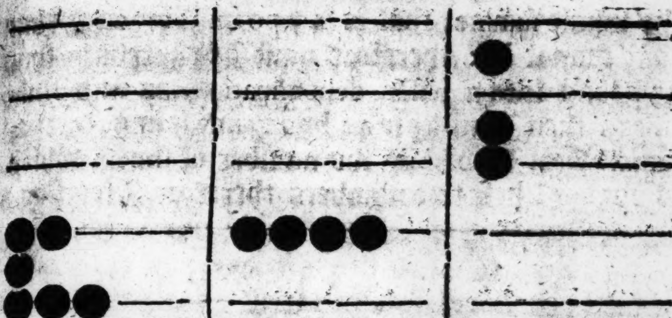
Then say, if 6 times four (reckoning that second line as the line of Unites) maketh 24, for which summe I should set two counters in the third line, and 4 in the second; therefore do I set two in the third line, and let the 4 stand still in the second line thus.



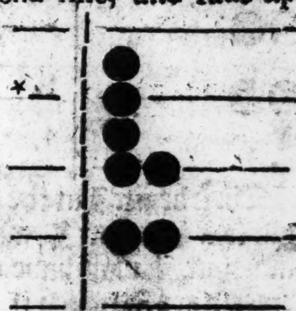
So appeareth the whole days wages to be 240 pence, that is 20 shillings.

Then do I multiply again the same summe by the number of days: and first I set the numbers thus: then because there are counters in divers lines, I shall begin with the highest, and take them up, setting for them the Multiplier so many times as I took up counters, that is twice: then will the summe stand thus:

Then



Then come I to the ſecond line, and take up thoſe 4 counters, ſetting for them the Multiplier 4 times: ſo will the whole ſumme appear thus:



So is the whole wages of 40 workmen for 28 days after 6 pence each day for a man 6720 pence, that is, 560 ſhillings, or 28 pound.

Maſter. Now if you would prove Multiplication, the ſureſt way is by Diviſion: therefore will I overpaſs it till I have taught you the Art of Diviſion, which you ſhall work thus.

Diviſion.

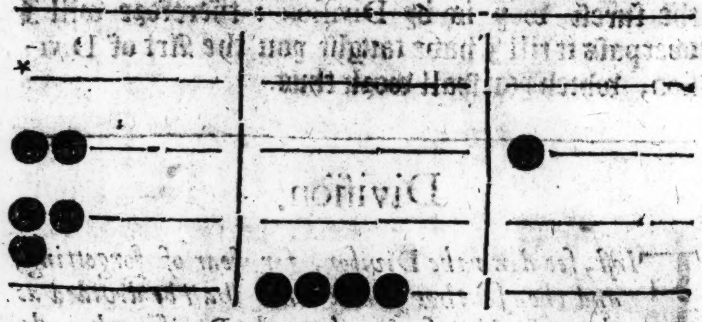
Firſt, ſet down the Diviſor, for fear of forgetting, and then ſet that number that ſhall be divided at the right ſide, ſo far from the Diviſor, that the Quotient may be ſet between them: as for example:

If 225 ſheep coſt 45 pound, what did every ſheep coſt? To know this, I would divide the

An example of Sheep.

the whole summe, that is 45 pound, by 225; but that cannot be: therefore must I first reduce that 45 pound into a lesser denomination, as into shillings; then I multiply 44 by 20, and it is 900: that summe shall I divide by the number of sheep, which is 225. These two numbers therefore I set thus:

Then begin I at the highest line of the dividend, and seek how oft I may have the divisor therein; and that I may have four times: then say I, four times 2 are 8, which if I take from 9, there resteth but 1, thus:



And because I found the divisor 4 times in the dividend, I have set, as you see, 4 in the middle room, which is the place of the quotient: but now

must I take the rest of the divisor as often out of the remainder, therefore come I to the second line of the divisor, saying, two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

Then come I to the lowest number, which is 5, and multiply it 4 times, so is it 20; that take I from 20, and there remaineth nothing; so that I see my quotient to be 4, which are in value shillings, for so was the dividend: and thereby I know that if 225 sheep cost 45 pound, every sheep cost 4 shillings.

Scholar. This can I doe, as you shall perceive by the example. If 160 Souldiers do spend every month 68 pound, what spendeth each man?

Example of souldiers wages.

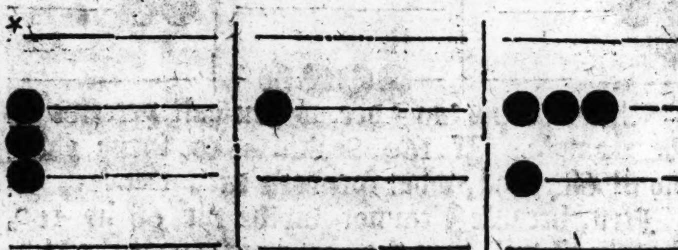
First, because I cannot divide the 68 by 160, therefore I will turn the pounds into pence by Multiplication, so shall there be 16320 pence: now must I divide the summe by the number of Souldiers, therefore I set them in order thus:

Then begin I at the highest place of the dividend, seeking my division there, which I find once, therefore I set 1 in the nether line.

Master. Not in the nether line of the whole summe, but in the nether line of that work, which is the third line.

Scholar. So standeth it with reason.

Master. Then thus do they stand:



Scholar. Then seek I again the rest, how often I may find my divisor; and I see that in 300 I might find 100 three times; but then the 60 will not be so often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there resteth 100, out of which with the 20, that maketh 120, I may take 60 also twice, and then stand the numbers thus:

Where

Where I have set the quotient 2 in the lowest line: so is every Souldiers portion 102 pence, that is 8 shillings 6 pence.

Master. But yet because you may justly perceive the reason of division, it shall be good that you set your divisor still against those numbers from which you do take it, as by this example I will declare.

If the purchace of 200 acres of ground did cost 290 pound, what did one acre cost?

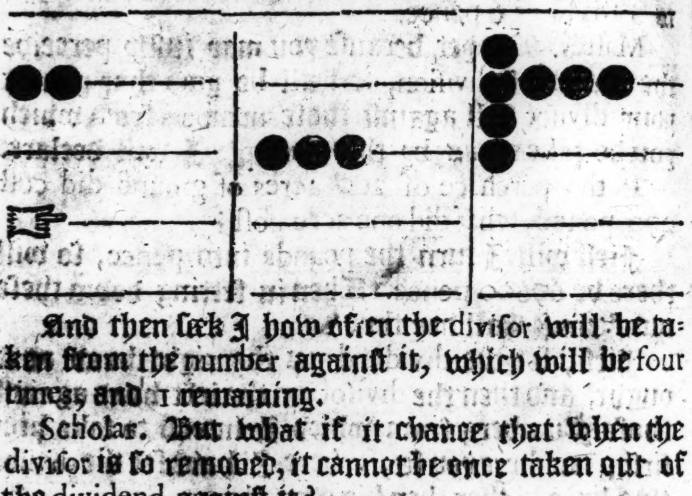
First will I turn the pounds into pence, so will there be 69600 pence. Then in setting down these numbers I shall doe thus: An example of Purchace.

First, set the dividend on the right hand as it ought, and then the divisor on the left hand against those Numbers from which I intend to take him first; as here you see, where I have set the divisor two lines higher then his own place.

Scholar. This is like the order of diviſion by the Pen.

Maſter. Truth you ſay; and now I muſt ſet the quotient of this work in the third line, for that is the line of unites in reſpect of the diviſor in this work.

Then I ſee how often the diviſor may be found in the dividend, and that I find 3 times: then ſet I 3 in the third line for the quotient, and take away that 60000 from the dividend: and farther, I ſet the diviſor one line lower, as you ſee here.



And then ſeek I how often the diviſor will be taken from the number againſt it, which will be four times, and 1 remaining.

Scholar. But what if it chance that when the diviſor is ſo removed, it cannot be once taken out of the dividend againſt it?

Maſter. Then muſt the diviſor be ſet in another line lower.

Scholar. So was it in diviſion by the pen, and therefore was there a cypher ſet in the quotient: but how ſhall that be noted here?

Maſter. Here needeth no token, for the lines do repreſent the places, onely look that you ſet your

quotient

quotient in that place which ſtandeth for unites in reſpect of the diviſor. But now to return to the example: I find the diviſor four times in the dividend, and 1 remaining; for 4 times 2 make 8, which I take from 9, and there reſteth 1, as this figure following ſheweth: and in the middle ſpace for the quotient I ſet 4 in the ſecond line, which is in this work the place of unites.

Then remove I the diviſor to the next lower line, and ſeek how often I may have it in the dividend, which I may have here 8 times juſt, and nothing remain, as in this form.

Where you may ſee that the whole quotient is 348 pence, that is 29 ſhillings: whereby I know that ſo much coſt the purchace of one Acre.

Scholar. Now resteth the proofs of Multiplication, and also Division.

Master. Their best proofs are each one by the other; for Multiplication is proved by Division, and Division by Multiplication, as in the work by the pen you learned.

Scholar. If that be all, you shall not need to repeat again that which was sufficiently taught already: and except you will teach me any other feat, here may you make an end of this Art, I suppose.

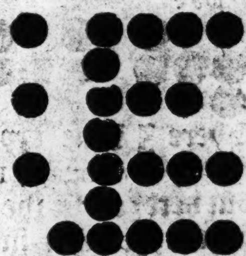
The reason of all the former Rules.

Master. So will I doe as touching whole number, and as for broken number I will not trouble you with it, till you have practised this so well that you be full perfect, so that you need not to doubt in any point that I have taught you, and then may I boldly instruct you in the Art of Fractions or broken numbers; wherein I will also shew you the reasons of all that you have now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, proceeding by no grounded reason, but onely by a received form, and that diversly, of divers men: for the Merchants use one form, and Auditors another.

Merchants

Merchants uſe.

BUT firſt for Merchants form, mark this example here, in which I have expreſſed this ſumme, 198 pounds, 19 ſhillings, 11 pence. *So that you may ſee that the loweſt line ſerveth for pence, the next above for ſhillings, the third for pounds, and the fourth for ſcores of pounds.*



Merchants Accompt.

And farther, you may ſee that the ſpace between pence and ſhillings may receive but one counter, (as all other ſpaces likewiſe do) and that one ſtandeth in that place for 6 pence.

Likewiſe between the ſhillings and the pounds one counter ſtands for 10 ſhillings.

And between the pounds and 20 pounds one counter ſtandeth for 10 pounds.

But beſides theſe, you may ſee at the left ſide of ſhillings, that one number ſtandeth alone, and betokeneth 5 ſhillings.

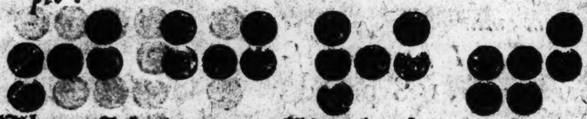
So againſt the pounds, that one counter ſtandeth for 5 pound; and againſt the 20 pounds, the one counter ſtandeth for five-ſcore pounds, that is 100 pounds. So that every ſide-counter is five times ſo much as one of them againſt which he ſtandeth.

Auditors

Auditors Accompt.

Auditors accompt.

NOW for the accompt of Auditors, take this example:



Where I have expressed the same summe 198 pound, 19 shillings, 11 pence.

But here you see the pence stand towards the right hand, and the other increasing orderly towards the left hand.

Again you may see that Auditors will make two lines (yea and more) for pence, shillings, and all other values, if their summs extend thereto. Also you see that they set one Counter at the right end of each row, which so set there standeth for five of that room, and on the left corner of the row it standeth for 10 of the same row.

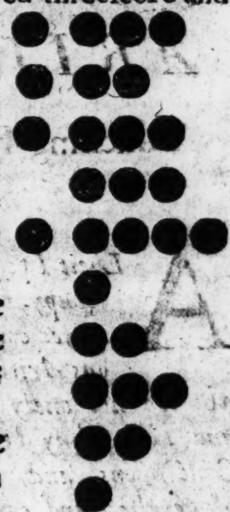
But now if you would adde or subtract after any of both these sorts, if you mark the order of the other feat which I taught you, you may easily doe the same here without much teaching: for in Addition you must first set down one summe, and to the same set the other orderly, and in like manner if you have many; but in Subtraction you must set down first the greatest summe, and from it must you abate the other, every Denomination from his due place.

Scholar. I do not doubt but with a little practice I shall attain these both: but how shall I multiply and divide after these forms?

Master. You cannot duely doe any of both by these sorts: therefore in such case you must resort to your other Arts.

They

They that use such Accounts that it exceeds 200 in the summe, they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space over they set the 10000, and in a higher row 20000: all which I have expressed in this example, which is 97869 pounds 12 shillings 9 pence ob. q. Ninety seven thousand, eight hundred threescore and nine pounds, twelve shillings and nine pence half-peny farthing. For I had not told you before where, neither how, you should set down farthings, which (as you see here) must be set in a void place sideling beneath the pence, for a farthing one counter, ob. two counters, for ob. farthing 3 counters, and more there cannot be; for 4 farthings make 1 peny, which must be set in his due place.



And if you desire the same summe after Auditors manner, lo here it is.

But in this thing you shall take this for sufficient, and the rest you shall observe as you may see by the working of each sort; for the divers wits of men have invented divers and sundry ways, almost innumerable.

THE SECOND PART OF *ARITHMETICK*,

Touching Fractions, briefly set forth.

Scholar.

Arithmeticall Fractions.

ALbeit I perceive your manifold businesses do so occupy, or rather oppress you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, which you have prepared, wherein not onely sundry works of Geometry, Musick and Astronomy be largely set forth, but also divers Conclusions and natural works touching mixtures of metalls, and compositions of Medicines, with other strange examples; yet in the mean season I cannot stay my most earnest desire, but importunately crave of you some brief preparation toward the use of Fractions, whereby at the least I may be able perfectly to understand the common works of them, and the vulgar use of those Rules, which without them cannot well be wrought.

Master. If my leisure were as great as my will is

is good, you should not need to use any importunate craving for the attaining of that thing whereby I may be perswaded that I shall any way profit the Commonwealth, or help the honest studies of any good Members in the same: wherefore while mine attendance will permit me to walk and talk, I am well willing to help you as I may.

Therefore, first to begin with the explication of this name fraction, what take you it to be?

Scholar. Marry, Sir, I think a Fraction (as I have heard it often named) to be a broken number, that is to say, to be no whole number, but part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of another number; but that must be understood of such another number as cannot be divided into any other parts then fractions: for although I may take the third part of 60, or the fourth part of it, and so of other parts diversly, yet those parts be not properly, nor ought to be called, fractions, because they may be expressed by whole numbers; for the 3 part of it is 20, the 4 part is 15, the 12 part is 5, and so forth of other parts, all which be whole numbers.

What a Fraction is.

Wherefore properly a fraction expresseth the parts or part onely of a unite, that is to say, that the number which is the whole or entire summe of any fraction may not be greater then one: and therefore it followeth, that no one fraction alone can be so great that it shall make 1; as by example I will declare, as soon as I have taught you to know the form how a fraction is expressed or represented in writing.

What a Fraction is properly.

Nume-

Numeration.

The expressing of fractions.

BUT first to begin with expressing of a Fraction, which is the Numeration of it; you must understand that a Fraction is represented by two numbers set one over the other, and a line drawn between them, as thus, $\frac{1}{3}$ $\frac{3}{4}$ $\frac{2}{5}$ $\frac{10}{17}$: which four fractions you must pronounce thus; $\frac{1}{3}$ one third part, $\frac{3}{4}$ three quarters, $\frac{2}{5}$ two fifth parts, $\frac{10}{17}$ ten seventeen parts.

Scholar. I understand this form of their expression and pronunciation, but their meaning or valuation seemeth more obscure. Yet I think that by the two first Fractions I understand the valuation of the two latter Fractions, and consequently of other.

Master. Value them then, that I may perceive your taking of them.

Scholar. $\frac{2}{5}$ betokeneth two fifth parts, that is to say, if one be divided into 5 parts, that Fraction doth express two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them; so $\frac{3}{4}$, that is three quarters, doth declare that one being divided into four quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficulty in this Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of works. For I understand that the same kinds

of

of works be in fractions that be in whole numbers.

Master. There are the same kinds of works in both, albeit the order of them is diverse, as I will anon declare: but yet more in Numeration before we leave it. You must understand that those two numbers which expresse a Fraction have severall names, the over most which is above the line is called the Numerator, and the other beneath the line is called the Denominator. Numerator. Denominator.

Scholar. And what is the reason of their divers names? For (in mine opinion) both be Numerators, seeing both do expresse the numeration of the fraction.

Master. You are deceived; for one onely (which is the over most) doth expresse the Numeration, and the Denominator doth declare the number of parts into which the Unite is divided: as in this example, when I say, divide a pound weight of gold between four men, so that the first man shall have $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now do you perceive that by the Denominator (which is one in all four fractions) it is intended that the pound weight should be divided into so many parts, I mean 15, and by the 4 severall Numerators is limited the divers portion that each man should have; that is, that when the whole is parted into 15, the first man shall have two of those 15 parts, the second man three of them, the third man four, and the fourth man six. And so may you see the severall offices (as it were) of those two numbers, I mean of the Numerator and the Denominator.

And hereby you perceive that a man can have no more parts of any thing then it was divided

into, neither yet aptly so many: so that it were unaptly said, You shall have $\frac{15}{15}$, that is fifteen parts of any thing, seeing it were better said you shall have the whole thing.

Scholar. So doth it appear reasonably, for the labour is vain to divide any thing, and then to apply the Division to no use. And much less reasonable were it to say $\frac{16}{15}$: for if the whole be divided into 15 parts onely, it is not possible to take 16 of them, that is to say, more then all together.

Master. This is true touching the proper and apt use of the name of a Fraction; yet improperly (and after a vulgar acceptation for easiness in work) both those forms be called Fractions, because they be written like Fractions, although they be none indeed: for $\frac{15}{15}$, and generally in such other where the Numerator and Denominator be equal, are not Fractions, but the whole thing with all his parts: and so $\frac{14}{12}$ is not to be called a Fraction, but a mixt number of a whole number and a Fraction; for it is as $1\frac{2}{12}$, that is, one whole and $\frac{2}{12}$ parts, as shall be declared in Reduction. Therefore they do abuse the names that call them Fractions, where the Numerator is either equal or greater then the Denominator.

An improper fraction, or a mixt number.

Scholar. But is there any needfull cause why they should so abuse the name?

Master. There is cause why they shall sometimes for easiness in work write some numbers after that sort like Fractions: but they needed not to call them Fractions, but (as they be) whole numbers, or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions, or as improper Fractions.

Now

Now must you understand, that as no Fraction properly can be greater then one, so in smalness under one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase above one infinitely; so that not onely one may be divided into infinite Fractions or parts, but also every Fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of Fractions: and they be expressed diversly, as for example, of $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of 2 third parts of one half part. Whereby is signified, that if one be divided into 2 halfs, and the one half into three parts, and two of those three parts be divided joyntly into four quarters, this fraction of fractions doth represent three of those quarters. Fractions of fractions.

Scholar. I pray you let me prove by an example in common money whether I do rightly understand you or no. One Crown, which I take for an unite, doth contain 60 pence; therefore the half of it is 30 pence, $\frac{2}{3}$ of that half is 20 pence, whereof $\frac{3}{4}$ is fifteen pence: so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crown: and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceive this well enough: yet this note I give you by the way, that the form of expressing the fractions is voluntary, and hath no other reason then the will of the Divisor, which form many follow: for some express them thus, $\frac{3\,2\,1}{4\,3\,2}$ without any figure of distinction between them, which form also many follow; some other do make lines between every fraction, and adde words of distinction, after this sort, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which form is best.

Some other express them thus in slope form, to distinguish them from fractions of whole numbers; for if they were in one right line thus, $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$, then ought it to be pronounced, three quarters and two third parts and a half, which maketh almost two whole unites, lacking but one twelfth part; and so is it nothing agreeable with the other fraction of fractions: wherefore it is a great oversight in certain learned men, which do express them so confusedly with such severall fractions, that a man cannot know the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

Therefore some men (as *Stifelius*) do express without a line numbers of proportion, being applied to *Addition* or *Subtraction*, because they must be taken as two, where the line in fractions maketh them tobe taken for one: for of the *Numerator* and *Denominator* is made one number.

Three severall varieties.

Scholar. Then I perceive there be three several varieties in fractions. First, when one onely fraction is set for one number; as $\frac{4}{5}$, that is four fifth parts. The second is, when there be set two or more severall fractions of one number; as $\frac{4}{9}$, $\frac{2}{5}$, that is, four ninth parts, and two fifth parts. The third sort is fractions of fractions; as $\frac{4}{9}$ of $\frac{2}{5}$, that is, four ninth parts of two fifth parts.

Master. You have said well, if you understand well your own words.

Scholar. If it shall please you, I will by an example in the parts of an old English Angel express my meaning.

Master. Let me hear you.

Scholar.

Scholar. The old English Angel did contain 7 shillings 6 pence, that is 90 pence: now $\frac{4}{5}$ of it is 72 pence: And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is four ninth parts and two fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is, four ninth parts of two fifth parts, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yield but 16, for $\frac{1}{9}$ of 36 is but 4, and that taken four times maketh 16.

Master. This is plainly expressed and truly, and hereby I doubt not but you do perceive, that as great a difference as is between 16 and 76, so much difference is between those two fractions $\frac{4}{9}$ and $\frac{2}{5}$, and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that ye understand these varieties, I will proceed to the rest of the works; first admonishing you, that there is another order to be followed in Fractions then there was in whole numbers: for in whole numbers this was the order, Numeration, Addition, Subtraction, Multiplication, Division, and Reduction; but in Fractions (to follow the same aptness in proceeding from the easiest works to the harder) we must use this order of works, Numeration, Reduction, Addition, Subtraction, Multiplication, and Division. The order of works in fractions.

Scholar. That Addition and Subtraction should go together, and Division to follow Multiplication, natural order doth persuade: but why Reduction should be first in order here next to Numeration, and Addition and Subtraction in the middle, I desire to understand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so

goe forward by degrees to the hardest: even reason teacheth in Fraction the like order. And consider that Addition or Subtraction of Fractions can very seldome be wrought without Reduction; and contrariwise, Reduction may be wrought without this form of Addition or Subtraction: therefore was it orderly required that Reduction should go before Addition and Subtraction; and this reason serveth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the easiest, I pray you declare the form of it, first by rule, and then by example.

Master. Your request is good.

Reduction of Fractions.

Of Reduction of Fractions there are five varieties.

Therefore will I now declare the diversities of Reduction of Fractions, which commonly hath five varieties or forms.

1. First, when there be sundry Fractions of one intire Unite, they must be reduced to one Denomination, and also into one Fraction.

2. Secondly, when there be propounded fractions of fractions, they must be reduced likewise into one fraction; for otherwise they cannot be brought into one Denomination.

3. Thirdly, when an improper fraction is propounded, that is to say, a fraction in form, which indeed is greater

greater then an unite, it must be reduced into apt form, expressing the unite or unites of it, and the proper fraction distinctly. And sometimes also it shall be needful to convert such a mixt number of unites with fractions into the form of a fraction, that is, into an improper fraction: which two forms I esteem but as one, because they work one kind of number.

Fourthly, there happeneth sometimes fractions to be written in great numbers, which might be written in lesser numbers: therefore is there a mean to reduce such great numbers into their smallest terms. 4.

Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certain parts, but none of like *Denomination* with that fraction, then may you reduce the said fractions into another, whose *Denomination* shall express the common parts of that whole thing. 5.

Scholar. **This distinction in Doctrine delighteth me much, but more with hope then present fruit: for as yet I do scarcely understand the varieties, and much less the practice and use of their works.**

Master. Reduction **is an orderly alteration of** Numbers **out of one form into another, which is never done orderly but for some needful use; as in every of the said five severall forms I will distinctly declare.**

First therefore, when two or more several fractions of any unite be propounded, as for example $\frac{2}{3}$ and $\frac{4}{5}$, because it is hard to tell what proportion of the intire number those two fractions do express, therefore was *Reduction* devised to be a mean whereby these several fractions might be brought into one Denomination and fraction. The first form of Reduction.

And in these fractions this is the Art for bringing them to one Denomination.

How to reduce fractions of divers denominations into one denomination.

Multiply first the *Denominators* together, and the totall thereof you shall set twice down under two severall lines for two new *Denominators*, or rather for one common *Denominator*. Then multiply the *Numerator* of the first fraction by the *Denominator* of the second, and set the totall thereof for the *Numerator* over the first line. Likewise multiply the *Numerator* of the second fraction by the *Denominator* of the first, and set that totall over the second line for the *Numerator* of that fraction. And so are these two first Fractions of severall *Denominations* brought to one *Denomination*.

Scholar. If I understand you, as I think I do, my example shall declare the same. The Fractions which you propounded were these, $\frac{3}{16}$ and $\frac{4}{6}$, whose Denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set under two lines, thus; $\frac{}{96}$ $\frac{}{96}$.

Then I multiply the Numerator of the first Fraction by the Denominator of the second, saying, 3 into 6 maketh 18; that I set over the first line for a new Numerator, and it will be thus, $\frac{18}{96}$.

Likewise I multiply the Numerator of the second Fraction by the Denominator of the first, saying 4 times 16 maketh 64; that I set for the second Numerator, and the Fraction will appear thus, $\frac{64}{96}$.

So that both Fractions brought to one Denomination must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after my

my accustomed form, by common parts of coyn or other measure.

Master. Go to.

Scholar. I have a piece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$: again, $\frac{1}{6}$ of the same piece of gold is 16 pence, so that $\frac{4}{6}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I find the summs to agree with the other before.

Master. So have you now the Art to bring two such fractions into one Denomination: And if there be more then two, then must you multiply all the Denominators together, and set the totall thereof so many times down as there be fractions; and then to get for each one a new *Numerator*, multiply the Numerator of the *first* by the Denominator of the *second*, and the totall thereof multiply by the Denominator of the *third*, and so forth, if there be more. Likewise multiply the Numerator of the *second* by the Denominator of the *first*, and the totall thereof by the Denominator of the *third*. And in the same sort multiply the Numerator of the *third* into the Denominator of the *first*, and the totall thereof into the Denominator of the *second*, and so forth, if there were moe. So these three fractions $\frac{2}{5}\ \frac{3}{4}\ \frac{2}{3}$ do make by Reduction these other three fractions of Denomination, $\frac{24}{60}\ \frac{45}{60}\ \frac{40}{60}$. All which you may bring into one fraction, by adding the Numerators together, and putting the totall for the totall Numerator, reserving still that same common Denominator. And those three fractions make one improper fraction thus, $\frac{19}{60}$.

Note the Reduction of three fractions or more into one.

Scholar. All this I perceive, and also that this

last

last Fraction is more then an unite, and therefore you did call it an improper Fraction.

Master. There be certain other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to exercise your wit therein.

The first variety of Reduction.

The first variety is this: When you have made and written down your *common Denomination*, (as I have taught before) then to get a *numerator* for the first doe thus; Divide the *common Denominator* by the *denominator* of the first Fraction, and the quotient multiplied by the *numerator* of the same yieldeth a new *numerator* for the first new fraction. So likewise doe with the second and the third, and with all the residue, if there be more.

Scholar. That will I prove in your last example of these three fractions, $\frac{2}{5}\,\frac{3}{4}\,\frac{2}{3}$. When the Denominators be multiplied they make 60, for 5 into 4 maketh 20, and 20 by 3 yieldeth 60: that I set down three times thus; $\overline{60}\;\overline{60}\;\overline{60}$: then to have a numerator for the first, I must divide 60 by 5, (the Denominator of the first) and the quotient is 12, which I must multiply by 2 (the numerator of the first,) and that maketh 24, and so have I for the first Fraction $\frac{24}{60}$.

Likewise for the second Fraction, I divide 60 by 4, and there cometh 15, which I multiply by 3; and so have I 45, for the second Fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{45}{60}$.

The second variety.

Master. Another way is this: If it happen so that the lesser *denominator* can by any *multiplication* make the greater, then note the *multiplier*, and by it multiply the *numerator* over that lesser *denominator*, and for the lesser *Denominator* put the greater; as thus in

these

these two fractions $\frac{3}{12}$ and $\frac{2}{3}$, three being the lesser Denominator multiplied by 4 will make 12, which is the greater Denominator: therefore by the same 4 I do multiply 2, which is the numerator over 3, and that maketh 8; under which I do put 12, being the greater Denominator, which is also made by multiplication of 4 into 3. And so have I these two fractions $\frac{3}{12}\frac{8}{12}$ thus shortly reduced, without altering the one Fraction.

Scholar. This I understand.

Master. Then mark this third way: If the *Denominators* do not happen so, that one by *Multiplication* may make the other, then look whether they both may be parts of any other one Number; as in $\frac{5}{12}$ and $\frac{7}{18}$, although the lesser taken but twice be too much to make 18, yet they both may be parts to 36; therefore look how many times twelve is in 36, and that quotient being multiplied by the *Numerator* over 12, the totall shall be put in stead of the *Numerator* over 12, and for 5 put 15, thus, $\frac{15}{36}$. So likewise look how often is 18 in 36, because it is twice, therefore by 2 multiply 7, which is over 18, and it will be 14: set that for the *Numerator*, and in stead of 18 put 36: and then your *Fractions* reduced stand thus, $\frac{15}{36}\ \frac{14}{36}$, in stead of $\frac{5}{12}$ and $\frac{7}{18}$. The third variety.

And if you will prove whether you have wrought well or no, that may be proved by Reduction of them again to their former Denominations; which Art shall be taught in the fourth kind of Reduction, where greater terms of Fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction the same terms or numbers come again that were before, then is the work good, else not. Proof.

Scholar.

Scholar. Sir, I hear your words, but I do not understand many of them; which, if it please you, declare.

Master. With a good will, when convenient place serveth, but that must be in the said fourth kind of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one Denomination.

The second form of Reduction of fractions of fractions into one fraction and Denomination.

When Fractions of Fractions be propounded, you shall multiply the *Numerators* of each into other, and set the totall for the new *Numerator*; and then multiply all the *Denominators* likewise, and take their totall for the new *Denominator*; and so are they speedily reduced.

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one denomination and proper simple fraction.

Therefore begin I with the Numerators, and multiply them together, saying, 3 by 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7 yieldeth 252: that I set over a line for the Numerator thus: $\frac{252}{}$

Then I multiply the denominator 4 by 3, maketh 12, and that by 7 bringeth 84, which multiplied by 9 yieldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand yet. $\frac{252}{756}$

Master. You think so, and no marvell, but anon you shall learn to judge it easily; for this fraction is no more indeed then $\frac{1}{3}$ although it be in greater terms, and therefore more stranger & more obscure. And this sufficeth for this Reduction, save that I will

I will ſhew you by a figure of meaſure the juſt rate and reaſon of this kind of fractions, and alſo the due underſtanding of their Reduction.

The entire meaſure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
1	2	3	4	5	6	$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{3}{4}$					

Here you ſee the longeſt meaſure (which ſtandeth for the whole and intire quantity) firſt parted into 9 diviſions, whereof 7 are ſevered by the ſecond meaſure, and thereof again are parted out 6, and that 6 being diſtinct into three parts, two of them are parted by the fourth meaſure, of which fourth meaſure being divided into four parts, the loweſt meaſure doth contain $\frac{3}{4}$, ſo that the ſame $\frac{3}{4}$ muſt be named not $\frac{3}{4}$ of the whole meaſure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is ſo ſenſible, that I cannot chuſe but ſee it. And furthermore I ſee alſo, that the ſame fraction is equal to $\frac{3}{9}$ of the entire meaſure, as the lines which run up and down do expreſly ſet forth. Alſo I ſee here that $\frac{2}{3}$ of $\frac{6}{7}$ is equal to $\frac{4}{9}$; and farther yet, that $\frac{2}{3}$ of $\frac{7}{9}$ is equal to $\frac{6}{9}$ or $\frac{2}{3}$.

Maſter. I am glad that you ſee it ſo well, not doubting but you will gather greater light of knowledge hereby.

But now it is time that we come to the third form of *Reduction*, which teacheth of improper Fractions, that

that is to say, mixt numbers of unites and fractions although they appear like fractions, as the $\frac{26}{5}$, which doth conclude 5 unites wholly and $\frac{1}{5}$ over. Wherefore first you shall know them by this, that the Numerator is greater then the Denominator.

The third form of Reduction of improper fractions.

Scholar. Indeed, Sir, that appeareth reasonable, that if the Numerator do expresse more parts to be taken of any unite then the Denominator doth signifie that unite to be divided into, it must needs follow, that such a Fraction importeth more then the whole, that is to say, the whole with certain parts over: but what Reduction is there in it?

Two severall waies in this Reduction.

Master. There be two severall kinds of Reduction concerning such fractions. Sometimes it shall be needful to convert these fractions into unites and the proper fractions that will remain. And sometimes, contrariwise, it shall be meet to reduce mixt numbers, that is, unites written with fractions, into the form of one simple fraction. And so be there two waies.

The first way.

Scholar. What is the meaning of the first way, to turn improper Fractions into unites with their proper Fractions?

Reduction of improper fractions into unites with their proper fractions.

Master. That is thus: Your *numerator* being greater then the *denominator*, must be divided by the same denomination, and the quotient thereof expresseth the unites; the remainer shall be put for the numerator of the fraction that resteth, and the denominator must be the same that was before.

Scholar. For example, I take $\frac{17}{5}$, and dividing 17 by 5, the quotient will be 3, and there will remain 2.

Master. That you must write thus, 3 $\frac{2}{5}$; where (you see) I have written 3 without any line, as

entire

intire numbers ought to be written, and the 2 that remained I have set over the former denominator with a line, as a proper fraction. And this number doth signifie now three unites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by unites here understand Crowns, so it were 3 Crowns and $\frac{2}{5}$, that is 2 s.

Master. Even so: and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes, that when the Reduction is so wrought, there remaineth nothing: and then it is not a mixt number, but a simple intire number, represented like a Fraction.

Scholar. As $\frac{15}{5}$ will make 3 just, and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the second form of reduction that you spake of for these sorts of fractions. The second way.

Master. Whensoever you have any of these two sorts of *numbers*, that is to say, whole *numbers* without *fractions*, or whole *numbers* with *fractions*, and you would turn them into the form of a *fraction*, you must multiply the whole *number* by that *denominator* which you will have to remain still, and to the totall thereof adde the *numerator* which you have already, and all that you shall set for the new *numerator*, keeping still the former *denominator*: As if you have 6 $\frac{3}{4}$, which you would convert into an improper *fraction*, you must multiply 6 by 4, whereof cometh 24, and thereto adde the *numerator*, which is 3, and so have you 27 for the *numerator*, and 4 still for the *denominator*. Reduction of whole numbers either alone or joyned with fractions into improper fractions.

Scholar. Then is $\frac{27}{4}$ equal to 6 $\frac{3}{4}$.

Master. Even just; and so backward (as appeareth by the former Reduction) 6 $\frac{3}{4}$ maketh $\frac{27}{4}$. Note.

And

And this one of their Reductions may be the proof of the other work.

Scholar. This I perceive: But now if you would turn whole numbers without fractions into any fractions, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master. That is well marked: but this you know, that no man intendeth to turn any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication must be made: for the proof whereof I set down 7, which is a whole number; and if you will have this number converted into any certain fraction, will me to doe it.

Scholar. I pray you reduce 7 into a Fraction.

Master. Then you care not what the Fraction be, so it be some Fraction.

Scholar. No, I pass not for the sort of the Fraction.

Master. Then how can you think that you require me to doe any thing certain, when you leave me to doe as I list? And seeing you stand at that stay, whether think you that I must first intend in minde what fraction I will make of it before I can doe it indeed?

Scholar. Else you should doe ignorantly.

Master. Then will I limit my self (seeing you will not) to turn it into quarters. And therefore I multiply 7 by 4, (which is the denomination by quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the denominator, and the fraction will be thus, $\frac{28}{4}$.

Scholar.

Scholar. Indeed I perceive this to be reasonable; for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turn 8 into 5 parts, it will make $\frac{40}{5}$, which is all one with 8: for 8 Crowns turned into 5 parts (that is into Shillings) will make 40 shillings, that is $\frac{40}{5}$ of a Crown.

Master. Seeing you understand now these three kinds of *Reduction*, I will declare unto you the fourth kind; that is, when *Fractions* be written in greater terms then they need, how they may be brought to lesser terms.

The fourth form of Reduction.

Scholar. To write any thing in greater terms then needeth, seemeth to be a fault; and so this Rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must be redressed; but in this case it is not so: neither did I say absolutely (as you do) that it needeth not to expresse those Fractions in so great terms; but that the Fractions do not need, I mean for their value, to be understood: but yet it may be needfull for the ease of these works whereto they be applied. As for example, In the first kind of Reduction this was your own example, $\frac{3}{16}$ and $\frac{4}{6}$, which when you would reduce, you were fain to turn them first into one denomination, and so appeared they thus, $\frac{18}{96}$ and $\frac{64}{96}$; where the Fractions (for their own understanding) needed not to be turned out of smaller terms into greater, but yet the easiness of working needed it.

Scholar. Sir, I understand now, not onely the difference of this need, (for the Fractions might better be understood as Fractions severall,

each in his value, when they were in lesser terms, although they could not so well be reduced) but also I understand what you mean by greater terms and lesser terms, whereof before I was in doubt: for I see you call the Numerator and Denominator the terms of the Fraction.

Terms of Fractions.

Master. I am glad you understand it so well. Now then when you would value any *fractions*, because they may best be done when the terms are smallest, you shall reduce them to the smallest that you can; which thing you may doe thus: Divide the greatest of any such two terms by the lesser, and if any thing remain, by that remainer divide the last *Divisor*; and if any thing remain now, by that divide the *first divisor* (which was before the remainer of the last *division*) and so continue still, till nothing do remain in the *division*: and then mark your last *divisor*; for it is the *number* that will easily reduce your *fractions*, if you divide both the *numerator* and the *denominator* by the same *number*, and put for the *numerator* the *quotient* of his *division*, and for the *denominator* also his *quotient* that riseth by his *division*.

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, and the quotient is 5, and there resteth 6: what shall I doe with this quotient?

Master. Nothing in this work; but now seeing there remaineth somewhat, by that remainer must you divide the last divisor.

Scholar. If I shall divide 18 which was the last divisor by 6 that was the remainer, so is the Quotient 3, and nothing resteth.

Master. As for the Quotient, I omit him yet: but

but because there doth remain nothing, therefore is 6 (which was your last divisor) that number by which you may reduce the fraction propounded.

Scholar. Then, as you taught me, I must divide the Numerator 18 by 6, and the quotient is 3, which I must put for the numerator over a line thus: and then by the said 6 must I divide also the denominator 96, and the Quotient will be 16, which I must take for the denominator; and so is the Fraction $\frac{3}{16}$. And so methinketh this Rule doth prove the work of the first Reduction.

$$\frac{3}{16}$$

Master. That is true, if the first Reduction were made of fractions into their least terms; and else not, without some help, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$, which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it again into the least terms, I must divide 96 by 64, and there remaineth 32; wherefore I must take that 32 for the divisor, to reduce the said Fractions. Then do you divide 64 by 32, and the Quotient is 2, which I set for my numerator. Again, I divide 96 by 32, and the Quotient will be 3, and so have I but $\frac{2}{3}$.

Master. Muse not at the matter, for you have done well enough: but you think you have not the fraction that you looked for, that is, $\frac{4}{6}$; yet have you one equal to it, as by the parts of a shilling you may prove.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$ be all three equal. And now I perceive that because $\frac{4}{6}$

was not written in the least terms that it might be, therefore this Reduction brought forth not it, but that other which is written in the least terms. Now understand I this Rule well. But is there any other way to work this Reduction?

[A]nother [w]ay to [w]ork this [R]eduction.

Master. Yes: But first note this, That if you find no such Divisor, to reduce the fraction till you come to 1, because one doth make no division, therefore that fraction is already in his least terms, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{98}$, and many other like.

Note that to mediate any number is to divide by two.

But now for your better aid to find the due proportion in less terms, with more ease for a young learner, you shall mediate or take the half of the Numerator, and also of the Denominator, as long as you may upon a line, alwaies parting them with a right-down dash of your pen as you work: which may easily be done, if the numbers be even, as 2, 4, 6, 8, or 10; but if they be odde, (though it be but one of them) then must you abbreviate them by 3, 5, 7, or 9, &c.

And because examples do most instruct, I have here set down the manner of two or three, whose last number at the end of the line sheweth the least term of valuation of that Fraction.

As for example; I would reduce $\frac{288}{576}$ into his least term or value, whereupon I set forth $\frac{288}{576}$ with a long line drawn from it, thus:

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator and the Denominator end in even numbers, I see this may be abbreviated by 2, or 4, or 6, &c. Therefore on the

the other side of the right-down dash toward the right hand, I first take the half of the Numerator, saying, the half of 2 is 1, the half of 8 is 4, and again, the half of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right-down dash as before.

Then do I also take the half of 576, in saying, the half of 5 is 2, and the half of 17 is 8, and the half of 16 is 8; and so have I 288 for a new Denominator.

Then beginning again, saying, the half of 144 is 72, and the half of 288 is 144. Thus continuing the mediation of division by 2, untill you come to the last work; as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, which is equal to $\frac{288}{576}$.

So the second example $\frac{28}{112}$ first abbreviated by 2, and again by 2, and last by 7, is reduced to $\frac{1}{4}$, which is equal to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Again, $\frac{1465}{4395}$ abbreviated first by 5, then by 293:

$\frac{1465}{4395}$	$\frac{293}{879}$	$\frac{1}{3}$

Scholar. Sir, I thank you much, this is very easie and good for a young learner.

Master. So it is: but yet notwithstanding, if you can, without that division, by memory espy the greatest number that may divide exactly both terms of your Fractions proposed, then need you not to use that division; as in this Fraction $\frac{60}{98}$, I see

I see that 12 is the greatest number that can divide them both, and therefore without any work, by memory onely, I turn that into $\frac{3}{4}$. But this ability in knowledge is got by exercise.

Yet one other way of easie Reduction in this kind there is: When your fraction hath any cyphers in the first places of both terms, then may you by casting away the Cyphers make a brief Reduction, as thus, $\frac{300}{400}$. Here take away the Cyphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I have $\frac{400}{650}$, it will be $\frac{4}{65}$.

Master. You are deceived, for you take away more cyphers from the Numerator then you do from the Denominator, which you may not doe.

Scholar. I confess my fault, which came of too much hast; I was gladder of the Rule then wise in using it: but now I understand it, I trust.

Master. Then may I go in hand with the fifth or last kind of Reduction, which teacheth how to turn any fraction proposed into any other Denomination that you list, or into any part of common coins, weights, or measures, or such like.

The fifth kind of Reduction.

For declaration whereof, first you shall mark whether your *fraction* be a simple *fraction*, or else a *fraction* of sundry parts, I mean of more terms then two. And if your *fraction* be a *fraction* of *fractions*, or otherwise compound, you must reduce it to one simple *fraction*. And then mark well the *denomination* of that other *fraction* into which you would turn this: for by that *denominator* you must multiply the *numerator* of your first *fraction*, and the total *Product* thereof shall you divide by the *denominator* of your first *fraction*, and that *quotient* shall be the *numerator* of the

To reduce fractions to a denomination appointed.

deno-

denominator proposed. As for *example*, I have this *fraction* $\frac{3}{5}$, which I would turn into ten parts: therefore I multiply this 10 by 3, that is the *numerator* of my *fraction*, & there ariseth 30, which I divide by 5, and the *quotient* is 6, which must be the *numerator* to 10; and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholar. This is easie enough to doe.

Master. Then shall you see another example of the same fraction that is not so easie; as if I would turn $\frac{3}{5}$ into 8 parts: prove you that work.

Scholar. I must multiply 8 by 3, and there amounteth 24, which I divide by 5, and the quotient is 4; then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this work?

Scholar. I see that when 24 was divided by 5, there remained 4, which I did not pass of, because you spake nothing of any remainer, but onely of the quotient.

Master. By likelihood you remember what I said to you in Division of whole numbers, that you should not pass of the remainer there, but onely note it as a summe that could not be divided without knowledge of Fractions. Wherefore now mark this, That in all divisions of whole numbers, when there is any remainer, you shall set it over a line as a Numerator, and set the divisor for the denominator, and that Fraction doth make the Division compleat, and is part of the quotient. As if I would divide 48 by 5, the quotient will be 9 $\frac{3}{5}$: so in your former work when 24 was divided by 5, the quotient should be 4 $\frac{4}{5}$, and so the new Fraction should be thus, $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$

part of any thing, which you may prove by example of some Coin.

Scholar. Then I take a Crown, whose $\frac{3}{5}$ is 3 s. Now I would prove whether the 3 s. be $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$. I shall have a cumbrous work to doe.

Master. Indeed for whole pence your example is a little troublesome: yet turning the Crown into half-pence, it is easie enough.

Scholar. That will I try.

¶ First, I see that $\frac{3}{5}$ of a Crown is 3 shillings, which is 36 pence, or 72 half-pence. Now if I can find that this Fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equal unto 3 shillings, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a Crown, I turn the Crown into half-pence, as you willed me, which makes 120, which I divide by 8; my Quotient is 15, which taken 4 times make 60 ob. Now resteth me to have $\frac{4}{5}$ of the $\frac{1}{8}$ part of a Crown, whereof $\frac{1}{8}$ part is 15 ob; the 15 being parted into 5 parts the Quotient is 3, which taken four times maketh 12 ob, which with my 60 before amounteth to 72, which are then equal to $\frac{3}{5}$, my desire.

Master. I commend you for your diligence, you might have wrought it thus either. $\frac{4}{8}$, being abbreviated as before I taught, is $\frac{1}{2}$: now half a Crown is 2 shillings 6 pence: now $\frac{4}{5}$ of $\frac{1}{8}$ is a Fraction of Fractions, which if you do reduce into one entire Fraction, as before you have learned, in saying, 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new numerator, it maketh $\frac{4}{40}$, and abbreviated also maketh $\frac{1}{10}$: now the tenth part of a Crown is 6 pence, which put to 2 shillings 6 pence make also 3 shillings, your desire.

But

But now one example more for this Rule, and then we shall end it. If I have $\frac{7}{15}$ of a Soveraign, (accounting the Soveraign 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholar. I must multiply 7 by 20, and that maketh 140, which I shall divide by 15, and the quotient will be 9 $\frac{5}{15}$, or in lesser terms $\frac{1}{3}$.

Master. That is 9 shillings, and one third part of a shilling, that is 4 pence, as by the same Rule you may prove. And this for this time shall suffice for Reduction: And now I will proceed to Addition.

Addition.

Addition of fractions of one denomination.

W*Hensoever you have any* Fractions *to be added, you must consider whether they be of one* denomination, *or not; and if they be of one* denomination, *then adde the* Numerators *together, and set that that amounteth for the* numerator *over the common* denominator, *and so have you done. The reason is, because that such differ little in* Addition *or* Subtraction *from the work of vulgar* denominations, *where the* denominators *be of the* number; *as* 3 pence *and* 5 pence *make* 8 pence, *where the* denomination *is not altered. But if the* fractions *be not of one* denomination, *or any of them be mixt of whole* numbers *and* fractions, *then must you first reduce them to one* denomination, *and after adde them. And if they be many, then adde first two of them, and so the* summe *that doth amount of the* Addition, *and the third, and then the* 4, *&c. if you have so many.*

Scholar.

Scholar. This seemeth easie enough, now that I have already learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you did place Reduction before Addition.

Master. It is well considered; but yet refuse not to expresse your understanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$; and because the denominators are like, (and so need no Reduction) I adde 7 to 5, which maketh 12; and then is my summe $\frac{12}{18}$, that is, in smaller numbers, being abbreviated, $\frac{2}{3}$.

To adde fractions of divers Denominations.

And if you have many numbers to be added, as here $\frac{3}{8}$, $\frac{4}{5}$, $\frac{9}{10}$, first I must reduce them (because they have divers denominators) into one denomination, and then they will be thus, $\frac{150}{400}$, $\frac{320}{400}$, $\frac{360}{400}$, or, in lesser terms, $\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$, that is $2\frac{3}{40}$.

Master. Now may we go to Subtraction.

Subtraction of Fractions.

Subtraction of Fractions.

Subtraction *hath the same precepts that* Addition *had: For if the* denominators *be like, then must you subtract the one* numerator *from the other, and the rest is to be set over the common* denominator, *and so your* subtraction *is ended: but and if you have many* fractions *to be subtracted out of many, then must you reduce them to one* denomination, *and into two several* fractions, *that is,*

all

all that must be subtracted into one fraction, *and the residue into another* fraction, *and then work as I said before.*

Scholar. For the first example I take $\frac{11}{12}$ to be subtracted out of $\frac{13}{12}$, and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be this, $\frac{24}{32}$ and $\frac{28}{32}$.

Then do I subtract 24 out of 28, and there resteth 4, which I set over the common denominator for a Remainer, thus, $\frac{4}{32}$; that is, $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ and $\frac{9}{10}$; and because their denominators be divers, I do reduce them into one denomination thus, $\frac{38}{24}$ and $\frac{142}{180}$ $\frac{3408}{1920}$ and $\frac{3040}{1920}$.

Then do I adde the two first, and they make $\frac{3040}{1920}$. Also I adde the two last, and they yield $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368. So is the remainer $\frac{368}{1920}$, that is in smaller terms $\frac{23}{120}$. And thus have I done with Subtraction, except you have any more to teach me.

Master. Prove one example or more out of Fractions of divers Denominations.

Scholar, I take two Fractions $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced will stand thus, $\frac{168}{192}$ and $\frac{72}{192}$: Now would I subtract 168 out of 72, but I cannot.

$$\begin{array}{ccc} 168 & & 72 \\ \frac{7}{8} & & \frac{9}{24} \\ & 192 & \end{array}$$

Master. Then may you perceive that you mistook the Fractions: for you can never subtract the greater out of the lesser, although you may adde, multiply or divide the greater with the lesser.

And

The greateſt of two fractions.

And albeit that $\frac{7}{8}$ hath both his terms leſſer then $\frac{9}{24}$, yet is $\frac{9}{24}$ the leſſer Fraction: for generally if you multiply the numerator and the Denominators of two Fractions croſs-waies, that fraction is the greateſt of whoſe Numerator cometh the greateſt ſumme: as in this example, 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yieldeth but 72; therefore is the firſt fraction 7 the greateſt of theſe two; ſo can you not ſubtract it out of a leſſer Fraction.

But if you ſhould ſubtract a Fraction out of a whole number, what ſhould you doe?

Scholar. Marry I would reduce the whole number into a Fraction of the ſame Denomination that my Fraction is, and then work by Subtraction.

Maſter. So may you doe; but it is much eaſier, if your Fraction be a proper Fraction, that is to ſay, leſs then an unite, to take an unite from the whole number, and then turn it into an improper Fraction, and ſo work your Subtraction. As if I would ſubtract $3\frac{3}{5}$ from 4, I may take 1 from 4, and turn it into $\frac{5}{5}$, from which I abate $3\frac{2}{5}$, there will remain $\frac{3}{5}$. And if the firſt be an improper Fraction, then may I take ſo many unites from the whole number, that they may make an improper Fraction greater then that firſt, and then work by Subtraction. As if there be propoſed $\frac{10}{3}$ to be ſubtracted from 6, becauſe $\frac{10}{3}$ is more then 3, and not ſo much as 4, I muſt take 4 from 6, and turn them into thirds thus, $\frac{12}{3}$; then abate $\frac{10}{3}$ from $\frac{12}{3}$, there reſteth $\frac{2}{3}$: ſo the whole remainer is $2\frac{2}{3}$. Or elſe you may at your pleaſure take $3\frac{1}{3}$, which is $\frac{10}{3}$, from 6 whole; then ſet 1 under 6, as thus, $\frac{6}{1}$: And then

then to reduce those two Fractions into one Denomination, as here appeareth, $\frac{10}{3}$ from $\frac{6}{1}$: Then $\frac{10}{3}$ from $\frac{18}{3}$, resteth $\frac{8}{3}$, which maketh 2 $\frac{2}{3}$, your desire. And thus will I make an end of the work of subtraction of Fractions, and proceed to Multiplication.

8

$\frac{10}{3}$ X $\frac{18}{3}$

3

Multiplication of Fractions.

Therefore when any two fractions *be proposed to be multiplied together, the* numerator *of the one must be multiplied by the* numerator *of the other; and the summe that amounteth thereof must be set for a new* numerator *: likewise the* Denominator *of the one must be multiplied by the* Denominator *of the other, and that that amounteth shall be set for the* Denominator : *and this new third* Fraction *expresseth the* Product *of the* Multiplication *of the two first* Fractions *proposed: Whereof take this* Example, $\frac{3}{5}$ *multiplied by* $\frac{5}{12}$ *doth make* $\frac{15}{60}$. Multiplication of Fractions.

15

$\frac{3}{5}$ X $\frac{5}{12}$

60

Scholar. I perceive then that 3 being the Numerator of the first Fraction is multiplied by 5 being the Numerator of the second Fraction, whereof amounteth 15, the Numerator of the third Fraction; and so likewise 5 being the Denominator of the first Fraction is multiplied by 12 the Denominator of the second Fraction, whereof amounteth 60, the new denominator: so that

that I perceive how the work is done. I do not perceive how $\frac{15}{60}$ is greater then $\frac{3}{5}$: for if I shall use my former manner of examination by the parts of some coin, I see that $\frac{3}{5}$ of a Crown is 36 pence, and $\frac{5}{12}$ of a Crown is 25 pence, whereof the one multiplied by the other doth make 900 pence, which is 15 Crowns; but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much less then any other of both the first Fractions.

Master. That difference is between multiplication in whole numbers, and multiplication in broken numbers: that in whole numbers the summe that amounteth is greater then both the other whereof it came; but in fractions it is contrariwise, for the summe that amounteth is lesser then any of the other two fractions whereof it is produced.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserve the reasons of works Arithmeticall for the perfect Book of Arithmetick, yet I will shew you this, because of the strangeness of the work.

You see in whole numbers, that of two numbers being multiplied together is made the third number, which third number doth bear the same proportion to the number multiplied that the multiplier doth bear to an unine. And so in Fractions, the third number which amounteth of Multiplication beareth the same proportion to each of the two first Fractions that the other of those two fractions doth bear to an unite.

Scholar. Sir, I understand your words thus: when 40 is multiplied by 12, there doth amount 480,

which

which 480 doth contain 40 so many times in it as 12 doth contain Unites, that is to say, twelve times; and so it appeareth that 480 doth contain twelve so many times also as 40 doth contain unites, that is, 40 times. But now I see not how the third number in this example of Fractions can contain any of the two former, (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to be seen of any man, how the third number in multiplication of Fractions should be greater then any of the two former Fractions: but yet this may you see, (which I said) that the third number in fractions so multiplied doth bear the same proportion to any of the two former Fractions that the other of those two Fractions doth bear to an unite. As in your example, $\frac{3}{5}$ being multiplied by $\frac{5}{12}$ doth make $\frac{15}{60}$: now I say that $\frac{15}{60}$ doth bear the same proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth bear to a unite, as you may in your own form of examination by Coin try it: for in an old Angel (which in times past was currant for 7 shillings 6 pence) are 180 half-pence, which I set for the intire unite, whose parts (according to the Fractions aforesaid) are these, for $\frac{15}{60}$ set 45 half-pence, for $\frac{3}{5}$ take 108 half-pence, and for $\frac{5}{12}$ put 75 half-pence: now doth 45 bear the same proportion to 108 that 75 doth bear to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserved till another time, when the knowledge of proportions in due order shall be taught: yet in the mean season I will

I will ſhew you how it cometh to paſs, that i Fractions the third ſumme muſt needs be leſſer th any of the other two.

Conſider this, that when a Fraction is propoſe as in the former example $\frac{3}{5}$, if it be multiplied b more then 1, it will make more then one enti number: as if I multiply $\frac{3}{5}$ by 5, that is to ſay, i I take it five times, it will make three entire unite Example: in a Crown $\frac{3}{5}$ of it maketh 3 ſhilling which if I take five times, it will amount to 1 ſhillings, that is, 3 entire Crowns; ſo if I ta the ſame $\frac{3}{5}$ but twice, it will yield 6 ſhillings, tha is, one entire Crown and $\frac{1}{5}$: now if I take it bu once, it cannot be more then it was before, that is 5 ſhillings; and if I take it leſs then once, it canno be ſo much as it was before. Then ſeeing that Fraction is leſs then one, if I multiply a Fraction by another Fraction, it followeth that I do take the firſt Fraction leſs then once, and therefore the ſumme that amounteth muſt needs be leſs then the firſt Fraction.

Scholar. Sir, I thank you much for this reaſon: and I truſt I do perceive the thing, as by example of this ſame Fraction $\frac{3}{5}$ I will expreſs. If I take $\frac{3}{5}$ of a Crown once, that is to ſay, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 ſhillings; ſo if I do multiply it by $\frac{1}{2}$, that is, if I take but half one time, then will it be but half ſo much; likewiſe if I multiply it by $\frac{1}{3}$, that is, if I take but the third part of one, it will yield but 12 pence, that is, the third part of the firſt Fraction.

And

And so, to make an end, if I take but the twelfth part of one, that is, if I do multiply it by $\frac{1}{12}$, it will yield but the twelfth part of the first Fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ must needs make five times so much, that is 15 pence, which was the summe that hath been the occasion of all this doubt.

Master. Whereby I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will proceed to the rest.

To multiply a whole number into a fraction.

In *Multiplication* it happeneth sometime that there be whole numbers to be multiplied with Fractions, and may be in two sorts: for either the whole number is several from the fraction, and is the *multiplier*; or else the whole number is joyned with one or both of the fractions, and so maketh a *mixt number* thereof. If it be in the first sort, then needeth there no *Reduction*, but onely multiply the *numerator* of the fraction by that whole number, and the total thereof set for the new *numerator*.

Scholar. I understand you thus: If I have $\frac{6}{23}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator, whereof cometh 96, and that must I set for the new Numerator, keeping still 23 for the Denominator; and so the Fraction will be $\frac{96}{23}$, that is $4\frac{4}{23}$.

Master. And in this sort of work you may abridge the labour thus: If it happen the denominator to be such a number as may evenly be divided by the said whole number proposed, then divide it thereby, and set the quotient of that division for the former denominator, but reserve still the numerator; and so is the multiplication ended.

Scholar.

Scholar. Then seeing this example $\frac{7}{20}$ to be multiplied by 5, and because 5 will justly divide 20, therefore I take the Quotient of that division, which is 4, and set it in stead of 20, and so the Fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$, that would have followed of the other sort of work.

Scholar. I perceive it well.

How to multiply mixt numbers. *Master.* Now then for the other sort, where the number is *mixt*, take this way: first to reduce the said whole number and Fraction into one improper Fraction, (as I shewed you in *Reduction*;) and then multiply them together, as if they were proper Fractions.

Scholar. $13\frac{3}{5}$ being set to be multiplied by $\frac{5}{8}$, first I must reduce the mixt number, as in this example appeareth, by multiplying 13 by 5, and that makes 65, whereto I must adde the numerator 3, and so the Fraction will be $\frac{68}{5}$; which two Fractions now I shall multiply after the accustomed form, and it will be $\frac{340}{40}$, or $\frac{34}{4}$.

$13\frac{3}{5}$	340
68 by	5
5 40	8

Master. You have done well: and so may you see, that although most part of the forms of multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

Duplation. And yet one note more I will tell you of *Multiplication* before we leave it: that is, Whensoever you would multiply any Fraction by 2, which commonly is called *Duplation*, you may doe it not onely by doubling the *numerator*, but also by parting the *denominator* into half, if it be even.

Scholar. Then if I would double $\frac{5}{12}$, I may chuse

chuse whether I will make it $\frac{2}{4}$ or else $\frac{1}{2}$. And indeed I see that is all one, but the dividing of the Denominator seemeth the better way to make smaller terms of the Fractions, and so they shall need the less Reduction.

Master. It is so. And now I shall not need to tell you that Multiplication is proved by Division, and Division likewise by Multiplication: but the like work that I shewed you in multiplication, will I shew you in division.

Division of Fractions.

WHensoever two *Fractions* be proposed, that one should be divided by the other, I must set down first the *Fraction* that shall be divided, (which is called the *dividend*) and then after it the other, which is the *divisor*. Then shall I multiply the *numerator* of the *dividend* by the *denominator* of the *divisor*, and that which amounteth I must put for a new *numerator*. Again I shall multiply the *denominator* of the *dividend* by the *numerator* of the *divisor*, and the *number* that amounteth thereof I must put for the new *denominator*. And this third *fraction* is the *Quotient* of the said *division*.

Division of Fractions.

Scholar. This seemeth easie in form, as by example thus: If I would divide $\frac{5}{8}$ by $\frac{2}{6}$, first I multiply 5 (being the numerator of the dividend) by 6, which is the denominator of the divisor, and

and thereof riseth [illegible] then I multiply 8 (being the denominator of the dividend) by 2, being the numerator in the divisor, and so riseth 16, the which I must make a third Fraction thus, $\frac{30}{16}$.

	30	
$\frac{5}{8}$		$\frac{2}{9}$
	16	

Master. Methinketh you are quicker in understanding now then you were when I taught you the Art of whole numbers, but that is no marvel; for the more knowledge that a man getteth, the readier shall be and his wit, and be quicker in understanding: but yet of two things I will admonish you, which you might have observed here for the ease of work, and lightness of understanding the nature of the Quotient.

Whensoever you divide one Fraction by another, either they be both equal together, or else the one is greater then the other: if they be equal, their quotient shall be such that the Numerator and the Denominator of it shall be equal also; and if the two first Fractions be unequal, their quotient shall declare the same by the inequality of the Numerator and Denominator: as in these examples following shall appear.

First, if equal Fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equal together, and if the one be divided by the other, the quotient will be $\frac{108}{108}$; as you may perceive by that Rule aforesaid.

Now in the unequal Fractions, as $\frac{4}{9}$ and $\frac{3}{10}$, the quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Master.

Master. The reason is this; when any Fraction is divided by another, the Quotient declareth what proportion the Dividend beareth to the Divisor: so $\frac{1}{2}$ divided by $\frac{1}{4}$ maketh 2, which must be so named, not 2, but twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

Note how to know the proportion between two numbers.

And note this, that the Numerator in the Quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is always true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be greatly known till you have learned the Art of proportions; notwithstanding somewhat of it I have declared in the Rule of Reduction. But now for the safe remembrance of the Quotient in division, as soon as you [illegible] down your two Fractions the one against [illegible], then make a straight line for the Quotient, and as soon as you have multiplied the Numerator of the dividend by the denominator of the divisor, set the Number that amounteth over the said line, and then multiply the other two Numbers, and set their total under the same line.

Scholar. I perceive you would not have me trust to memory till I were better expert, lest oftentimes I happen by false remembrance to be abused. This Example I take for that declaration.

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as here doth appear) and then make another line for the quotient in some good distance,

$$\frac{2}{3} \text{ by } \frac{3}{4}$$

distinctly, where I may set the numbers of the Quotient as soon as any of them is multiplied. So that as soon as I have multiplied 2 by 4, which maketh 8, I shall set that 8 over that line, thus; and then multiply 3 by 3, which yieldeth 9; and that 9 must be set under the same line: and then will the whole Quotient appear thus, $\frac{8}{9}$. [illegible] it appeareth (as I remember your words) [illegible] in proportion [illegible] but how may I perceive that [illegible]

Master. Although you might better perceive it by the [illegible] of Reduction, [illegible] example may be [illegible] common coins, as in a common shilling [illegible] pence, [illegible] 8 pence, and 4 [illegible] you may easily see that [illegible] And if you had taken this example [illegible] the example of [illegible] Quotient would appear (as this doth) more [illegible] whereas that Quotient, being [illegible] is not an easy proportion for you to perceive, being yet little acquainted with proportions.

To divide a whole number by a fraction.

Scholar. If there be whole numbers to be divided by a Fraction, how shall I perform it?

Master. When any whole number shall be divided by a *Fraction*, you must multiply the said whole number with the *Denominator* of the *fraction*, and set the totall thereof for the new *Numerator*, and for the *Denominator* set the *Numerator* of the *fraction*.

To divide the fraction by the whole number.

Scholar. Then 20 divided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth, $\frac{80}{3}$. $\frac{20}{1}$ by $\frac{3}{4}$

Master. Even so: but if you would divide the *Fraction* by the whole number, then multiply the *Denominator* by the same whole number, and set the totall for

for the Denominator, without changing the Numerator.

20
20 by 4
23 4
92

Scholar. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth; $\frac{20}{23}$ by $\frac{4}{1}$ in this Example, $\frac{20}{92}$.

Master. You say well. And by the same Example you give me cause to remember another brief way to do the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not onely as well done, but also in a Fraction of lesser terms. Another brief way.

Scholar. I guess it to be even so by a like work that you taught me in multiplication: And for proof thereof, $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set for 20 over 23, thus, $\frac{5}{23}$; and I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these terms by 4, and so reducing them to their least Denomination, I may easily prove: as appeareth by this example, $\frac{20}{92}$ $\frac{5}{23}$.

Master. You conceive it well. And if there be mixt numbers, (either one or both) you must first reduce that mixt number into an improper Fraction, and then work as you have learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Division: If the denominators be like, then divide the numerators, as if it were in whole numbers, and the Quotient, whether it be Fraction, whole number, or mixt, is a good

good Quotient for that Division. And generally, if one of the numerators may justly divide the other, by that Quotient, multiply the Denominator of the lesser numerator, and set it that doth amount in the room of the same denominator; and then for a numerator to it, set the denominator of the other Fraction.

Scholar. Then if I would divide $\frac{3}{4}$ by $\frac{12}{17}$, I see that 3 will divide 12, and the Quotient will be 4, by which I must multiply the other 4, that is the denominator under 3, and then it is 16, which I set for the denominator 4, and over it in stead of 3 I must set 17, the other denominator; and so it is thus, $\frac{17}{16}$.

Master. And so is $\frac{17}{16}$ in stead of $\frac{51}{48}$ which would have risen by the common work, as here appeareth.

$$\begin{array}{ccc} & 51 & \\ 3 & & 12 \\ \hline 4 & \text{by} & 17 \\ & 48 & \end{array}$$

And now for *Mediation*, (which is to divide by 2) mark this, If the *Numerator* be an even number, set the half of it in his place without the *divisor*, and so have you done: and if the *numerator* be not even, then double the *denominator*.

Scholar. That is, if I would mediate $\frac{16}{11}$, I may make the Quotient $\frac{8}{11}$, and if I would mediate $\frac{7}{11}$, I must make it $\frac{7}{22}$.

Master. And thus will I make an end of the works of common fractions for this time, not doubting but you can apply them both to the Rule of progression, and also to the Golden Rule, without any other teaching then you have learned before, which might seem tedious to repeat, in regard you have sufficient knowledge in Reduction,

Addition,

Addition, Subtraction, Multiplication, and Division: and therefore will I goe in hand with the Rule of proportion, or Golden Rule, which now will appear more plainly.

The Golden Rule direct in Fractions.

THerefore as touching the Golden Rule, for the placing of the three numbers proposed in the question whereby to find the fourth, and for the form of their work, with other like things, I referre you to that which you have already learned. The rule of proportion in Fractions.

But this same form of working by fractions shall you note, that if your three numbers be fractions, for apt work and certain, multiply the denominator of the first number in the question by the denominator of the second, and all that again multiply by the denominator of the third number; and the totall thereof shall you keep for to be the divisor. Then multiply the Denominator of the first number by the numerator of the second, and the whole thereof by the numerator of the third, and the totall thereof shall be your dividend. Note this for a general Rule.

Now divide this dividend by the divisor which you found out before, and that number shall be the fourth number of the question which you seek for: as in this example.

If $\frac{3}{4}$ of a yard of Velvet cost $\frac{2}{3}$ of a Soveraign, esteemed at 20 shillings, what shall $\frac{1}{6}$ cost? A question of Velvet.

Scholar. If it please you to let me make the answer, I would first place these three numbers as I learned in whole numbers, thus:

$$\begin{array}{ccc} \frac{3}{4} & & \frac{2}{3} \\ \frac{1}{6} & & \end{array}$$

And

And then, according to your new rule, I must multiply 3, being numerator in the first number, by 3, the denominator of the second, and thereof cometh 9; which I multiply again by 6, the denominator of the third number, and so have I 54, which I keep for the divisor. Then multiply I 4, the denominator of the first, by 2, the numerator of the second, and thereof riseth 8; which again I multiply by 5, the numerator of the third, and it maketh 40. Then must I divide 40 by 54, and it will be $\frac{40}{54}$, that is $\frac{20}{27}$ in lesser terms; and then the figure will stand thus:

$\frac{3}{4}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{20}{27}$

But what that is in money I cannot tell, except I shall work it by Reduction, as you taught me.

Master. It needeth not now, you may reduce it when you list, but it were disorderly done here to mingle divers works together, where we do not seek the value of the thing in common money, but in apt number, which you have well done: and therefore will I now shew you another like way of easiness in work, how you may change your three Fractions into three whole numbers, by which you shall work as if the question was proposed in whole numbers. The first number you shall find as I taught you. Now to find the divisor of the second number, take the numerator for the second fraction; and for the third number, take that that ariseth of the multiplication of the denominator of the first by the numerator of the third, and then work your question.

A question of Silver.

Scholar. For example hereof, I put this question: If $\frac{11}{12}$ of 1 pound weight of silver be worth [illegible] of a Soveraign, what is [illegible] of 1 pound weight worth?

For

For the Answer, first I place the Fractions in order thus:

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{2}$

Then to turn these Fractions into whole numbers, I multiply 11, which is the numerator of the first, by 4 the denominator of the second, and there cometh 44, which I multiply by 2, the denominator of the third, and so amounteth 88, which I set for the Divisor in the first place. Then in the second place I set 12, which is the numerator of the second fraction; and in the third place I set the summe that amounteth of 12, being the denominator of the first number, multiplied by one, being numerator in the third number: and so the figure will stand as here you see.

88 Z 12
12

Then to work it forth, I multiply 12 by 12, and there amounteth 144, which I divide by 88, and the quotient will be 1 $\frac{56}{88}$, or, in lesser terms, 1 $\frac{7}{11}$, and then the figures will stand thus.

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{2}$ 1 $\frac{7}{11}$

Master. These two forms now you understand well enough, and as for any other at this time I will not repeat; onely this shall you mark for the proof of this Rule, whether your work be well wrought or no: Multiply the first number by the fourth, and note what amounteth; then multiply the second by the third, and mark what amounteth also. Now if those two numbers so amounting be equal, then is your work well done, else you have erred. And this shall suffice for the former Rule.

The proof of the Golden rule.

The

The Backer Rule, or Reverse Rule in Fractions.

The backer Rule in Fractions.

BUt in the Backer Rule this shall you note for your ease of work, that you multiply the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof shall be the Dividend. Then multiply the Denominator of the first by the denominator of the second, and the whole by the Numerator of the third, and that that ariseth thereof shall be the Divisor. Example of this:

Note this also for a general Rule.

A question of Loan.

I did lend my friend $\frac{3}{4}$ of a *Porteguise* seven Months, upon promise that he should do as much for me again; and when I should borrow of him, he could lend me but $\frac{5}{12}$ of a *Porteguise*: now I demand how long time I must keep his money in just recompence of my loan, accounting 13 Months in the year.

Scholar. The first number must be the first money borrowed, that is $\frac{3}{4}$ of the Porteguise: the second number the 7 months, that is $\frac{7}{13}$ of a year: and the third number the mony that was lent in recompence, that is $\frac{5}{12}$ of a Porteguise. Then I set the numbers thus:

$$\begin{array}{ccc} \frac{3}{4} & & \frac{7}{13} \\ & Z & \\ \frac{5}{12} & & \end{array}$$

Then (as you taught me) I multiply 3 (being Numerator in the first number) by 7, the Numerator of the second number, and it maketh 21, which I multiply by 12, the Denominator of the third; and so have I 252 for the dividend: then I multiply 4, the Denominator of the first, by 13, the Denomina-

tor

tor of the 2d, and it yieldeth 252, which I multiply again by 5, the Numerator of the third, & it will make 160, that is the divisor. Then must I divide 252 by 160, so it will be in the small Fraction $\frac{63}{40}$ of a year.

Master. And thus do you see some ease in working, better then to multiply and divide tediously so many Fractions.

Another question yet I will propose, to the intent you may see thereby the reason of the Statute of Assise of Bread and Ale, which in all Statute-Books, in Latine, French, and English, is much corrupted for want of knowledge in this Art: for the right understanding whereof I propose this question. Statute of Assise of bread and ale.

When the price of a quarter of Wheat is 2 *shillings*, the farthing white loaf shall weigh 68 *shillings*: then I demand what shall such a loaf weigh when a quarter of Wheat is sold for 3 *shillings*. Question of bread.

Scholar. This question must be wrought as it is proposed, in whole numbers, and not in Fractions.

Master. You seem to say reasonably; howbeit in the Statute of Assise the rate is made by the proportion of parts in a pound-weight Troy, else could it not be a Statute of any long continuance, seeing the shillings do change often as all other moneys do: but this Statute, being well understood, is a continuall Rule for ever, as I will anon declare by a new Table of Assise, converting the shillings into ounces and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound weight, and so by a peny $\frac{1}{20}$ of an ounce: wherefore although you might work this question proposed by whole numbers well enough, for that time when the Statute was made,

yet

yet to apply it to your time, and make it serve for all times generally, it is best to work it by fractions, setting for 2 shillings $\frac{2}{20}$ and for 68 shillings $\frac{68}{20}$ and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus: In which question, because all the Denominators be like, you shall work onely with the Numerators.

$\frac{2}{20}$ $\frac{3}{20}$

Scholar. Then shall I multiply 68 by 2, whereof cometh 136; which if I divide by 3, the quotient will be 45 $\frac{1}{3}$: but how shall I make a fraction of that to stand with the other?

Master. Have you so soon forgotten what was taught you so lately? This is his form. $\frac{45\frac{1}{3}}{20}$

Scholar. I remember it now, and then it signifieth 45 twenty parts, and the third deal of one twentieth part.

Note what a shilling is. Master. So is it that it maketh in shillings 54 shillings 4 pence; whereby you may note one great errour in the Statute-Books, which have constantly 48 shillings in that Assise. And by this Rule if you examine the Statute, you shall find many summs false. Wherefore for the true understanding of that Statute and such like, as I have made mention of it, and somewhat recognized it, so do I wish that all Gentlemen and other Students of the Laws would not neglect this Art of Arithmetick as unneedfull to their studies. Wherefore to encourage them thereto, and to gratifie both them and all other in general, I will exhibit a Table of that part of the Statutes in two Columns, and in a third Column I will adde the correction of those errours which have crept into it.

Here followeth the Table.

The price of a quarter of Wheat.		The weight of a farthing white loaf, by the Statute-Books.			The Correction by just Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	[illegible]
2	0	2	8	0	3	8	0
2	6	2	14	4 $\frac{1}{3}$	2	14	4 [illegible]
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 [illegible]
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	[illegible]
5	0	1	8	2 $\frac{1}{2}$	1	7	2 $\frac{2}{3}$
5	6	1	4	8 [illegible]	1	4	8 $\frac{8}{11}$
6	0	1	2	8	1	2	8
6	6	0	16	11	1	0	11 $\frac{1}{13}$
7	0	0	19	1	0	19	5 [illegible]
7	6	0	18	[illegible] $\frac{1}{2}$	0	18	1 $\frac{3}{7}$
8	0	0	7	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 $\frac{1}{4}$	0	15	1 $\frac{1}{3}$
9	6	0	14	0 [illegible]	0	14	3 $\frac{15}{19}$
10	0	0	13	7 $\frac{1}{2}$	0	13	7 $\frac{2}{5}$
10	6	0	12	11 $\frac{1}{2}$	0	12	11 [illegible]
11	0	0	12	4 $\frac{1}{4}$	0	12	4 [illegible]
11	6	0	11	10	0	11	9 $\frac{21}{23}$
12	0	0	11	4	0	11	4

In the common Books there is no farther rate of Assise made then unto 12 s. the quarter of wheat, but in an ancient copy of 200 years old (which I have) there is added the rate of Assise unto 20 s. the quarter: but yet was that Assise also either wrong cast in the first penning, or else corrupt since that time, for lack of just knowledge in the *Rule of Proportion*; which I will adde here also, to gratifie such as be desirous to understand truth exactly.

The price of a quarter of Wheat.		The weight of a farthing white loaf by the Statute-Books.			The Correction by just Assise.		
s.	d.	l.	s.	d.	l.	s.	d.
12	6	0	11	0	0	10	10 [illegible]
13	0	0	11	0 2/7	0	10	5 7/[illegible]
13	6	0	10	1 1/2	0	10	0 [illegible]
14	0	0	9	7	0	9	8 [illegible]
14	6	0	9	2 1/2	0	9	4 1/[illegible]
15	0	0	9	1 1/2	0	9	0 [illegible]
15	6	0	9	1 1/4	0	8	9 [illegible]
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	2 10/1[illegible]
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	9 9/16
18	0	0	7	6	0	7	6 2/3
18	6	0	7	3	0	7	4 8/37
19	0	0	7	2	0	7	1 17/19
19	6	0	[illegible]	10	0	6	11 9/13
20	0	0	[illegible]	6	0	6	9 3/5

These

These two Tables I have set several, because no man should think that I would either adde or take away from any Law, those parts which might of right seem either superfluous or diminute: but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but every good Subjects duty with sobriety to correct. And for avoiding of offence, I have rather done it in this private Book, then in any Book of the Statutes it self, trusting that all men will take it in good part.

Scholar. I could wish so, but I dare not so hope, sith never good man that would reform errour could reform the venemous tongues of envious detractors, which because they either cannot or list not to doe any good themselves, do delight to bark at the doings of others: but I beseech you to stay nothing for their perverse behaviour.

Master. I consider many things that some may object, whereunto I am not unprovided of just answers; but I will not seem so hasty to make the answers, before I hear their Objections: but as I trust that men are of a better nature and more gratefull now then some have been in times past; as I have done in the Statute of Assise of bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applied to all times. But I mean not by this to alter any word of the Statute, (being so good an Ordinance, and of so great commodity) but onely to make it as a kind of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood, and consequently

Concerning the following Tables.

T   sequently

A Pound weight.

sequently better put in execution. And here you shall note, that I have accounted the shillings after the rate of 60 shillings to the pound weight, because I esteem it the most apt for our time. Wherefore in the first Column you find the price of Wheat directly against it; in the second Column you may find the weight of a farthing white loaf in this our time; and if you double the number, (as I have done in the third Column) then have you the weight of the half-peny white loaf; and so in the fourth Column is set the weight of a peny white loaf. It needeth not to tell, that the sight doth testifie how that every Column is parted into three smaller pillars, whereof the first Column hath these three titles, pounds, ounces, and peny weights. And as in the first Column 12 pence make a shilling, and 20 shillings make a pound; so in the other three Columns 20 pence weight maketh an ounce, and 12 ounces make a pound.

Gentle Reader, touching the understanding of the *Table* following; Therein, according to our time, Master *Record* alloweth 60 *pence* to the *ounce*, and 3 *pound* or 60 *shillings* to the *pound*, and thereupon, after the rate of 60 *shillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *shillings* the *quarter*, till it come to 40 *shillings* 6 *pence* the *quarter*. And this his proportion (for that he hath not set down any one *Example* to continue the work) hath been hard for many to conceive or comprehend; and therefore the onely chief cause why I have written this *digression* is for the better understanding of him therein.

The first thing therefore that is sought for in this *Table*, as in the other aforesaid, is a *Maxime* grounded upon the *Statute*, which

which is this: When the *quarter of Wheat* is sold for *two shillings*, then the *farthing white loaf* shall weigh 68 *shillings*; where by a *shilling* is $\frac{1}{20}$ meant of a *pound*, and by a *peny* $\frac{1}{20}$ of an *ounce*. Now therefore for a generall Rule, to find what weight the *farthing white loaf* shall weigh at 3 *shillings* the *quarter*, till you come to 40 *shillings* 6 *pence* the *quarter*, it is thus to be wrought. Coming to the *first ground*, and working by the *Backer Rule*, say, if *two shillings* the *quarter* give or allow the *farthing white loaf* to weigh 68 *shillings*, what weight ought the *farthing white loaf* to weigh at 3 *shillings* the *quarter*? Work, and you shall find 45 *shillings* 4 *pence*, as before in the correction of the *first Table* is noted.

Then for the *second work*, say by the *Rule of three direct*, if 20 *pence* give one *ounce*, what giveth 45 *shillings* 4 *pence*? Multiply and divide, and you shall find 544 *ounces*; which 544 *ounces* being multiplied by 3, for 3 *pounds* or 60 *shillings* yieldeth 1632 *ounces*, which divided by 20 produceth 81 *ounces*, and $\frac{12}{20}$ or rather $\frac{3}{5}$ of an *ounce*, equal unto 12 *peny weight*, which is *half an ounce* and 2 *peny weight*, and so maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces*, and 2 *peny weight*. Now the next way to continue this *Table*, to know the weight of the *half-peny white loaf*, is this; Multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and divided by 20 it yieldeth 163 *ounces* and $\frac{1}{5}$, which is equal to 13 *pounds*, 7 *ounces*, and 4 *peny weight*, as M. *Record* his *Table* noteth.

Thirdly, for the *weight* of the *peny white loaf*, multiply 1632 *ounces* by 4, and divide by 20, and after by 12, as before; and you shall find 27 *pounds*, 2 *ounces*, and 8 *peny weight*, &c. This *Method*, or else by doubling the *farthing white loaf* for the *weight* of the *half-peny white loaf*, and so doubling the *half-peny white loaf* for the *weight* of the *peny white loaf*, is the order to continue the *Table* to the end thereof.

The price of a quarter of Wheat. l.	s.	d.	The weight of a farthing white loaf. Po.	Ounces	Peny weight.
0	3	0	6	9	12
0	4	6	4	6	8
0	6	0	3	4	16
0	7	6	2	8	12 $\frac{4}{5}$
0	9	0	2	3	4
0	10	6	1	11	6 $\frac{2}{7}$
0	12	0	1	8	8
0	13	6	1	6	2 $\frac{2}{3}$
0	15	0	1	4	6 $\frac{2}{5}$
0	16	6	1	2	16 $\frac{8}{11}$
0	18	0	1	1	12
0	19	6	1	0	11 $\frac{1}{13}$
1	1	0		11	13 $\frac{1}{7}$
1	2	6		10	17 $\frac{3}{5}$
1	4	0		10	4
1	5	6		9	12
1	7	0		9	1 $\frac{1}{3}$
1	8	6		8	11 $\frac{15}{19}$
1	10	0		8	3 $\frac{1}{5}$
1	11	6		7	15 $\frac{3}{7}$
1	13	0		7	8 $\frac{4}{11}$
1	14	6		7	1 $\frac{21}{23}$
1	16	0		6	16
1	17	6		6	10 $\frac{14}{25}$
1	19	0		6	5 $\frac{7}{13}$
2	0	6		6	0 $\frac{8}{9}$

The

The price of a quarter of Wheat.			The weight of a half-peny white loaf.			The weight of a half-peny white loaf.		
l.	s.	d.	Po.	Ounces.	Peny weight.	Po.	Ounces.	Peny weight.
0	3	0	13	7	4	27	2	8
0	4	6	9	0	16	18	1	12
0	6	0	6	9	12	13	7	4
0	7	6	5	5	5 3/5	10	10	11 1/5
0	9	0	4	6	8	9	0	16
0	10	6	3	10	12 4/7	7	9	5 1/7
0	12	0	3	4	16	6	9	12
0	13	6	3	0	5 1/3	6	0	10 2/3
0	15	0	2	8	12 4/5	4	5	5 3/5
0	16	6	2	5	13 5/11	4	11	6 10/11
0	18	0	2	3	4	4	6	8
0	19	6	2	1	2 2/13	4	2	4 4/13
1	1	0	1	11	6 2/7	3	10	12 4/7
1	2	6	1	9	15 1/5	3	7	10 2/5
1	4	0	1	8	8	3	4	16
1	5	6	1	7	4	3	2	8
1	7	0	1	6	2 2/3	3	0	5 1/3
1	8	6	1	5	3 11/19	2	10	7 3/19
1	10	0	1	4	6 2/5	2	8	12 4/5
1	11	6	1	3	10 6/7	2	7	1 1/7
1	13	0	1	2	16 8/11	2	5	13 5/11
1	14	6	1	2	3 19/23	2	4	7 15/23
1	16	0	1	1	12	2	3	4
1	17	6	1	1	1 8/25	2	2	2 6/25
1	19	0	1	0	11 1/13	2	1	2 2/13
2	0	6	1	0	1 7/9	2	0	3 1/9

Having,

Having spoken before, for the understanding of the *Table* placed by M. *Record*, a man indued with rare knowledge in *Arithmetical* and *Geometrical Proportions*, touching the *Statute* of *Coynage* and the *Standard* thereof, as appeareth in his Epistle of this *Book* dedicated to King *Edward* the Sixth, insinuating unto his Highness, that the *Standard of Coyn* is much altered from the 14 year of King *Edward* the Third (when this *Statute* and *Assise* was confirmed) to the *Standard* of this *our time*: for it appeareth that in King *Edward* the Third's time, when the *Assise* of *Bread* and *Drink* was established, that a *Sterling peny, round without clipping*, did then weigh 32 *corns of wheat* dried, and taken out of the middle of the *ear*, and 20 of these *pence* made an *ounce*, and 12 *ounces* made a *pound Troy*; and so from the *weight* of a *peny* to 20 *shillings sterling*, which then weighed 12 *ounces*, took *Bread* his *weight* and proportion: and now finding 60 *pence* is an *ounce*, that onely cause (I perceive, for the zeal of the Commonwealth) moved him to set down the same *Table* in this private Book; meaning not thereby to alter any word of the *Statute*, being so good an Ordinance and of so long continuance, but as a kind of exposition by the way, that thereby the *Statute* may be better understood, and so consequently better put in execution; which *Assise* of his is three times greater then the *Statute* now alloweth: Therefore also, to gratifie such as are desirous of knowledge, according to these *prices of a quarter of wheat*, I have added to this *Author* these *three* other *new Tables* following, and reduced their *prices* into their just proportions of *Sterling mony*, and also reduced the *mony* into known *weight Troy*, according to the *Statute*; and thereafter, according to proportion in my other three *Tables*, have I noted the *just weight* that a *Farthing*, *Half-peny*, and *Peny white loaf*, ought to weigh by the *Statute*.

The

The price of a Quarter of Wheat. l.	s.	d.	The weight of a farthing white loaf in Sterling mony by Assise. l.	s.	d.	The weight of a farthing white loaf in Troy-weight by Assise. Po.	Ounces.	Peny weight.
0	3	0	2	5	4	2	3	4
0	4	6	1	10	2 2/3	1	6	2 2/3
0	6	0	1	2	8	1	1	12
0	7	6	0	18	1 3/5		10	17 1/2
0	9	0	0	15	1 1/3		9	1 1/3
0	10	6	0	12	15 3/7		7	15 3/7
0	12	0	0	11	4		6	16
0	13	6	0	10	0 8/9		6	0 8/9
0	15	0	0	9	0 4/5		5	8 4/5
0	16	6	0	8	2 11/10		4	18 10/11
0	18	0	0	7	6 2/3		4	10 2/3
0	19	6	0	6	11 9/13		4	3 7/13
1	1	0	0	6	9 3/5		4	1 1/2
1	2	6	0	6	0 8/13		3	12 8/15
1	4	0	0	5	8		3	8
1	5	6	0	5	4		3	4
1	7	0	0	5	0 4/9		3	0 4/9
1	8	6	0	4	9 5/19		3	17 5/19
1	10	0	0	4	6 2/7		2	14 2/5
1	11	6	0	4	3 17/21		2	11 17/21
1	13	0	0	4	2 1/11		2	9 1/2
1	14	6	0	3	11 7/23		2	7 7/23
1	16	0	0	3	9 2/3		2	5 2/3
1	17	6	0	3	7 13/25		2	3 13/25
1	19	0	0	3	5 11/13		2	1 11/13
2	0	6	0	3	4 8/17		2	0 8/27

The price of a Quarter of Wheat. | The weight of a half-peny white loaf in Troy-weight by Assise. | The weight of a half-peny white loaf in Troy-weight by Assise.

l.	s.	d.	Po.	Ounces.	Peny weight.	Po.	Ounces.	Peny weight.
0	3	0	4	6	8	9	0	16
0	4	6	3	0	5 1/3	6	0	10 2/3
0	6	0	2	3	4	4	6	8
0	7	6	1	9	15 1/5	3	7	10 2/5
0	9	0	1	6	2 2/3	3	0	5 1/3
0	10	6	1	3	10 6/7	2	7	1 5/7
0	12	0	1	1	12	2	3	4
0	13	6	1	0	1 7/9	2	0	3 5/9
0	15	0	0	10	17 3/5	1	9	15 1/5
0	16	6	0	9	17 8/11	1	7	15 7/11
0	18	0	0	9	1 1/3	1	6	3 2/3
0	19	6	0	8	7 1/13	1	4	14 2/13
1	1	0	0	8	3 1/7	1	4	0 2/7
1	2	6	0	7	5 1/15	1	2	10 2/15
1	4	0	0	6	16	1	1	12
1	5	6	0	6	8	1	0	16
1	7	0	0	6	0 8/9	1	0	1 7/9
1	8	6	0	5	14 10/19	0	11	9 1/19
1	10	0	0	5	8 4/5	0	10	17 3/5
1	11	6	0	5	3 14/21	0	10	7 5/12
1	13	0	0	4	19	0	9	18
1	14	6	0	4	14 14/23	0	9	9 5/23
1	16	0	0	4	11 1/3	0	9	2 1/3
1	17	6	0	4	7 8/25	0	8	14 2/25
1	19	0	0	4	3 9/26	0	8	7 5/28
2	0	6	0	4	0 16/27	0	8	1 5/29

Scholar.

Scholar. Sir, I do thank you most heartily for this, not onely in mine own name and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in Bread, which long time hath been abused. And if you know any thing more wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtain great thanks of all good-hearted men that love the Commonwealth.

Master. I have sundry things to declare, but I have reserved them for a private Book by it self: yet notwithstanding, because the Statute of the rate of measuring of grounds is so common, that it toucheth all men, and yet no more common then needfull, but so much corrupt, that it is too far out of all good rate, not onely in the English Books of Statutes commonly printed, but also in the Latin Books and in the French also, (for I have read of each sort, and conferred them diligently;) I will give you such a Table for the restitution of these errours as may suffice for this present time. And first I will propose one question to you touching the use of that Statute, whereby you may perceive the order how to examine the whole Statute, and every parcell thereof: and the question is this.

When the Acre of ground doth contain 4 Perches in breadth, then must it contain 40 Perches in length. Then do I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches: But before you shall answer to this question, I will declare unto you another Statute, which is the ground of the former Statute: and that Statute is this. A question of measuring of ground.

It

Statute measure. It is ordained, that three Barly-corns dry and round shall make up the measure of an inch, 12 inches shall make a foot, and 3 foot a Yard, (the common English books have an Elne) five yards and a half shall make a Perch, and 40 Perches in length,

An Acre. and 4 in breadth, shall make an Acre. This is that Statute, (whereby you may perceive that the intent of the Statute is, that one Acre should contain 160 square Perches.) Now let me hear you answer to the question.

Scholar. As I perceive by the words of the Statute a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so work the question: but seeing I may doe it also in whole numbers, I take that form for the most ease: therefore thus I set the question in form. Then do I multiply 40 by 4, and it maketh 160; which I divide by 13, and the Quotient is $12\frac{4}{13}$.

4	40
13	12

Master. Now turn that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former Statute. Howbeit it shall be best to take one of the least parts in Denomination, for avoiding of much labour, as Feet, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to return $\frac{4}{13}$ into Feet, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must divide by 13; and the Quotient is $5\frac{1}{13}$.

Master. So I find, that if the Acre hold in breadth 13 Perches, it shall contain in length 12 Perches, 5 foot, and $\frac{1}{13}$ of a foot, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a foot. But here all the

Note this errour. Statute-Books in Latin and English (that I have seen) do note it to be 13 Perches, 5 foot and one

one Inch, which make above 13 Perches too many in the Acre: so that I would have thought the errour to have crept into the Printed Books by the great negligence that Printers in our time do use, save that in written copies of great antiquity I do find the same: yet have I one French copy which hath 12 Perches ¼ and one foot, and that misseth very little of the truth.

Scholar. Then I see it is true that I have often heard say, that the truest Copies of the Statutes be the French copies.

Master. That is often true, but not generally, as I have by conference tried diversity: but in this Statute the French Book is most corrupt in all other places lightly.

But now, to perform my promise, I will set forth the Table for measuring of an Acre of ground onely by such parts as the Statute doth mention, because at this time I do of purpose write it for the better understanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this *Table* following I have not done as in the other *Statute* before, compared by restitution with the faults crept into the *Statute*, but onely have written that true measure which the equity of the *Statute* doth pretend. For it were vile to judge of so noble Princes and worthy Counsellors as have authorized and set forth this *Statute*, that they would make one *Acre* in any form greater then another, but every one to be just and equall with each other; which is the ground also of my work. And hereby may all men perceive how needful *Arithmetick* is to the *Students of Law*. But

now

now I think best to make an end of these matters for this present time, sith the *Table* hath in it no obscurity that I should need to declare.

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feet.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	6	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The

The breadth of the Acre.	The length of the Acre.			
Perches.	Perches.	Feet.	Inches.	Parts of an inch.
28	5	11	9	3/7
29	5	8	6	13/29
30	5	5	6	0
31	5	2	7	29/31
32	5	0	0	0
33	4	14	9	9
34	4	11	7	13/17
35	4	9	5	1/7
36	4	7	4	0
37	4	5	4	8/37
38	4	3	5	13/19
39	4	1	8	4/13
40	4	0	0	0
41	3	14	10	28/41
42	3	13	4	2/7
43	3	11	10	32/43
44	3	10	6	0
45	3	9	2	0

Scholar. Indeed, Sir, I understand the Table (as I think) by those other which you set forth before. For in the first column is set the Perches of the breadth

breadth of an Acre; and then in the 2 Columns following appeareth, how many Perches and how many foot that same Acre must have for his length.

Master. You take it well: howbeit to speak exactly of breadth and length, the first Column doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth. Yet it is no great abuse in such Tables, where a man cannot well change the Title, to let the name remain, although the proportions of the numbers do change: for still by the first Column is expressed the measure of the one side, and by the two other pillars in one Column is set forth the measure of the other side. And this shall be sufficient now for the use of the Golden Rule.

The Rule of Fellowship.

NOW will I touch certain other Rules, which for their several names may seem divers Rules, and distinct from this, but indeed they are but branches of it: yet because they have several workings in appearance, but all pleasant in use, I will give you a tast of each of them. As for the *Rule of Fellowship* both single and double, with time and without time, I shall need to say little more then I have already said in teaching the works of *whole numbers*: yet an example or two will we have, to refresh the remembrance of the same, and to declare certain proper uses & applications of it, as this for one.

The Rule of Fellowship without time.

Four men got a booty or prize in time of warre, the prize is in value of mony 8190 pound; and because the

A question of inequal society.

the men be not of like degree, therefore their shares may not be equal: but the chiefest person will have of the booty the third part, and the tenth part over; the second will have a quarter, and the tenth part over; the third will have the sixth part; and so there is left for the fourth man a very small portion, but such is his lot, (whether he be pleased or wroth) he must be content with one 20 part of the prey. Now I demand of you, what shall every man have to his share?

Scholar. You must be fain to answer to your own question, else it is not like to be answered at this time.

Master. The form to understand the solution of this question, and all such like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of some other of them, for all such parts you may overpass, and take for them all those numbers whose parts they be: as in this example, the shares be these, $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{20}$; if I multiply all the Denominators together, beginning with 3, and so goe on unto 20, it will make 144000; but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may overpass also, and then is there but 3 Denominators to multiply, that is, 4, 6, and 20, which make 480; which summe I take for my work, because all the Denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$; the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$, which is 48, and that maketh in the

whole

whole 168. Now for the third man, which must have $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe. So that if the whole prey had been but 480 pounds then were the question answered: but because the summe was of greater value, by this means now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty: And look what proportion is between the first number and the second, the same proportion shall be between the parts of that first number and the parts of the second, comparing each to his like. Therefore I must put in the third place one of the parts or shares, and then work by the former Rule of Proportion, or Golden Rule. And because I have 4 several parts of the first number, by which I would find out four like parts of the second number, therefore must I make four several figures.

The reason of this rule.

Scholar. Now I trust I can answer to your question, as by your favour I will prove.

And to try it, I set the four figures thus, marked with A, B, C, D, to shew their order.

A, B,

A: 480, 208, 8190

B: 480, 168, 8190

C: 480, 80, 8190

D: 480, 24, 8190

and then in each of them I multiply the second number by the third, and divide their totall by the first, and so commeth the fourth summe, which I [illegible] for: as if I do multiply 8190 by 208, it maketh 1703520, which being divided by 480, maketh in the Quotient 3549 for the first mans portion. [illegible] with the other three figures: I [illegible] for the second [illegible] and for the third [illegible] 1365, [illegible] the fourth man 409 1/2: and [illegible] is [illegible] in the [illegible] here [illegible].

A: 480, 208, 8190, 3549

B: 480, 168, 8190, 2866 1/2

C: 480, 80, 8190, 1365

D: 480, 24, 8190, 409 1/2

And [illegible] I think I have done well.

Master. [illegible] to prove it, adde all the [illegible] together, and if they make the totall, then certainly it is [illegible]. The proof of Addition.

Scholar. I may set them thus: and then by Addition the full summe doth amount, that is, 8190, and therefore (as you say) it seemeth to be well wrought.

3549
2866 1/2
1365
409 1/2
————
8190

But, I beseech you, is there any doubt in this triall, that you use that word seemeth?

Master. You may easily conjecture, that if you did assign the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appear all one, and therefore is not the proof exact.

The just Proof.

But if you will make a just proof, for the first mans part take [illegible] and [illegible] of the whole summe, and [illegible] it again with the number in the figure, then it is well done. And so [illegible] for the second, third, and fourth summs, and this [illegible] satisfy you. Now will I propound certain other questions, which [illegible] set [illegible] certain [illegible], albeit not [illegible] some [illegible]. [illegible] I do not [illegible] whose [illegible], whole [illegible] house [illegible] and greatly [illegible] in; but [illegible] my prediction, to [illegible] our [illegible] to remove all [illegible] of [illegible] lieth; and [illegible] I will [illegible] name the questions, without putting the Author's name.

The [illegible] question is this.

A question of building.

Four men did build a house, which cost them 3000 crowns, [illegible] such that one man should pay [illegible] summe and [illegible] crowns over, the second the [illegible] and [illegible] crowns over, the third man must lay [illegible] crowns, and the fourth man should pay [illegible] and [illegible] crowns more. Can you answer to this question?

Scholar. No, I cannot, sir, [illegible] you have better [illegible]; for I know no more than you have taught me.

Master.

Master. When I haue say you cannot doe it, nei- [illegible] that euer was proposed it: for the question is impossible. For declaration whereof I will be able to use [illegible] of the Numbers in their right forme (although I haue not yet taught that manner of worke) because it may appeare plainly that the question is [illegible]. For which I haue set the [illegible] and then make the whole [illegible] An impossible question.

[illegible] gains [illegible]

		6
		12
		8
		20
	14	30

Scholar. [illegible]

Master. You say truth; and (because you shall perceiue it the better) I will set it after the vulgar forme, as in this figure you see: where the ½ doth amount to 1506, for the which, as you heard before, 3000: the same the 12 make is 1012, the ⅔ amount to 2000, but there is [illegible] but 1992 and then last of all, the ¼ [illegible] and the 20 more maketh 770: which all together added in one summe, doe make 5280, where the totall summe should be but 3000: which summe of 3000 if you diuide by [illegible] you shall haue [illegible] that is 1250, and thereto adde 30 more, then will those two summes make 5280. Whereby you may see how this forme (as well as the other) doth declare, that the particulars in that question would make more than the whole summe by ¾ and 30 more, and therefore can that question not be accepted as a possible

1506
1012
1992
770
5280
3000
2000
2290
5280

possible thing. But yet do [illegible] men pro-pound such questions, and answer to them. Where-fore [illegible] to [illegible] excuse (rather of them [illegible] doing) I will [illegible] what may [illegible] for their defence, but in the mean season I will propound the question as it may be [illegible] by any possibility.

The former question of building now possible.

As, Foure men [illegible] build a house [illegible] and it cost them [illegible] Crowns, and when [illegible] they agree thus [illegible] as the first man shall pay 6 Crowns [illegible] the second man shall pay [illegible] the third man 8, and the fourth man [illegible] like [illegible] but the first man shall pay double [illegible] is the fourth, and the second man shall [illegible] half the first mans charge, the third man shall pay double [illegible] the second: (and these two ways do [illegible] to one end.) But farther for their agreement [illegible] also that the first shall give 6 Crowns [illegible] and the second [illegible] and the fourth shall give [illegible] the 3d man shall give no o-verplus, but shall [illegible] Crowns [illegible] his charge.

Now is the question possible to be [illegible], and this is the way [illegible] it. Mark the proportion of the severall charges, and set out [illegible] numbers in that rate, by which you may reduce the work to the Golden Rule: as here in the first [illegible] the numbers are [illegible] named, 6, 4, 8, 3, and in the second [illegible] although they be not plainly named, yet they may be the same numbers: for 6 is double to 3, and 4 is [illegible] of 6, and again 8 is double to 4. Now adde them together, and they make 21; which 21 must be set for the first number in the Golden Rule: for it [illegible] with the overplus of each mans charge, would make the totall summe of the

charges,

charges, then were those severall summes the charges of each man, beside his overplus; but now it is not so.

But yet this is true, (so excellent are conclusions Arithmeticall) that look what proportion each of their severall summes doth bear to 21, the same proportion doth the just charges of every man (beside his overplus) bear to the totall of the charges, the overplus being deducted. Wherefore this may you note, that before you do apply the totall of his charges to the Golden Rule, you must deduct the overplus, which is 6, 12, and 20, that is in the whole 38; but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule for the second summe; and for the third summe I must put each of the small numbers before mentioned, which although they be not severall charges, yet they represent them in proportion. And so making for every mans charge a severall question, the figures will be 4, which I mark with four letters, A, B, C, D, thus.

A

21 2970

6 $848\frac{4}{7}$

B

21 2970

4 $565\frac{5}{7}$

C

21 2970

8 $1131\frac{3}{7}$

D

21 2970

3 $424\frac{2}{7}$

Where I have set for briefnesse the summe of every mans charge in the fourth place, presupposing that you can tell how to try out the fourth summe by

by so many Examples as ye have had?

Scholar. As I trust that I understand this [illegible], so I desire much to know what may be said for the [illegible] that missed this Question.

Master. [illegible] seem so [illegible] to know this error, that you have forgotten to examine whether [illegible] [illegible] faults.

Scholar. [illegible] this work to be well done, because [illegible] of the four several numbers doth make [illegible] sum of [illegible], which was to be divided into those four parts.

Master. But [illegible] have you forgotten that the first man [illegible] crowns more besides his share, and the second [illegible] 12 crowns more, the third man 8 crowns [illegible] the fourth man 20 crowns more; [illegible] first [illegible] of 3000 crowns [illegible]

Scholar. [illegible] the first man's [illegible] 4 [illegible] and to the second's sum [illegible] it will be 577 5/7; [illegible] the third [illegible] I [illegible] 8, and then will the summe be 1123 [illegible], then [illegible] into the fourth's summe 20, it will be 444 2/7. And these four summes will make 3000, which is the whole charge. [illegible] in this example it may appear; where first I gather the 4/7, [illegible] and so proceed I in the Addition to the [illegible]

854 4/7
577 5/7
1123 3/7
444 2/7
3000

Master. Now have you well done, and this [illegible] in the [illegible] [illegible] of other [illegible] for the [illegible] of the question, [illegible] (as I [illegible]) was impossible.

possible. And now examine by these severall summs, and see whether it doth agree with the summs th this Question proposed.

The first man must pay $\frac{1}{2}$ and 6 over of the totall summe: how think you, is $854\frac{2}{7}$ the half and 6 more of 3000?

Scholar. No that it is not, for it should be 1506, and for the second man 1012, and for the third man 1992, and for the fourth man 770: whereof not one summe agreeth to this work. But I marvell that so wise men could be so much overseen.

Master. It is commonly seen, that when men will receive things from elder writers, and will not examine the things, they seem rather willing to erre with their Ancients for company, then to be bold to examine their works or writings. Which scrupulosity hath ingendred infinite errors in all kinds of knowledge, and in all civil administration, and so in every kind of Art. But these learned men did not mean any other thing by this question, then to find such numbers as should bear the same proportion together, as those numbers in the question proposed did bear one to another: which thing you shall perceive more plainly by another question of theirs, that is this:

A man lying upon his Death-bed bequeathed his goods (which were worth 3600 Crowns) in this sort: because his Wife was great with child, and he yet uncertain whether the Child were Male or Female, he made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife have half his goods, and the Daughter $\frac{1}{3}$: but if she were delivered of a Son, then that Son should have

A question of a Testament.

have $\frac{1}{2}$ of the goods, and his Wife but $\frac{1}{3}$. Now it chanced her to bring forth both a Son and a Daughter. The question is, How shall they part the goods agreeable to the Testator his Will?

S. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite void, and so the man to die intestate, because the Testament was made insufficient, with this condition not expressed in it: and also it might have chanced that she should have brought forth neither Son nor Daughter, as often hath been seen; so is the Will insufficient in that point also.

Master. Such Scanners should seem too cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken favourably for the aid of the Legataries, when there ariseth such doubt. But let us try this work, not by force of Law, but by proportion Geometricall, seeing the Testator did minde to provide for each sort of them.

Scholar. If the Son shall have $\frac{1}{2}$ by force of the Testament, so must the Mother have $\frac{1}{3}$. Again, because she hath a Daughter also, therefore ought she to have $\frac{1}{2}$ and the Daughter $\frac{1}{3}$, that is both ways $\frac{1}{2}\frac{1}{3}$, and $\frac{1}{2}\frac{1}{3}$, which cometh to the whole goods and $\frac{2}{3}$ more.

Wherefore it seemeth also impossible.

Master. In this matter the minde of the Testator is to be understood, that such proportion should be between the portion of the Wife and the Son, as is between $\frac{1}{2}$ and $\frac{1}{3}$, that is, the Son must have $\frac{3}{5}$ for $\frac{2}{5}$ to his Mother, so shall he have 3 to 2, that is as much as his Mother and half as much more; and the Mother must have the like rate in comparison to her Daughter,

Then

Then must I find out 3 numbers in such proportion, that the first may have as much as the second, and half as much more, (that is in proportion sesquialtera) and the second to the third in that same proportion: such numbers be 9, 6, 4.

Scholar. I pray you, Sir, how shall I find out these numbers?

Master. That will I gladly tell you.

Whatsoever the proportion be of any three numbers, multiply the terms of that proportion together, and the number that amounteth shall be the middle number of the three: then multiply that middle number by the lesser term, and divide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extream, and divide the totall by the lesser extream, then will the greatest number of that Progression amount.

To find three numbers in any proportion.

Scholar. Then in this example to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must divide (as you taught me in Division) $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$: whereby I perceive that the proportion in this question is as three to 2. Therefore, as you taught me even now, I multiply 3 by 2, and the summe is 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least term, and the summe is 12: that I do divide by 3, being the greater term, and the Quotient is 4: so is 4 the least number of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and so have I 9, which is the greatest number of the three.

To find the proportion between two numbers.

Master. Another way yet may you find the

third

third number in any Progression, if you have two of them: for if the middle number be one of them which you have, then multiply it by it self, (as in this example, 6 by 6 maketh 36) and that totall divide by the other number which you have, and the third number will be the Quotient.

Scholar. Then if I divide 36 (which cometh of 6 multiplied by it self) by 4, the Quotient will be 9: and if I divide 36 by 9, the Quotient will be 4. But when I know the first number and the third, and would have the middle number?

Master. Multiply the 2 numbers together, and in their totall you must seek the root of that number, and it shall be the middle number: but because as yet you have not learned to extract Roots, therefore use the first form which I have taught you, till I teach you to extract Roots. And now go forwards with the answer to the same question.

Note.

Scholar. I perceive then that the Son must not have $\frac{1}{2}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{4}$: but yet must the goods be divided into such proportion, that the Son shall have 9 crowns for 6 to the Mother, and the Mother shall have 6 crowns for 4 to her Daughter. Then I apply it to the Golden Rule in these examples, as followeth.

Where the first number is the Addition of those three numbers, 9, 6, 4; and the third is one of them severally; the second is the totall of the goods in that Testament: and then by the work of the Golden Rule I find out the fourth number

19	3600
9	
19	3600
6	
19	3600
4	

number in their kinde, that is, for the Son 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and for the Daughter 757 $\frac{17}{19}$, the which summes added together do make the summe of the whole goods, as may be seen by this Example.

Another like question.

1705 $\frac{5}{19}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (you thinketh) I no perceive, because in this case there is a necessary remedy which is against an urgent inconvenience: therefore those learned men thought they might use the like liberty in that other question.

Master. Your guess is good, but they had so good reason for them in the one as they have in the other: as in another example of theirs it may better appear; as in this.

A man left unto his three Sons 7851 crowns to be parted in such sort, that the first Son should have $\frac{1}{2}$, the second Son $\frac{1}{3}$, and the third Son $\frac{1}{4}$, which is not possible; for $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$ do make $\frac{26}{24}$, or $\frac{13}{12}$, that is, 1 $\frac{1}{12}$, so it is more then the whole: but reduce these *Fractions* into one denomination, the least that they will come to, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and so may you part the goods into such proportion as these 3 Numerators bear together; that is, the first to have 6 for every 4 to the second, and the second to have 4 as often as the third hath 3; and so their portions will be, for the first 3623 $\frac{7}{13}$, for the second 2415 $\frac{9}{13}$, and for the third 1811 $\frac{10}{13}$; and these 3 shares added together will make the totall summe of the whole goods, as you may easily see in this example.

Another question of a Testament.

3623 $\frac{7}{13}$
2415 $\frac{9}{13}$
1811 $\frac{10}{13}$
7851

Another

Another like question.

Another question is there proposed thus:

There are 450 crowns to be divided between three men, so that the first man must have $\frac{1}{2}$ and $\frac{1}{3}$, the second man $\frac{1}{3}$ and $\frac{1}{4}$, the third man shall have $\frac{1}{4}$ and $\frac{1}{5}$.

Scholar. I marvell that any man would be so overseen to propose that question as seeing [illegible] with the [illegible] make $1\frac{52}{60}$ that is almost double the whole summe.

But I perceive it might be thus proposed: that as often as the first man did receive 50 crowns so often the second man should receive 35, and the third man 27; for $\frac{1}{2}$ and $\frac{1}{3}$ is equal to $\frac{50}{60}$, so is $\frac{1}{3}$ and $\frac{1}{4}$ equal to $\frac{35}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$; and so working the question, the three figures will appear in this form: whereby the first man's portion is found to be $200\frac{50}{56}$, the second man's part is $140\frac{35}{56}$, the third man's share $108\frac{27}{56}$, which in the whole doth make 450 crowns to be divided between them.

112		450
50		$200\frac{50}{56}$
112		450
35		$140\frac{35}{56}$
112		450
27		$108\frac{27}{56}$

Master. And thus you are (I think) sufficiently instructed in the Rule of Fellowship.

The

The Rule of Alligation.

The Rule of Mixture

Now will I goe in hand with the *Rule of Alligation*; which hath his name, for that by it there are divers parcells of sundry prices, and sundry quantities [illegible] bound, or mixed together, whereby also it might be well called the Rule of Mixture; and it hath great use in composition of [illegible], and also in mixtures of Metals, [illegible] some use it hath in mixtures of Wines; but I [illegible] with [illegible] less used therein then it is now-a-days. The order of this rule is this.

The reason of this Rule.

[illegible] summs are propounded to be mixed, set these numbers one under another, and the common number (whereunto you will reduce them) set on the left hand: then mark which summes be lesser then that common number, and which be greater, and with a draught of your pen evermore link two numbers together, so that one be lesser then the common number, and the other greater (for two greater or two smaller cannot well be linked together.) And the reason is this, that one greater and one smaller may be so mixed, that they will make the mean or common number very well; but two less can never make so many as the common number, being taken severally: no more can two summs greater then the mean ever make the mean in one [illegible], as it shall appear better to you hereafter. And as it is of necessity to link every smaller (once at the least) with one greater, and every greater with one smaller; so it is at liberty to

to link the [illegible] together [illegible] once, and so many [illegible] be to one question [illegible] solutions. [illegible] when you have so linked them, the [illegible] each the lesser number [illegible] smaller then the mean or common number, and [illegible] difference set against the greater number [illegible] linked with those lesser, each with but [illegible] the right hand [illegible] the [illegible] number [illegible] mean [illegible] number [illegible] com[illegible]

☞ [illegible] the Ad[illegible] differences [illegible] in [illegible] Rule [illegible] num[illegible] ber [illegible] Those [illegible] the first [illegible] differences [illegible] and [illegible] portion of [illegible] the [illegible]

A question of mixing of Wines.

There [illegible] four sorts of [illegible] of [illegible] prices, one of [illegible] pence the gallon, another of [illegible] pence, the third of [illegible] pence, and the fourth of [illegible] pence the gallon: of all these [illegible] would I have a mixture made to the summe of fifty gallons, so that the price of each gallon may be [illegible] pence. Now demand I, how much must be taken of every sort of Wine?

Scholar. It it shall please you to make the first example, that I may [illegible] the applying of it to the Rule, then I trust I shall be able not onely to doe the like, but allow [illegible] the reason in the order of the work.

Master.

Master. Mark then this form, and the placing of every kind of number in it.

The prices severall. | The differences.

9 { 6, 8, 11, 15 } | 6 A, 2 B, 1 C, 3 D

Here (you see) I have set down the severall prices, which are 6, 8, 11, 15, and have linked together 6 with 15, and 8 with 11. The common price 9 I have set on the left side, and the difference between it and each particular price I have set on the right hand, not against the summe whose difference it is, but against the summe that it is linked withall. So the difference of 15 above 9 (which is 6) I have set, not against 15, but 6, that is linked with 15: and the difference between 6 and 9 (that is 3) I have set against 15. So likewise the difference between 8 and 9 is but 1, that I have set against 11: and the difference of 11 above 9 (which is 2) I have set against 8. Then adde I all those four differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of

[illegible] tions that I would haue, and the third num[illegible] [illegible] ular difference. Now if you [illegible] [illegible] Rule, you shall find the numb[illegible] [illegible] that shall be taken of each sort of [illegible]

The proofe of this rule.

[illegible] better vnderstanding whereof I haue [illegible] A, B, C, D, both against the Number [illegible] to serue, and ouer the work [illegible] for each of them. And [illegible] the [illegible] of these [illegible] and then will [illegible] that [illegible] I would haue [illegible] this example [illegible] how [illegible] multiply [illegible] common [illegible] 450 [illegible] then [illegible] numbers [illegible] shall [illegible] same [illegible] with [illegible] [illegible] of [illegible] [illegible] 66 2/3 [illegible] former [illegible] this work to be good and well done.

25
8
450
66 2/3
45 5/[illegible]
187 1/2
450

And

And now to prove how you can doe the like, I propound the same Question, onely willing you to use some other form of combining or linking the summs. *The variation of this question.*

Scholar. That shall I prove with your favour, and therefore I combine 8 with 15, and 6 with 11: and then the form will be as followeth.

				A			B	
9	6	2 A	12		50	12		50
	8	6 B	2		$8\frac{2}{6}$	6		25
				C			D	
	11	3 C	12		50	12		50
	15	1 D	3		$12\frac{3}{6}$	1		$4\frac{1}{6}$
		12						

Whereby amounteth the same summe in totall of the differences as did before; and yet now the differences be altered as the combination is changed: whereof I understand the reason by your former work. And therefore here appeareth no strange thing, but that now I have $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence, and 12 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{2}{6}$ by 6, it maketh 50, and then 25 multiplied by 8 maketh 200; likewise $12\frac{3}{6}$ multiplied by 11 yieldeth $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplied by 15 maketh $62\frac{3}{6}$; which 4 summs added into one will yield in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or mean price.

50
200
$137\frac{3}{6}$
$62\frac{3}{6}$
450

Master. Seeing you conceive this work so

well, I will propound another example unto you of more variety in the Alligations or combinings, as thus.

A queſtion of Spices.

A Merchant being minded to make a bargain for Spices in a mixt maſſe, (that is to ſay) of Cloves, Nutmegs, Saffron, Pepper, Ginger and Almonds, the Cloves being at 6 ſhillings, Nutmegs at 8 ſhillings, Saffron at 10 ſhillings, Pepper at 3 ſhillings, Ginger at 2 ſhillings, and Almonds at 1 ſhilling:

Now would he have of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another to bear in price five ſhillings: How much ſhall he have of each ſort?

Scholar. That will I try thus:

Firſt I ſet down thoſe 6 ſeveral prices, and at the left hand I ſet the common price five ſhillings. Then I link them thus, 1 with 10, 2 with 6, and 3 with 8; as in the example following.

Maſter. I had minded to have combined them in more variety: but I am content to ſee your

own

own work first, and then more varieties in combination may follow anon.

Scholar. Then, to continue as I began, I seek the difference between 1 and 5, (which is 4) and that I set against 10; then against 1 I set 5, which is the excess of 10 above 5: so I gather the difference between 2 and 5, which is 3, and that I set against 6, because it is combined with 2; and likewise the difference of 6 above 5, which is 1, I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8: and before that 3 I set the difference of 8 above 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first Number in the Golden Rule. And so appeareth by those works, that of Almonds I must take 83 $\frac{1}{3}$ pound, of Ginger 16 $\frac{2}{3}$ pound, of Pepper 50 pound, of Cloves 50 pound, of Nutmegs 33 $\frac{1}{3}$ pound, and of Saffron 66 $\frac{2}{3}$ pound.

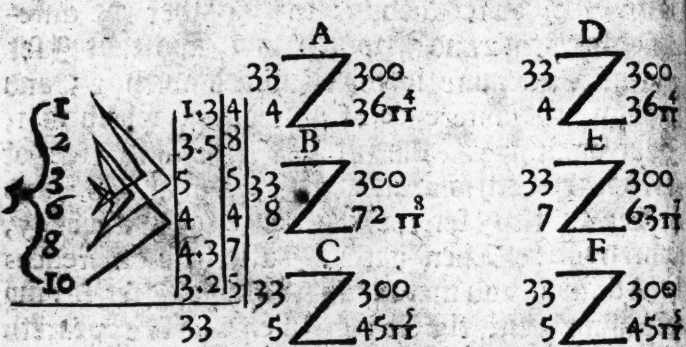
83 $\frac{1}{3}$
33 $\frac{1}{3}$
150
300
266 $\frac{2}{3}$
666 $\frac{2}{3}$
1500

Then for triall thereof, I multiply every parcell by his several price, as 83 $\frac{1}{3}$, which is the summe of Almonds, I multiply by 1, which is their price.

Also 16 $\frac{2}{3}$, the summe of Ginger, I multiply by 2, which is the price of it; and so each other in his kind, as this Table annexed doth represent: and then adding them all together, I find the totall to be 1500, which also will amount by the multiplication of the gross mass of 300, by the common price 5: wherefore it appeareth well wrought.

 Master.

Master. Now I will make the alligation, to proue your cunning ſomewhat better: but becauſe you ſhall not think your ſelf preſſed too much, I will alſo note the differences. As by this Example you may ſee, where I haue alligated 1 with 6



and 8, and therefoꝛe haue I ſet againſt 1 both their differences, that is 1 and 3: Likewiſe, becauſe 2 is combined with 8 and 10, I ſet befoꝛe him their differences, 3 and 5. Againſt 3 I haue ſet onely 5, which is the difference of 10, with whom 3 is combined onely. Likewiſe 6 is onely alligate to 1; and therefoꝛe is the difference of 1 from 5, which is 4, onely ſet againſt it: 8 is linked with 1 and 2, and therefoꝛe hath ſet againſt him both their differences, 4 and 3: and 10 is joyned with 2 and 3, therefoꝛe hath he their differences, 3 and 2. And becauſe of eaſe foꝛ you, in another column I haue ſet the differences reduced into one number, foꝛ every ſeveral ſoꝛt, and haue alſo added them together; whereby appeareth that they make 33: and ſo conſequently you ſee the woꝛks of the Golden Rule ſet foꝛth. Foꝛ the ſix Drugs

I haue

I have added the letters A, B, C, &c. as before.

But I would not wish you to cleave still to these Elementary aids, but accustome Memory to trust her self: so shall occasion of negligence best be avoided. And as for the proof, try it at more leisure, because the time now is short, and you sufficiently instructed in that proof; and there rest divers things behind yet, of which I would gladly give you some tast before your departure, Note.

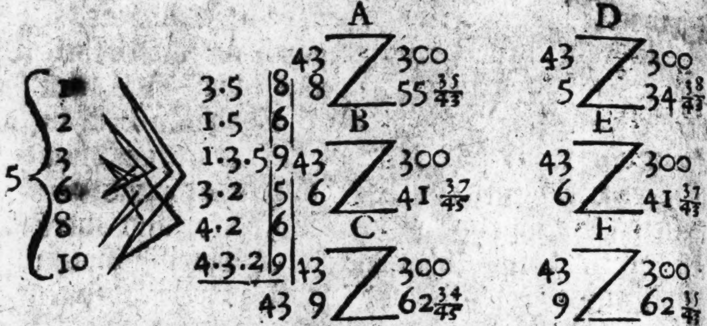
Scholar. But if it may please you, let me see all the variations of this question, before you go from it, for methinketh I could vary it two or three ways more yet.

Master. I am content to see you make two or three variations, but I would be loth to stay to see all the variations; for it may be varied above 300 ways, although many of them would not well serve to this purpose.

Scholar. I thought it impossible to make so many variations.

Master. Marvell not thereat, for some questions of this Rule may be varied above 1000 ways; but I would have you forget such fantasie till a time of more leisure. And now go forward with some variation of this question. Note.

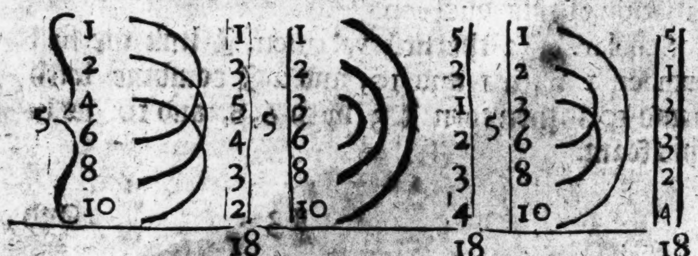
Scholar. For the first variation, I link the first number 1 with 1 and 10, and 2 I combine with 9 and 10; then joyn I 3 with 6, 8, and 10, as in this form.



And ſo doth there appear the proportion of weight for every kind of Drug in this mixture. Now for the triall.

Maſter. Nay ſtay there: you ſhall not need to make triall in one example ſo often; or if you liſt to doe it by your ſelf, I am content. But now ſet forth (for declaration that you conceive the Rule) two or three examples of ſeveral Combinations; & then will we paſs to ſome other example, and ſo end this Rule.

Scholar. As it pleaſeth you, ſo will I doe. And theſe be the varieties: in which as the combinations are ſeveral, ſo doth it plainly appear that the differences by which the proportion of each ſeveral kind is taken are alſo ſeveral. And yet I ſee in the three firſt of theſe five varieties, and in the one

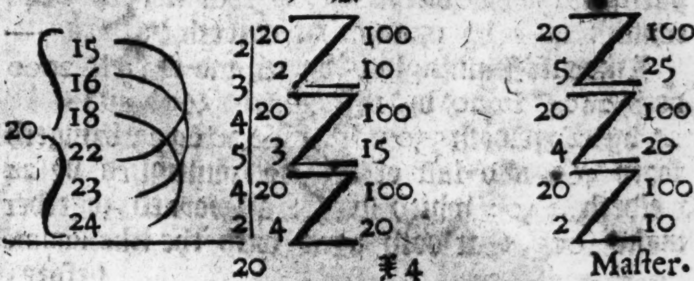


other

other befoꝛe, the totall ſumme of the differences to be one, that is to ſay, 18; whereby I perceive that the variety of their mixture doth depend on the variety of their differences ſeverall, and not on the variety of their totall ſumme.

Maſter. So is it. And ſeeing you conceive it ſo well, I will make an end of this Rule, onely exhibiting unto you one Queſtion oꝛ two of the mixture of Metalls, that by it you may deviſe others like, and exerciſe your ſelf therein alſo, becauſe the uſe of it ſerveth often in buſineſs of charge, not ſo much foꝛ Goldſmiths, as of coinage in Mints. Firſt, I demand of you this queſtion: If a Mint-maſter have Gold of 22 Karects, and ſome of 23 Karects, ſome of 24, again, ſome 15, ſome 16, and ſome of 18 Karects, and would mix them, ſo that he might have 100 Ounces of 20 Karects; how much muſt he take of each ſort?

Scholar. To know that, I anſwer in oꝛder thus:



Master. You have wrought the question well; but how chanced you made no doubt of that new name Karect?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much hast to end: and again, in this case the proportion of the number is sufficient for my purpose in this work, trusting that another time you will instruct me as well of this, as of sundry other things, which as I have heard you talk of, so I have a great desire to them.

Master. Your answer is reasonable, and 10
your request and trust (with Gods help) I intend 15
to satisfie: & now to go forward with this mat- 20
ter, let me see your Examination of this last work. 25

Scholar. First, for the one part I adde toge- 20
ther all the particular summs, as they appear in 10
the work, and they make 100, as here by their ——
Addition doth appear. 100

And so it seemeth that the summs are well gathe-
red. But for the farther trial of them, I mul- 150
tiply first 20, which is the common or mean 240
summe of the Karects, by 100, which is the 360
summe of the whole mass which I would have, 550
and it maketh 2000. Then I multiply every 460
particular summe by the Karects that it doth 240
contain, as 10 by 15, and that maketh 150. ——

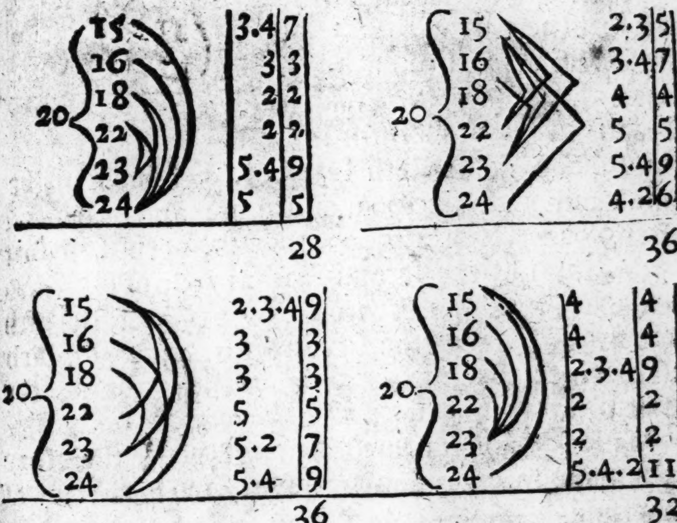
Likewise I multiply 15 by 16, and it yiel- 2000
deth 240; so 20 by 18 maketh 360, and 25 by 22 yieldeth 550; likewise 20 by 23 bringeth forth 460, and last of all, 10 multiplied by 24 yieldeth 240: which summs all joyned together make 2000, that doth agree with the like summe

before;

before; wherefore I may well ſay that the work is good. And now (if it pleaſe you) I would ſet forth ſome varieties of this queſtion, to prove my wit.

Maſter. Go to, let me ſee.

Scholar. Here be four varieties.



And more yet could I make, but not like to the number that you ſpake of in the variation of the other queſtion.

Maſt. That will I teach you at more leiſure, ſeeing it is a thing rather of pleaſure then of any neceſſity.

But now, for your exerciſe in this Rule, one other *queſtion* I will propoſe. A Mint-maſter hath ſix Ingots of ſilver, of ſundry fineneſs, ſome of four Ounces fine, and ſome of five Ounces, ſome of ſix, and other of eight, ſome of 11, and other of 12; and his deſire is to mix 500 pounds weight, ſo that in the whole maſs every

A queſtion of mixing of Silver.

every pound-weight ſhould bear 9 Ounces of fine ſilver: How much ſhall he take, ſay you, of every ſort of ſilver?

Scholar. To find out that, I ſet the numbers thus in order:

9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differences, it will appear that of the firſt ſort there muſt be 43 $\frac{11}{23}$, of the ſecond as much; of the third ſort 65 $\frac{5}{23}$, and of the fourth ſort as much; of the fifth ſort 195 $\frac{15}{23}$, and of the ſixth ſort 86 $\frac{22}{23}$; which in the whole will make 500 pound weight: and in ounces, after 9 ounces fine, 4500; that is, of the firſt ſort 173 $\frac{21}{23}$, and of the ſecond ſort 217 $\frac{9}{23}$, of the third ſort 391 $\frac{7}{23}$, of the fourth ſort 521 $\frac{17}{23}$, of the fifth ſort 2152 $\frac{4}{23}$, and of the ſixth ſort 1043 $\frac{11}{23}$; which all together do make 4500 ounces, agreeable to the multiplication of 9 by 500.

Maſter. This is well done of you; therefore now make three or four varieties, and ſo an end of this Rule.

Scholar. Theſe four varieties I ſet for examples.

9	4	3	3
	5	3	3
	6	3	3
	8	2	2
	11	1	1
	12	5.4.3	12
			24

9	4	2.3	5
	5	2	2
	6	2	2
	8	2	2
	11	5.4.3.1	12
	12	5	5
			29

Maſter

	4	2.3	5
	5	3	3
	6	2	2
9	8	2	2
	11	5.3.1	9
	12	5.4	9
			30

	4	3	3
	5	3	3
	6	2.3	5
9	8	2.3	5
	11	3.1	4
	12	5.4.3.1	13
			33

Master. And by these it appeareth that you can find out more, with which I will not now meddle; save onely (for to shew you an easie help in drawing the lines of combination) I will set forth 2 varieties here.

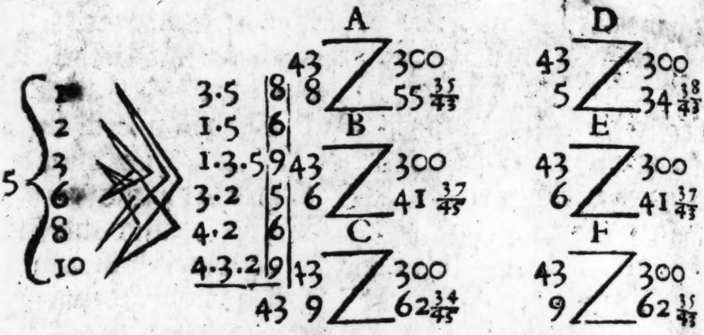
	4	2	2
	5	2.3	5
	6	3.2	5
9	8	3	3
	11	5.4.3	12
	12	4.3.1	8
			35

	4	3	3
	5	2.3	5
	6	2.3	5
9	8	2.2	5
	11	4.3	8
	12	5.4.2.1	12
			38

And this shall suffice now for the Rule of Alligation of mixture; for by these examples may you easily conjecture such other as do appertain to it, as well for the due working, as for variety of drawing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these Combinations, and termed them phantasies, yet my phantasie giveth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable: Therefore ye may well think, that at another time convenient

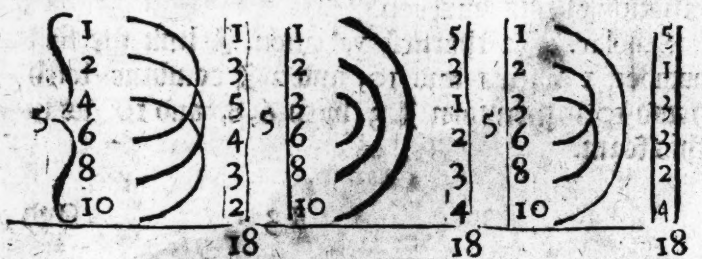
I will



And ſo doth there appear the proportion of weight for every kind of Drug in this mixture. Now for the triall.

Maſter. Nay ſtay there: you ſhall not need to make triall in one example ſo often; or if you liſt to doe it by your ſelf, I am content. But now ſet forth (for declaration that you conceive the Rule) two or three examples of ſeverall Combinations; & then will we paſs to ſome other example, and ſo end this Rule.

Scholar. As it pleaſeth you, ſo will I doe. And theſe be the varieties: in which as the combinations are ſeverall, ſo doth it plainly appear that the differences by which the proportion of each ſeverall kind is taken are alſo ſeverall. And yet I ſee in the three firſt of theſe five varieties, and in the one



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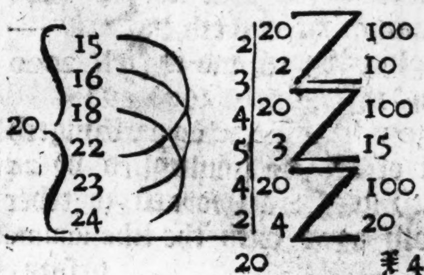
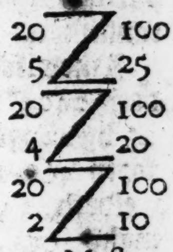
other befoꝛe, the totall ſumme of the differences to be one, that is to ſay, 18; whereby I perceive that the variety of their mixture doth depend on the variety of their differences ſeverall, and not on the variety of their totall ſumme.

Maſter. So is it. And ſeeing you conceive it ſo well, I will make an end of this Rule, onely exhibiting unto you one Queſtion oꝛ two of the mixture of Metalls, that by it you may deviſe others like, and exerciſe your ſelf therein alſo, becauſe the uſe of it ſerveth often in buſineſs of charge, not ſo much foꝛ Goldſmiths, as of coinage in Mints. Firſt, I demand of you this queſtion: If a Mint-maſter have Gold of 22 Karects, and ſome of 23 Karects, ſome of 24, again, ſome 15, ſome 16, and ſome of 18 Karects, and would mix them, ſo that he might have 100 Ounces of 20 Karects; how much muſt he take of each ſort?

Scholar. To know that, I anſwer in oꝛder thus:





Maſter. You have wrought the queſtion well; but how chanced you made no doubt of that new name Kareſt?

Scholar. Becauſe I thought it out of time to demand ſuch queſtions now, ſeeing you make ſo much haſt to end: and again, in this caſe the proportion of the number is ſufficient for my purpoſe in this work, truſting that another time you will inſtruct me as well of this, as of ſundry other things, which as I have heard you talk of, ſo I have a great deſire to them.

Maſter. Your anſwer is reaſonable, and 10
your requeſt and truſt (with Gods help) I intend 15
to ſatisfie: & now to go forward with this mat- 20
ter, let me ſee your Examination of this laſt work. 25
Scholar. Firſt, for the one part I adde toge- 20
ther all the particular ſumms, as they appear in 10
the work, and they make 100, as here by their ——
Addition doth appear. 100

And ſo it ſeemeth that the ſumms are well gathe-
red. But for the farther trial of them, I mul- 150
tiply firſt 20, which is the common or mean 240
ſumme of the Kareſts, by 100, which is the 360
ſumme of the whole maſs which I would have, 550
and it maketh 2000. Then I multiply every 460
particular ſumme by the Kareſts that it doth 240
contain, as 10 by 15, and that maketh 150. ——
Likewiſe I multiply 15 by 16, and it yiel- 2000
deth 240; ſo 20 by 18 maketh 360, and 25 by 22 yieldeth 550; likewiſe 20 by 23 bringeth forth 460, and laſt of all, 10 multiplied by 24 yieldeth 240: which ſumms all joyned together make 2000, that doth agree with the like ſumme

before; wherefore I may well ſay that the work is good. And now (if it pleaſe you) I would ſet forth ſome varieties of this queſtion, to prove my wit.

Maſter. Go to, let me ſee.

Scholar. Here be four varieties.

20	15	3.4	7
	16	3	3
	18	2	2
	22	2	2
	23	5.4	9
	24	5	5
			28

20	15	2.3	5
	16	3.4	7
	18	4	4
	22	5	5
	23	5.4	9
	24	4.2	6
			36

20	15	2.3.4	9
	16	3	3
	18	3	3
	22	5	5
	23	5.2	7
	24	5.4	9
			36

20	15	4	4
	16	4	4
	18	2.3.4	9
	22	2	2
	23	2	2
	24	5.4.2	11
			32

And more yet could I make, but not like to the number that you ſpake of in the variation of the other queſtion.

Maſt. That will I teach you at more leiſure, ſeeing it is a thing rather of pleaſure then of any neceſſity.

But now, for your exerciſe in this Rule, one other *queſtion* I will propoſe. A Mint-maſter hath ſix Ingots of ſilver, of ſundry fineneſs, ſome of four Ounces fine, and ſome of five Ounces, ſome of ſix, and other of eight, ſome of 11, and other of 12; and his deſire is to mix 500 pounds weight, ſo that in the whole maſs every

A queſtion of mixing of Silver.

every pound-weight should bear 9 Ounces of fine silver: How much shall he take, say you, of every sort of silver?

Scholar. To find out that, I set the numbers thus in order:

9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differences, it will appear that of the first sort there must be 43 $\frac{11}{23}$, of the second as much; of the third sort 65 $\frac{5}{23}$, and of the fourth sort as much; of the fifth sort 195 $\frac{15}{23}$, and of the sixth sort 86 $\frac{22}{23}$; which in the whole will make 500 pound weight: and in ounces, after 9 ounces fine, 4500; that is, of the first sort 173 $\frac{21}{23}$, and of the second sort 217 $\frac{9}{23}$, of the third sort 391 $\frac{7}{23}$, of the fourth sort 521 $\frac{17}{23}$, of the fifth sort 2152 $\frac{4}{23}$, and of the sixth sort 1043 $\frac{11}{23}$; which all together do make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you; therefore now make three or four varieties, and so an end of this Rule.

Scholar. These four varieties I set for examples.

9	4	3	3		9	4	2.3	5
	5	3	3			5	2	2
	6	3	3			6	2	2
	8	2	2			8	2	2
	11	1	1			11	5.4.3.1	13
	12	5.4.3	12			12	5	5
			24					29

Master

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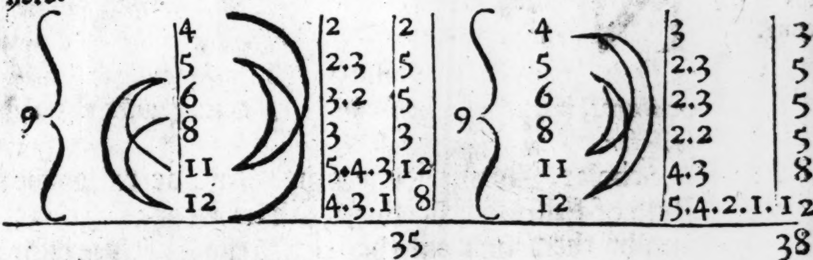


And this shall suffice now for the Rule of Alligation of mixture; for by these examples may you easily conjecture such other as do appertain to it, as well for the due working, as for variety of drawing the lines of Combination.

Scholar. Sir, albeit it pleased you erewhile to put me from my musing at the many varieties that may fall in these Combinations, and termed them phantasies, yet my phantasie giveth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable: Therefore ye may well think, that at another time convenient

I will

I will request you to aid me herein.

Master. Truth it is, that this consideration may fall in practice as well Politick as Philosophical, and sundry ways in them be applied: Therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The Rule of Falshood.

The occasion of the name.

NOw *will I briefly also teach you the* Rule of Falshood, *which beareth his name, not for that it teacheth any fraud or falshood, but for that by* false numbers *taken at all adventures, it teacheth how to find those* true numbers *you seek for.*

Scholar. So might any other Rule be called the Rule of Falshood, for they work by wrong numbers, and by them find out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship, and the Golden Rule, partly.

Master. In the Golden Rule, the Rule of Fellowship, and the Rule of Alligation, although the numbers that you work by be not the true numbers that you seek for, yet are they numbers in just proportion, and are found by orderly work; whereas in this Rule the numbers are not taken in any proportion, nor found by orderly work, but taken at all adventures. And therefore I sometimes being merry with my friends, and talking of such questions, do call unto them such Children or idiots as happen to be in the place, and so take their answer, declaring that

that I would make them solve those questions that seemed so doubtfull.

And indeed I did answer to the question, and work the triall thereof also, by those answers which they happened at all adventures to make. Which numbers seeing they be taken as manifest false, therefore is this Rule called The Rule of false Positions, and, for briefness, The Rule of Falshood. Which Rule, for readiness of remembrance, I have comprised in the few verses following, in form of an obscure Riddle.

Guess at this work as hap doth lead,
By chance to truth you may proceed.
And first work by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soon be found.
From many bate too many moe;
From too few take too few also:
With too much joyn too few again;
To too few adde too many plain.
In cross-wise multiply contrary kind,
All truth by falshood for to find.

The sense of these Verses and the summe of this Rule is this.

When any *question* is proposed appertaining to this *Rule*, first imagine any number that you list, which you shall name the *first position*, and put it in stead of the true number, and then work with it as the *question* importeth; and if you have missed, then is the last number of that work either too great or too little: that shall you note as hereafter shall be taught

The exposition of the Rule.

taught you, and you shall call it the *first errour*.

Then begin again, and take another number, which shall be called the *second position*, and work by the *question*: if you have missed again, note the excess or default as it is, and call that the *second errour*. Then multiply cross-wise the *first position* by the *second errour*, and again, the *second position* by the *first errour*, and note their totals severally by the names of totals. Then mark whether the two errours were both alike, that is to say, both too much, or both too little; or whether they be unlike, that is, the one too much, and the other too little. For if they be like, then shall you subtract the one totall from the other, (I mean the lesser from the greater) and the remainer shall be your *dividend*; so must you abate the lesser errour out of the greater, and the residue shall be the *divisor*. Now divide the *dividend* by that *divisor*, and the *quotient* will shew you the true number that you seek for. But and if the errours be unlike, then must you adde both those totalls (which you noted) together, and take that whole number for the *dividend*; so shall you adde both errours together, and that whole number shall be the *divisor*, and the *quotient* of that division shall give you the true number that the *question* seeketh for. And this is the whole Rule.

Scholar. This Rule seemeth so unlike any other, that without some example I shall not easily understand it.

Master. With a good will: propose half a score sundry questions and examples of variety, for the better understanding of the work hereof: and for the first take this example. A Mason

was bound to build a wall in 40 days, and it was covenanted ſo with him, that every day that he wrought he ſhould have for his wages 2 *ſhillings* 1 *peny*, and every day that he wrought not, he ſhould be amerced 2 *ſhillings* 6 *pence*; ſo that when the wall was made, and the reckoning taken of the days that he wrought, and of the other that he wrought not, the Maſon had clearly but *five ſhillings five pence* for the work. Now do I demand how many days he did work of thoſe 40, and how many he did not work.

A queſtion of Maſonry, the firſt Example.

Scholar. I pray you expreſs the order of the work, that I may partly by imitation, and partly by comparing it with the Rule, be able again to doe the like.

Maſter. This order ſhall you keep in the work of this Rule: firſt take ſome number (as you liſt) at adventure; as for example, I ſay he played 12 days, and wrought 28 days. Now caſt you the wages of every day, and ſee whether it will agree with the ſumme of 5 ſhillings 5 pence.

Scholar. The 28 days that he wrought after 25 pence the day yield 700 pence: then 12 days that he wrought not, at 30 pence each day, doth amount to 360 pence; which if I abate out of 700 pence, there reſteth 340. But you ſay he had not ſo much.

Maſter. He had but 65 pence, and by this ſuppoſition he ſhould have had 340; therefore is this ſumme too much by 275: which ſumme I muſt ſet down after this ſort as you ſee here; where firſt I have made a croſs, (commonly called S. Andrews croſs) and

12

275 †

at

at the over corner of the left hand I have set the first position 12; and at the other corner under it I have set 275, which is the first errour, with this figure †, which betokeneth too much, as this line ——— plain without a cross line betokeneth too little.

On the right hand of the cross I have left two like rooms for the second position and his errour. Therefore to prosecute the work, I suppose he played 16 days, and wrought 24.

Scholar. I was a while in doubt why you named the days of his working, seeing they be not set in the figure; and I doubted how you knew them, or else whether you did suppose them at all adventures, as you did the days that he played: but now I gather, that seeing 40 days is the whole time limited, then the days that he played being supposed, the rest of 40 must needs be the days that he wrought, and therefore 28 followeth 12 of necessity, and 24 followeth 16 also of necessity: but yet I scarce perceive why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here, both of working days, or both of idle. But now examine you this second position.

Scholar. If he played 16 days, then abating 16 times 30 pence, the summe will be 480 pence, and for 24 days that he wrought, every day yielding 25 pence, the totall is 600 pence; so that abating 480 out of 600, there resteth 120, and, as you say, it should be but 65, therefore it is too much by 55: that

that must be set on the right hand of the figure, at the neather part, and over it on the same side 16, which is the second position, thus:

And, as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you mark, that of what nature soever the two positions be, of the same nature is the quotient. Therefore when the positions in this question are 12 and 16, which both be numbers of the playing days, the quotient shall declare the true number of playing days: whereas if the positions had been 28 and 24, which are supposed to be the working days, then would the quotient declare the true number of the working days, and not of playing days, as it will doe now. And therefore to continue the work of this question, and to finde the true number of playing days, I must multiply cross-wise the first position by 55, that is the second errour, and the totall will be 660. Then I multiply 275 by 16, and it yieldeth 4400. Now because the errours are alike, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Again, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remain 220, which will be the divisor. Then dividing 3740 by 220, the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of days that the Mason played: and then it followeth that he wrought 23 days: and so is the question answered.

Now for the order of triall of this work, there needeth none other triall but onely this, to work with this number

The proof of this Rule.

number according to the question; and if it agree, then appears the number to be that you would have.

And here now seeing he wrought 23 days, and must have for every day 25 pence, the whole summe cometh to 575. Then again, seeing he played 17 days, and must abate 30 pence for every day, the whole summe of the abatement will be 510. Therefore I subtract 510 out of 575, and there will remain 65, which maketh 5 shillings 5 pence, the clear wages of the Mason for his work according to the question.

Scholar. Now I trust I understand the work and the Rule so well, (and the better by this proof) that I can be able to doe the like. And for a proof, I take the same question, all save the last number, where I will suppose that he had 10 shillings for his wages clear. And now to guess at the number of the days he wrought, I suppose first that he wrought 20 days; then say I, if he wrought 20 days, his wages must be 500 d; then did he play other 20 days, for which must be abated 600 d; and then he loseth 100 d. And so am I at a stay, for it is not like to your former work.

Master. You should have required of me some question, and not have taken a question of your own phansying, untill you were more expert in this Art, for so might you as well happen on an impossible question, as on a possible. But now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position he loseth 100, those both make 220, which you shall set down for the first errour, with this sign—, betokening too little, as

as here in this form following doth ap-pear.

20

—220

And now for the rest go forward your self once again.

Scholar. As my errour hath uttered my folly, so it hath procured me better understanding.

Now therefore considering this position not to salve the question, I take another, supposing that he wrought 30 days. Then for his wages he must be allowed 750 pence, and for the 10 days which he wrought not he must abate 300 pence, and so remaineth clear 450 pence; but it should be onely 120 pence, therefore it is too much by 330; which I set down in the figure with the former position and his errour, and the figure appeareth thus:

Now, first, I multiply in cross-ways 20 by 330, and it will be 6600.

20 30

—220—330+

Then again I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one out of the other, there will remain nothing to be the Dividend.

Master. In this you forget your self again: for inasmuch as the signs in the errours be unlike, therefore must you work by Addition, adding together those two totalls, to make the Dividend, and also adding the two errours, to make the Divisor. And because you shall no more forget this part of that Rule, take this brief remembrance;

Unlike require Addition,
And like desire Subtraction.

Scholar. You mean that if the errours have like signs, then must the Dividend and the Divisor

be made by Subtraction, as is taught before: and if those signs be unlike, (as in this last example they be) then must I by Addition gather the dividend and the divisor. Therefore must I adde 6600 to 6600, and it will be 13200, which will be the dividend.. Then again I adde 220 to 330, and it will be 550, which must be the divisor: wherefore dividing 13200 by 550, the quotient will be 24. Whereby I know that the Mason wrought 24 days, and then it followeth that he played 16 days.

Master. Examine your work, whether it be agreeable to the question or no.

Scholar. For 24 days work the wages must be 600 pence, and for 16 days which the Mason wrought not there must be abated 480 pence; and then remaineth clear to the Mason 120, as the question importeth: wherefore it is evident that 24 is the true number of days that he wrought.

Master. Although you seem now to understand this work, yet to acquaint your mind the better with the new Trade of this Rule, I think it good to propose to you 5 or 6 examples more, before I make an end of it.

Scholar. Sir, I thank you that you do so consider my commodity and profit in knowledge; for undoubtedly it is practice and exercise that maketh men prompt and expert in every kind of knowledge.

Master. You say well, so that they follow some certain precepts to govern and rule their practice by; else may practice procure custome of errour, and a repugnance to exactness of knowledge; namely, as long as the errour is not plainly known to the vulgar sort. But return to your work.

There

There is a ſervant that hath bought of Velvet and Damask for his Maſter 40 yards, the Velvet at 20 *ſhillings* a yard, and the Damask at 12 *ſhillings*, and when he cometh home, his Maſter demandeth of him how much he hath bought of each ſort. I cannot tell (ſaith he) exactly; but this I know, that I paid for Damask 48 *ſhillings* more then I paid for Velvet. Now muſt you gueſs how many yards there is of each ſort. A queſtion of wares, the ſecond example.

Scholar. Although the gueſs ſeemeth difficult, yet I will prove what I can doe: for I remember your ſaying, that it forceth not how fond or falſe the gueſs be, ſo it be ſomewhat to the queſtion, and not an anſwer of a contrary matter.

Therefore firſt I imagine that he bought 20 yards of Damask, for which he ſhould pay after the former price 240 ſhillings: then muſt he needs have of Velvet other 20 yards, (to make up the 40 yards) and that would coſt 400 ſhillings. So that the totall of the price of the Damask is leſs then the ſumme paid for Velvet 160 ſhillings, and ſhould be more by 48. Therefore the firſt errour is 208 too little. Then begin I again, and ſuppoſe he bought of Damask 30 yards, that coſt 360 ſhillings; then had he but 10 yards of Velvet, which coſt 200 ſhillings. And now the price of the Damask is greater then the price of the Velvet by 160 ſhillings, and ſhould be but 48. Therefore is the ſecond errour 112 too much, which I ſet in form of the figure as here doth appear. Then do I multiply in croſsways 208 by 30, and the ſumme will be 6240. Alſo I multiply 112 by 20, and there will amount 2240. And in as

20 30

208 —— 112 †

much as the signs of the errours be unlike, I know I must work by Addition: therefore adde I these two totalls together, and they make 8480, which is the Dividend: then adde I also the two errours together, 208 and 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be $26\frac{1}{2}$, which is the true summe of yards of Damask that he bought, and in Velvet 13 yards $\frac{1}{2}$; and that appeareth by examination, thus: $26\frac{1}{2}$ yards of Damask at 12 shillings the yard maketh 318 shillings: then in Velvet he had but 13 yards and $\frac{1}{2}$, which cost 270 shillings, at 20 shillings the yard. Now subtract 270 out of 318, and there will remain 48, which is the number of shillings that the Damask did cost more then the Velvet.

Master. Now shall you have a question of another kind.

A question of debt, the third example.

There are three men that do owe money to me, and I have forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace, you are too hasty, there is more help in it then yet you see. I have three severall notes, whereby it appeareth that I did confer their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third did rise to 88 pound. Now can you tell what every man did owe, and what was the whole summe?

Scholar. Nay, in good faith; but as I perceive that it must be found by conjecture, so will I guess at

at it, supposing that the first man did owe 20 pound, and the second man 30, and the third——

Master. Nay stay, there you are too far gone already: You may not suppose a severall summe for every man, for it is enough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20 pound, the second man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound; so must the second man his debt be but 27 pound. Now the second debt with the third must make 88: therefore subtract 27 out of 88, and there will remain 61, as the third man his debt. Then saith the declaration, that the first and third mans debts do make 71: but by this supposition they make 81, that is 10 too much, which I must set for the first errour. Now work you the second position.

Scholar. I suppose the first mans debt to be 24 pound: then must the second mans debt (by your declaration) be but 23 pound, seeing both they make but 47 pound. And the second man his debt with the third do make 88 pound, and the second man oweth but 23; therefore the third man must owe 65 pound. Now the third mans debt with the first should make by the declaration 71 pound, and they do make 89 pound, that is 18 pound too much, and that is the second errour, which I set down with the first, and their positions, in this form: and then I do multiply in cross ways 20 by 18, and it is 360, and 10 by 24 maketh 240. Also because the signs of the errours be

20	24
10+	18+

 like,

like, I must work by Subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the Dividend: then do I subtract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120. Therefore I say that the first man did owe 15 li, and then the second man must owe 32 li; for those two do make 47 li; and the third mans debt is 56, for so much remaineth if if I abate 15 out of 71, or if I take 32 out of 88.

The fourth example. *Master.* For the fourth example, take this easie question for the variety in work. Two men having several summs, which I know not, do thus talk together. The first saith to the second, If you give me two *shillings* of your mony, then shall I have three times so much mony as you. The second man answereth, It were more reason that our summs were made equall, and so will it be if you give me 3 *shillings* of your mony. Now guess what each of them had.

Scholar. I imagine that the first had 9 s.

Note. Master. Consider evermore in your imagination that you take a likely summe, as in this question, take such a summe, that having 2 added unto it may be divided into three parts even.

Scholar. Why? I remember you said before, it forceth not how fondly soever I guessed.

Master. As for the possibility of the solution it is truth; but for easiness in work, the aptest numbers are most convenient.

Scholar. I thought no less, and therefore I took 9 as an apt number to be parted into three: but I perceive I should have considered the aptness of that partition after the Addition of two unto

unto it, and then 7 had been more meet.

Master. That is truth, and then should the second man his summe be 5: for although he have now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

Scholar. Then to go forward: If the second man had three of the first man, then should he have 8, and the first man but 4; so hath he double to the first man: yet he said in the question they should have equal: wherefore it appeareth that he hath 4 too much.

Therefore I note that errour with his supposition, and guess again that he hath 10 shillings; whereunto I adde 2 shillings borrowed of the second man, and then he hath 12 shillings: so the second man hath remaining but 4, whereunto if I adde the 2 that he lent to the first man, so had he but 6 shillings at the beginning.

Then take 3 shillings from the first man, and give to the second, and then hath the first man but 7, and the second hath 9, which are not equal, but there are 2 too many; wherefore I set down both the positions with their errours, as before you see, and multiply a-cross, so cometh there 40 and 14: and because the signs be like, I take 14 out of 40, and so resteth 26 to be divided: then likewise I take 2 out of 4, and there resteth 2, by which I divide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that 2 being added thereto, the summe will be 15; so hath the second man but 5, and before he had 7. Then take 3 from the first, and put to this 7, and so have

7	10
4 †	2 †

each

each of them 10, and that is equall, as the question would.

The fifth example, a question of Lambs.

Master. For the fifth example take this question. One man said to another, I think you had this year two thousand Lambs. So had I, said the other; but what with paying of tithe of them, and then the severall losses, they are much abated: for at one time I lost half as many as I have now left, and at another time the third part of so many, and the third time $\frac{1}{4}$ so many. Now guess you how many are left.

Scholar. Because here is mention made of certain parts, I must take a number that may have all these parts, that is to say, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, which will be 24, howbeit 12 hath the same parts. Therefore I take first 12 to be the number that doth remain, so hath he lost 6, 4, and 3, that is 13, and the whole 25, but it should be 2000.

Master. You are deceived yet still, you have forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800: and now go on again.

Scholar. Then, to find the errour, I take 25 out of 1800, and there remaineth 1775, too few, which I set for the errour. Then for the second position I take 24, whose half is 12, the third part 8, and the quarter 6, whereby riseth 50, which is too little by 1750. Therefore I set down both the positions, with their errours, thus:

12	24
—1775	2750—

And multiply in cross waies 1775 by 24, whereof cometh 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signs are like,

like, I do subtract the one from the other, and so remaineth the Dividend 21600. Then do I subtract 1750 out of 1775, and there resteth 25; by which I divide 21600, and the quotient is 864; whereof the half is 432, and the third part is 288, the quarter is 216; which all being added together will make 1800. And if you adde thereto the tenth which was abated before, then will the whole summe be 2000.

864
432
288
216
1800

And now doth there come a question to my memory, which was demanded of me, but I was not able to answer to it: And now methinketh I could solve it.

Master. Propose your question.

Scholar. There is supposed a Law made, that (for furthering of tillage) every man that doth keep sheep, shall for every ten sheep ear and sow one Acre of ground: and for his allowance in sheep-pasture, there is appointed for every four sheep one Acre of pasture. Now is there a rich Sheep-master which hath 7000 Acres of ground, and would gladly keep as many as he might: by that Statute I demand how many sheep he shall keep.

A question of sheep and tillage, the sixth example.

Master. Answer to the question your self.

Scholar. First, I suppose he may keep 500 sheep, and for them he shall have in Pasture, after the rate of four sheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I guess again that he may keep 1000 sheep, that is, in Pasture 250 Acres, and in tillage 100 Acres, which make 350: that is too little by 6650. Both these errours with their positions I set down as you

you see, and multiply them cross, 6825 by 1000, and it maketh 6825000: also I multiply 6650 by 500, and there cometh 3325000; which summe I subtract out of the former, 6825—6650— and there remaineth 3500000 for the Dividend: likewise I subtract the lesser errour out of the greater, and there resteth 175; by which I divide 3500000, (the Dividend aforesaid) and the quotient will be 20000. So that by this rate he that hath 7000 Acres of ground may keep 20000 sheep.

500 ╳ 1000

Another way of working.

Master. You have done well, notwithstanding both this last question and the next before might be wrought without the second position by the Rule of proportion, as thus: When in this question you found in the first errour that for 500 sheep there must be 175 Acres, then might you reduce it to the Golden Rule, thus:

175 500
7000 20000

If 175 Acres will admit in allowance 500 sheep, then 7000 will have 20000. And so by one position, with the help of the *Golden Rule*, may you answer that question.

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have set the figure as followeth, and have said,

25 12
1800 864

If 25 do leave but 12, what shall 1800 leave? and it would appear to be 864.

Scholar. Sir, I thank you for this aid, for it doth much shorten the work of this Rule.

Another way yet.

Master. Yet again I will shew you another way to answer to this last question without the Rule of

of false position, and that by the Rule of Fellowship. For it appeareth in the proposing of the question, that ten sheep must have in pasture 2 Acres and $\frac{1}{2}$, and for them must there be eared but one Acre: so it followeth, that for 2 Acres eared there must be 5 set to pasture; and if you put them both into one summe, they will make 7. Therefore look what proportion 7 being this total doth bear to 5 and to 2, such proportion shall any totall in this question bear to the pasture ground and the eared ground.

Scholar. This serveth wondrous aptly. Therefore to prove it, I demand this by the former supposition: If a man have 300 Acres, how much shall he leave in pasture, and how much shall he turn to tillage? You say, that as 7 is to 5, so shall 300 be to the Acres of pasture; and as 7 is to 2, so is 300 to the Acres of tillage; whereof for both I have set examples here following: whereby appeareth that of pasture there shall be 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$; both which summs added together do make 300.

$$\begin{array}{lcl} 7 & & 5 \\ & Z & \\ 300 & & 214\frac{2}{7} \end{array}$$

$$\begin{array}{lcl} 7 & & 2 \\ & Z & \\ 300 & & 85\frac{5}{7} \end{array}$$

Master. Now take another Example. A man hath three silver Cups with one Cover, the Cover weigheth 18 ounces, the second Cup weigheth even half the weight of the first and the third. Now if the Cover be put to the first Cup, they weigh just as much as all the three Cups do weigh: and if the Cover be joyned with the second Cup, they weigh as much as the second twice, and the third: and if the Cover be put to the third Cup, they will make twice as much as the first and second Cup. Now try you what was the just weight of every Cup. Another question, the seventh example.

Scholar.

Scholar. I do set the weight of the first Cup to be nine ounces; then, inasmuch as these two (that is to say, the Cover and the first Cup) do weigh the weight of the three Cups, I see that the three Cups must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third do weigh double so much as the second, therefore it is the third part of that weight, that is 9; and then would it follow, that the third Cup also should weigh 9 ounces: but then the question saith, that the Cover being joyned to the second Cup, they weigh as much as the second twice, and the third once, that should be 27, and so it doth; that being joyned with the third Cup, they should weigh twice as much as the first and the second, that should be 36, and they weigh but 27; so is that errour 9 too little. Then begin I again, and say, that the first Cup doth weigh twelve ounces, which I joyn with the Cover, and they make thirty ounces: then seeing the second is of that weight, it must needs weigh ten ounces, and the third must weigh 8 ounces, seeing the first and the third must weigh 20 ounces. Now put I the Cover to the second Cup, and they weigh 28 ounces, which should be even so: then joyn I the Cover with the third Cup, and so should it weigh twice the first and the second, that is 44 ounces, and they weigh but 26, that is, 18 too little. These errours with their positions I set down, and multiply in cross-ways 9 by 12, whereof cometh 108; also 9 by 18, and that yieldeth 162: and inasmuch as the signs be like, I abate the lesser out of the greater, and there doth

9   12

9—   18—

remain

remain 54. Then do I also abate the lesser errour from the greater, and so remaineth 9, by which I divide 54, and the quotient is 6; which I take for the true weight of the first Cup, which being joyned with the Cover must weigh as much as the three Cups; so do they weigh but 24 ounces. Then seeing the second Cup is the third part of that weight, for the other two Cups (you say) must weigh double his weight, the weight of the second Cup is 8 ounces; and so the weight of the third Cup must be 10 ounces. Now put the Cover to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Again, put the Cover to the third Cup of 10 ounces, and they must weigh twice as much as the first and the second, that is, 28, and so is all agreeable.

Master. Then answer to this Question.

There is a Cistern with four Cocks, containing 72 barrels of water: and if the greatest Cock be opened, the water will avoid clean in six hours; at the second Cock it will ask eight hours; at the third Cock it will avoid in no less then nine hours; and at the smallest it will require twelve hours. Now I demand in what space will it avoid, all the Cocks being set open? A question of water, the eighth example.

Scholar. First, I imagine it will avoid in two hours.

Master. Then must there avoid by the first Cock $\frac{1}{3}$ of the water, that is 24 Barrels, and by the second Cock $\frac{1}{4}$, that is 18; and by the third Cock $\frac{2}{9}$, that is 16 Barrels, and by the smallest Cock $\frac{1}{6}$, that

that is 12 Barrels; all which summs put together do make 70, as by their Addition it doth appear: but it should be 72; therefore the errour is too few.

Scholar. Then will I begin again, by 24
your favour, because I think I understand the 18
work, and put three hours for the due time: 16
so shall there run out at the greatest Cock $\frac{1}{2}$, 12
that is, 36 Barrels, and at the second hole $\frac{3}{8}$, —
that is 27, and at the third Cock $\frac{1}{3}$, that is 70
24, and at the smallest hole $\frac{1}{4}$, that is 18 Barrels, which all together do make 105, and should be but 72, so is it too much by 33: therefore do I set the errours in order of the figure with their positions; and work by multiplication in cross, saying, 2 times 3 is 6, and 2 times 33 make 66; and because the signs are unlike, I must adde these 2 totalls together, which make 72; also I adde the two errours, and they make 35, by which I divide 72, and the Quotient riseth 2 $\frac{2}{35}$. Whereby I see that all the Cocks being set open, the water will avoid in two hours and $\frac{2}{35}$ of an hour.

2 3

2 — 33 +

Master. This exercise maketh you to grow expert in the Rule. Therefore I will enure you somewhat more with a question or two.

A question of partners, the ninth example.

There were two men that had been partners, and had in account between them 300 Ducats; whereof the one should have for his part 180, & the other 120: but in the parting of them they fell at variance, so that each of them catched as many as he could; yet afterward, being reconciled, they agreed that he which had gotten most part of them should lay down $\frac{3}{4}$ of them again, and he that had gotten least should lay down $\frac{1}{3}$ of those

which

which he had taken, and then parting them into two equall parts, each man to have half thereof: and so had they their just portions as they ought to have. Now I demand of you what each of them had gotten by the scrambling.

Scholar. I suppose he that had least got 108 Ducats, then the other had 192: wherefore in laying down again of the 192, there was put down $\frac{3}{4}$, that is 144, and so had he left but 48; also of the 108 there was laid down 36, that is $\frac{1}{3}$, and so he had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts even, and so cometh 90 to be given to each of them; which summe put to 72 maketh 162, and joyned to 148 it maketh 238. And now I doubt how I shall go forward.

Master. You need not to take but one of them, which you list, the greater or the smaller, for all cometh to one purpose; and so may you compare it that you take to any of the other summs, remembring that you make comparison to the same in the second work: As for example of the first part, if you compare 138 with the lesser summe due, that is 120, so is it 18 too much; and if you compare it with the greater summe, then is it 42 too little. Again, if you compare 162 to the greater summe, the errour will be 18, as it was in the other, but it will have a contrary sign; and if you compare it with the lesser summe, it will be 42 too much: so that the errour both waies is either 18 or 42. And as for the signs, it little forceth, for in them is nothing considered here but likeness and unlikeness, which in this case doth neither

Note.

Z further

further nor hinder. But now go on with the work.

Scholar. If it be so, then am I out of my greatest doubt. Then I joyn that 90 (which I found as the half of the latter partition) unto 48, which is left with the one man, and so hath he 138, which (I may say) is 18 too many, for the least should be but 120. That errour do I note, and then make a new position, supposing the one man to have 204, and the other to have 96: wherefore of the 204 there must be laid down 153, and so remaineth with him 51; also of the 96 there must be laid down $\frac{1}{3}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe, as 185, which I must divide into two equall parts, and so each man shall have $92\frac{1}{2}$; whereunto if I adde their former portions reserved, then the one shall have $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore I take the lesser summe now again, as I did before, that is, $143\frac{1}{2}$, and I find that he hath too many by $23\frac{1}{2}$, for he should have but 120. And so have I for my two positions two errours, which I set down as here may be seen, each errour under his position; and then by the Rule I do multiply in cross-ways 108 by $23\frac{1}{2}$, and there riseth 2538, which I note; then again I multiply 96 by 18, and thereof amounteth 1728.

108	96
18†	$23\frac{1}{2}$†

Now because the signs are both like, that is, both too many, I must work by Subtraction, and so abating 1728 out of 2538, there will rest for the Dividend 810: then for the Divisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth

maineth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{3}{11}$, which is the just portion of him that had the least summe. And if I do subtract it out of 300, being the totall summe, then will there remain $152\frac{8}{11}$, as the portion that the other did get.

Master. For the proof of this work, you may chuse whether you will examine those numbers according to the form of the questions, or else work by other two positions, for to find the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each work confirm other.

Scholar. By your patience, I will prove both ways, not onely to seek their agreement, but also to accustom my mind to those works: for I perceive it is exercise that must be the chief engraver of these Rules in my memory.

Master. You consider it well: then go to.

Scholar. First, I will by two other positions try to find the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your work the better, take the same positions that you did before, comparing them now to the greater, as you did before unto the lesser.

Scholar. Then I suppose that he that had most had 192, so had the other 108. Now if I take $\frac{3}{4}$ out of 192, that will be 144, and there will rest to that man but 48. And from the second, which had 108, if I take $\frac{1}{3}$, that is 36, there will remain to him 72. Then joyning 144 with 36, it will make 108, the half whereof, being 90, if I adde to

each of those two mens portions remaining with them, the one shall have 138, and the other 162; of which two I take the greater, (that is 162) and see it to be 18 too few, for it should be 180: that errour I note under this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will be 153, and there resteth to him 51; also of the 96 I take $\frac{1}{3}$, that is 32, and there remaineth to him 64. Now put I that 32 to 153, and it yieldeth 185; which being parted in equall values maketh $92\frac{1}{2}$ to be added to each mans remainder; and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$. Wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little; that do I note also, and set both these errours under their positions, as in this Example following doth appear.

And then multiplying 192 by $23\frac{1}{2}$, there doth arise 4512.

Again I multiply 204 by 18, and it maketh 3672, which I do subtract out of 4512, because the signs be like, and there resteth 840 for the dividend;

192 204

—18— $-23\frac{1}{2}$

then subtracting 18 out of $28\frac{1}{2}$, there will remain $5\frac{1}{2}$, which I must take for the Divisor. And so dividing 840 by $5\frac{1}{2}$, the quotient will be $152\frac{8}{11}$. Whereby I have found an agreeable summe to that which I found by the former positions, for him that had most, which I do subtract out of 300, that is the totall, and there will rest $147\frac{3}{11}$, which was the portion of him that had the least part.

Master. So by divers positions you see, that one

doth

doth confirm the work of the other. Now examine those two numbers by the form of the question, and so shall you prove your work good also.

Scholar. If that he which gat most had $152\frac{8}{11}$, then must he lay down $\frac{3}{4}$ of his summe, that is $114\frac{6}{11}$, and so shall remain with him but onely $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$, must put down of his summe $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doth there remain with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$ and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into equall parts, and that will be $81\frac{9}{11}$ to be given to each of them: putting $81\frac{9}{11}$ unto $38\frac{2}{11}$, there doth amount 120 just, which is the true portion of him that should have the lesser summe: and adding $81\frac{9}{11}$ to $98\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the work by this proof also tried to be good. And this I mark by the way, that in their scrambling he got most (as it chanceth often) that ought to have had least by just partition.

Master. Let your study be to learn truth and just Art of proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this Rule, save that I remember one pleasant question which I cannot overpass, which I will declare somewhat largely, because you shall as well understand some reason in the pleasant invention, as apt proceeding in the witty working thereof.

Hiero King of the *Syracusans* in *Sicilia* had caused to be made a Crown of Gold of a wonderfull weight, to be offered for his good success in wars: in making whereof the Goldsmith fraudulently took out a certain

The tenth example, of Gold and Silver.

portion of Gold, and put in Silver for it, so that there was nothing abated of the full weight, although there was much of the value diminished.

Which thing at length being uttered, (as no evil can alwaies lie hid) the King was sore moved; and being desirous to know the truth without breaking of the Crown, proposed the doubt to Archimedes, unto whose wit nothing seemed unpossible: which although presently he could not answer unto, yet he had good hope to devise some policy for that invention; and so musing thereon, as he chanced to enter into a Bain full of water to wash him, he observed, that as his body entred into the Bain, the water did run over the Tub: whereby his ready wit, of such small effects conjecturing greater works, conceived by and by a reason of solution to the King's question; and therefore rejoycing exceedingly, more then if he had gotten the Crown it self, forgat that he was naked, and so ran home, crying, as he ran, εὕρηκα, εὕρηκα, I have found, I have found. And thereupon he caused two massie pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier of nature then Silver, and therefore Gold of like weight with Silver must needs occupy less room, by reason it is more compact and sound in substance, he was assured that putting the mass of Gold into a vessell brimfull of water, there would not so much water run out, as when he should put in the silver mass of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time,

but

but also the difference or excess of the one above the other; whereby he learned what proportion in quantity is between Gold and Silver of equall weight. And then putting the Crown it self into the vessell of water brim-full, (as before) he marked how much water did run out then; and comparing it with the water that ran out when the Gold was put in, noted how much it did exceed that; and likewise comparing it to the water that ran out of the Silver, marked how much it was less then that: and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it. But seeing Vitruvius, which writeth this History, doth not declare the particular work of this triall, it shall be no inconvenience to suppose an example for declaration sake, wherein although the true and just proportion be not expressed, yet the form of triall shall be truly set forth. And for an example, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Mass of Gold was put into the water, I imagine that there ran out two pound of water; and when the mass of Silver was put in, I suppose there ran out 3 pound $\frac{1}{2}$; again, when the Crown was put in, there ran out two pound $\frac{1}{4}$. Now to know what quantity of Silver was in the Crown, work by the Rule of false position, and imagine that there was two pound of Silver, then must there be six pound of Gold: then say thus by the Rule of Proportion, If eight pound of Gold do expell two pound of water, what shall six pound expell? and it will be 1 pound $\frac{1}{2}$. Again, for the Silver; if eight pound of Silver

expell three pound $\frac{1}{2}$ of water, what shall two pound of silver put out? it will be $\frac{7}{8}$. Now adde those two weights of water together, and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{2}{4}$; so is it too much by $\frac{1}{8}$.

Scholar. Now do I understand the work, as I think, therefore I pray you let me work the rest of the question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the Silver to be but one pound, so must there be in Gold 7 pound. Then say I, if eight pound of Gold do yield two pound of water, what shall seven pound yield? and it will be 1 pound $\frac{3}{4}$. Again, if 8 pound of Silver expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{7}{16}$. Now must I adde those two summs together, and they make two pound $\frac{3}{16}$, and they should make two pound $\frac{1}{4}$; so is it too little by $\frac{1}{16}$. Therefore I set the positions with their errours in order as here followeth. And then I multiply in cross-ways 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewise 1 multiplied by $\frac{1}{8}$ maketh $\frac{1}{4}$. And because the signs be unlike, I must adde these two summs, which make $\frac{1}{8}$; and that is the dividend.

$$\begin{array}{ccc} 2 & & 1 \\ \frac{1}{8}\,+ & & -\,\frac{1}{16} \end{array}$$

Again, I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the Divisor. Now I shall divide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient will be $\frac{16}{12}$, that is, $1\frac{1}{3}$: whereby I knew that there was put 1 pound and $\frac{1}{3}$ of Silver into the Crown, and so much Gold taken out for it.

Master. Prove it now by examination, according to the question.

Scholar. If there were 1 pound $\frac{1}{3}$ of Silver, then

was

was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion, if 8 pound of Gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell? It will be 1 pound $\frac{2}{3}$.

$$\begin{array}{ccc} 8 & & 2 \\ 6\frac{2}{3} & & 1\frac{2}{3} \end{array}$$

Again, if 8 pound of Silver expell three pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$. Now must I adde together 1 pound $\frac{2}{3}$ and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is 2 pound $\frac{1}{4}$, according to the supposition of the question: whereby I perceive the work to be well done. And as I cannot but much rejoyce of this excellent invention, so my desire is kindled vehemently to be perfectly instructed in every part thereof, and namely in this point, Whether the proportion between water and Gold be such, that for 8 l. of Gold put into a vessel full of water there shall run out two pound of water, and for as much Silver, whether 3 pound $\frac{1}{2}$ of water would avoid.

$$\begin{array}{ccc} 8 & & 3\frac{1}{3} \\ 1\frac{1}{3} & & \frac{7}{12} \end{array}$$

Master. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact proportion between Gold and Silver and water, both in their weight and quantities, then could you easily find out the mixtures of them: which thing I have reserved for another work, that intreateth of such matters especially. And at this time you must consider that you learn Arithmetick, which intreateth of the manner to solve doubtful Questions touching Number, without regard what matter is signified by that Number: else were it necessary in Arithmetick to teach all Arts, seeing in it may be moved questions of all Arts.

But seeing you are so desirous to know these

things,

A question of the proportion of gold, silver, and quick-silver, unto water.

things, I will tell you in such a sort, that you shall practise your Art in finding it, and propose it in form of a question. Gold beareth a greater proportion to Water then Silver doth, and their two proportions be in proportion together as 48 to 25. But to help you somewhat in this Riddle, you shall note that the proportion of Quick-silver unto Water is the just middle number proportional in progression Geometrical between the proportion of Gold and Silver unto Water.

And this proportion is $\frac{220}{21}$. Now if you will know the just numbers of these 3 proportions, then must you find out 3 numbers in Progression Geometrical, whereof the middlemost must be $\frac{220}{21}$, & the first must be unto the last as 25 to 48. And thus I will leave you to find those numbers when you be at leisure.

Scholar. Yet, Sir, I thank you heartily for thus much, for now I see the possibility to find them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somewhat in the order of working for it, I should the more easily find the true working.

Master. You desire too much, if you will study for nothing: Therefore to occasion you to study the better, I will leave this doubt wholly to your own search. But as touching the generality of the Rule, Archimedes needed not to take two Masses of Gold and Silver equall in weight with the Crown, for the proportion might as well be found in any other weight, yea, although the Mass of Gold were of one weight, and the Mass of Silver of another. As for example; If the Crown were of 8 pound weight, as I did suppose, and I have not

so

so much other fine Gold, but onely one pound, and try that by water, and find that it doth expell but $\frac{3}{4}$ of an ounce of water; yet then by it I may inferre, that 8 pound of Gold would expell 6 ounces of water. And likewise of Silver, whereof if I had but two pound, and find that it doth expell three ounces of water, then might I affirm, that 8 pound would expell 12 ounces, that is, one pound weight: and so is it as good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmetick.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to look for the Art of Extraction of Roots, whereof hitherto I have learned nothing.

Master. I will not break my promise, but intend (God willing) to perform it within this three or four months, if I perceive this my pains to be well taken in the mean season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the mean time you shall take this Addition, not for the second part of Arithmetick which I promised, but for an augmentation of the first part; unto which I would have annexed the Extraction of Roots square and cube, namely for Examples of the Statute of Assise of Wood, but that in the Second Part I must write of divers other Roots, and thought it best to reserve those Rules also with their Examples unto the same Second Part.

Scholar,

Scholar. Sir, although I cannot recompense your goodness, yet I shall alwaies doe mine endeavour to occasion you not to repent your benefit on me thus imployed.

Master. That recompence is sufficient for your part.

FINIS.

THE THIRD PART,

OR,

ADDITION to this BOOK,

Entreateth of brief RULES,

CALLED

RULES of PRACTICE,

Of rare, pleaſant, and commodious effects; abridged into a briefer method then hitherto hath been publiſhed.

With divers other neceſſary RULES, TABLES & QUESTIONS, not onely profitable for Merchants, but alſo for Gentlemen, and all other Occupiers whatſoever, as by the Contents of this Book may appear.

Set forth by JOHN MELLIS, School-maſter.

The First Chapter of this Addition entreateth of brief Rules, called *Rules of practice*, with divers necessary questions, profitable not onely for *Merchants*, but also for all other *Occupiers* whatsoever.

THE working of *Multiplication in practice* is no other thing then a certain manner of Multiplying of one kind by another, whereupon is brought forth the Product of the proposed number: which is accomplished by the means of *Division*, in taking the *half*, the *third*, the *fourth*, the *fifth*, or such other parts of the summe which is to be multiplied.

And for the better understanding of such conversions, you shall understand, that in the manner and use of these Rules of Practice you ought first to know the even or aliquot parts of a shilling, which in this Table *following do appear.* Rule 1.

$$\textit{Item}\left\{\begin{matrix}6\\4\\3\\2\\1\end{matrix}\right\}\textit{pence is the}\left\{\begin{matrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\\\frac{1}{12}\end{matrix}\right\}\textit{of a shilling.}$$

Wherein, as you see according to the order of these Rules of Practice, at 6 pence the yard of any thing you must take $\frac{1}{2}$ of your number which is to be multiplied, and the Product that cometh thereof shall

be

be shillings: if any unite do remain, it is 6 pence.

For 4 d, take the $\frac{1}{3}$ of the number that is to be multiplied, and the product also produceth shillings: if any unites do remain, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

I. *Example.*

At 6 d. the yard, what	379 yards?
	189 s——6 d

II.

At 4 d the yard, what	104 yards?
	34 s——8 d

III.

At 3 d the yard, what	5014 yards?
	1253 s——6 d

IV.

At 2 d the yard, what	532 yards?
	88 s——8 d

V.

At 1 d the yard, what	409 yards?
	34 s——1 d

Here you may see in the first example, that 379 yards, at 6 d. the yard, are worth 189 s. 6, in taking the $\frac{1}{2}$ of 379. And in the second example, the 104 yards, at 4 d. the yard, are worth 34 s. 8 d, in taking the $\frac{1}{3}$ of 104. Likewise in the third example, 5014 yards, at 3 d. the yard, bring forth 1253 s. 6 d, in taking the $\frac{1}{4}$ of 5014. Also in the fourth example, 532 yards, at 2 d. the yard, maketh 88 s. 8 d. And lastly, in the fifth example, 409 yards, at 1 d. the yard, amounteth to 34 s. 1 d, in taking the $\frac{1}{12}$ of 409.

409. And so is to be done also of all other questions the like, when the number of the pence is any of the even or *aliquot* parts of 12 d.

Item, to bring the Products of these shillings, and all other the like, into pounds, is very easie in dividing of it in your minde by 20. For it is to be understood that as often as 20 is found in that Product, so many pounds doth it contain: which with facility to perform, always strike off the figure towards your right hand, with a right-down dash of your pen, for the o that appertaineth to the 20; and then begin at the left hand, in taking the half off the rest. And if that at the last any unite do remain, the same shall be joyned with the figure that is cut off, which shall represent the odde shillings contained in that work.

As for example, in your third question at 3 d. the yard, which amounteth to 1253 s. 6 d, the Product whereof maketh 62 li. 13 s. 6 d, as here you may see is easily performed by this example.

1	
125	3
62—13—6	

Also for the working of one peny the yard, it is something harsh and hard to take the $\frac{1}{12}$ of some Products: therefore to ease that hard work, you shall first bring your delivered summe into groats, by taking $\frac{1}{4}$ part of the Product, and if any unites remain of that $\frac{1}{4}$ part, as sometimes there may, they are pence, and must be signified with a line from the groats with their title of pence; and because that 60 groats maketh a *Pound* or twenty shillings, strike off the first figure toward your right hand for the o that appertaineth to 60, (as you did even now for the o that belongeth to 20.) Then in taking the $\frac{1}{3}$ of that product, if there do remain any unites, the same shall

you joyn with the figure that you cut off, esteeming them as groats, which keep in your minde, and by taking the $\frac{1}{3}$ part of them, you shall turn them into shillings, and so have you done. As for *example*, by a Question or two hereafter proposed, shall more plainly by the work appear.

At 1 d. the yard, what 54368 yards?

13592 groats.

$\frac{1}{6}$ li. —— 226 – 10 s. 8 d.

Here in taking the $\frac{1}{6}$ part of 1359, in coming to the last work, the $\frac{1}{6}$ part of 39 being taken, the remainder is 3, which joyned with the two that was cut off maketh 32 groats, which converted into shillings, by taking the $\frac{1}{3}$ part, maketh as appeareth 10 shillings 8 d. Many other ways there are, but none more apt for a young learner to understand then this: wherefore this one way well impressed in memory is better then 20 ways doubtfully understood.

At 1 peny the yard, what 4533 yards?

113 | 3 groats – – 1 d

$\frac{1}{8}$ li. 18 —— 17 —— 9 d

At 1 peny the yard, what 64768 yards?

1619 | 2 groats.

li. 269 —— 17 —— 4 d.

Now followeth also to be understood, that if the number of pence be not an aliquot *part of* 12, *you must reduce them into some* aliquot *part of* 12: *and after the aforesaid manner you shall make of them two or three Products, as need shall require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their*

parts,

parts, as they are to be taken.

for pence	*take*	*and*	*or*	& / *and*
5	3	2	4	1
7	4	3	6	1
8	4	4	6	2
9	6	3	4.4	1
10	6	4	4.4	2
1	6	4& 1	4.4	3

Here in the first note of this *Table* at 5 d. you shall first take for 3 d. the $\frac{1}{4}$ of the number that is to be multiplied; and likewise for 2 d. the $\frac{1}{6}$ of the same number, adding together both the Products. But if you will work by 4 and 1, you must for 4 d. first take $\frac{1}{3}$ of the number that is to be multiplied, and for 1 d. take the $\frac{1}{12}$ of the whole summe; or rather, which is better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence, because that 1 d. is the $\frac{1}{4}$ of 4 pence. The totall summs of these two numbers shall be the solution to the *Question.* And in like manner it is to be done of all others, as by these Examples following shall appear.

I.

At 5 d. the yard, what	758 yards ?
3 d	189 — 6 d
2 d	126 — 4 d
shillings	315 — 10 d.

Otherwise.

At 5 d. the yard, what	758 yards ?
4 d	252 — 8 d
1 d	63 — 2 d
shillings	315 — 10 d

II.

At 7 pence the Ell, what	563 Ells?
4 d	187—8 d
3 d	140—9 d
shillings	328—5 d

III.

At 8 d. the pound, what	112 pound?
4 d	37—4 d
4 d	37—4 d
shillings	74—8 d

Otherwise.

At 8 d. the pound, what	112 pound?
6 d	56—0
2 d	18—8
shillings	74—8 d

IV.

At 9 pence the Ell, what	356 Ells?
6 d	178—0
3 d	89—0
shillings	267—0 d

V.

At 10 pence the piece, what	795 pieces?
6 d	397—6
4 d	265—0
shillings	662—6

VI.

At 11 pence the pound, what	7576 pounds?
6 d	3788 —— 0
4 d	2525 —— 4
1 d	631 —— 4
	6944 —— 8 d
pounds	347 — 4 s — 8 d

1. Here in this first example, where it is demanded, at 5 d. the yard what will 758 cost? First, for 3 d. I take the $\frac{1}{4}$ of 758, and thereof cometh 189 s. 6 d. Then for 2 d. I take the $\frac{1}{6}$ of the same 758, which mounteth to 126 s. 4 d. These two summs added together do make 315 shillings 10 pence: and so much are the 758 yards worth at 5 d. the yard.

Item also for the same again: First for 4 d. I take the $\frac{1}{3}$ of 758, and thereof cometh 252 s. 8 d. Then for 1 peny I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s. 8 d, and it yieldeth me 63 s. 2 d. Which both added together make 315 s—10 d, as before.

2. *Item*, for 7 d. there is taken the $\frac{1}{3}$ and the $\frac{1}{4}$ of the whole summe which is to multiplied, and adde them together; that is to say, first, for 4 pence there is taken $\frac{1}{3}$ of 563, which comes to 187 s. 8 d, as appeareth by the work, and for 3 d. there is taken the $\frac{1}{4}$ of the whole summe, which amounts to 140s-9d. Both which products added together do make 328 s. 5 d. And so much comes 563 Ells to at 7 d. the Elle.

3. *Item*, for the first 8 d, there is taken for 4 d. the $\frac{1}{3}$ of the whole summe, and another $\frac{1}{3}$ for the other 4 d; which added together, as in the example doth

 evidently

evidently appear, amounteth to 74 s.-8 d.

Again, for the second work of 112 l. there is taken first the $\frac{1}{2}$ of the whole summe for 6 d, which comes to 56 s, then for that 2 d. you have to take $\frac{1}{6}$ of the whole summe, or, if you will, the $\frac{1}{3}$ of the product that came of 6 d, either of which maketh 18 s. 8 d. These two summs being added together do make 74 s. 8 d, as in the third example appeareth.

4. *Item* for 9 d, there is taken for 6 pence the $\frac{1}{2}$ of the whole summe, and the $\frac{1}{4}$ of the whole summe for 3 d; or otherwise for the 3 d. you may take the $\frac{1}{2}$ of the product that came of 6 d, because 3 pence is the $\frac{1}{2}$ of 6 d: which added together, as plainly appeareth in the fourth *example*, amounteth to 267 s. 0 d.

Item for 10 d, first there is taken for 6 pence the $\frac{1}{2}$ of the whole sum, which amounteth to 397 s. 6 d, then for 4 d. there is found 265 s; both which added together make 662 shillings 6 d, as appeareth in the fifth *example*. It may also be wrought, as appeareth by the second note in the *Table*, by 4 d. twice taken, and the $\frac{1}{2}$ of the product of 4 d, or else by the $\frac{1}{6}$ of the whole summe, &c.

Item for 11 d, there is first taken the $\frac{1}{2}$ for 6 d, then the $\frac{1}{3}$ of the whole summe for 4 d, lastly, the $\frac{1}{4}$ of the last product for 1 d. All which 3 summs added together make in shillings 6944-s-8 d, and in pounds 347-4 s.-8 d.

Item, likewise by the same reason, when you will multiply (by shillings) any number that is under 20, you shall have in the Product pounds, if you know the even or aliquot parts of 20, which are here in this little Table set down to sight.

Item,

$$\text{Item, s.}\left\{\begin{array}{c}10\\5\\4\\2\\1\end{array}\right\}\text{is the}\left\{\begin{array}{c}\frac{1}{2}\\\frac{1}{4}\\\frac{1}{5}\\\frac{1}{10}\\\frac{1}{20}\end{array}\right\}\text{of one pound.}$$

So that for 10 s, which is the $\frac{1}{2}$ of a pound, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you ſhall have in your product pounds: if an unite do remain, it ſhall be worth ten ſhillings.

Likewiſe for 5 ſhillings you muſt take the $\frac{1}{4}$ of the number which is to be multiplied: and if there do remain any unites, they ſhall be fourth parts of a pound, every unite being in value five ſhillings.

For 4 ſhillings take the $\frac{1}{5}$ of the number which is to be multiplied: and if there do remain any unites, they ſhall be fifth parts of a pound, each unite being in value 4 ſhillings.

For 2 ſhillings you muſt take the $\frac{1}{10}$ of the number to be multiplied. Wherefore to take the $\frac{1}{10}$ of any number, you muſt cut off the laſt figure of the ſame number (which is neareſt your right hand) from all the other figures with a ſmall right-down line or daſh with a Pen, and ſo have you done: for all the other figures which do remain toward your left hand from the ſame figure that you do ſeparate ſhall be pounds, and that figure ſo ſeparated towards your right hand ſhall be ſo many pieces of 2 s; the which figure you muſt double to make thereof the true number of ſhillings, as by the Example ſhall appear.

Finally, for 1 ſhilling needeth ſmall work, for it is ſo many ſhillings as be propoſed in the ſum, which to bring into pounds, hath been already taught in the firſt *Rule*.

Examples

At 10 s. the piece, what		6543 pieces?
$\frac{1}{2}$	li.	3271 10 s.
At 5 s. the Elle, what		4373 Ells?
$\frac{1}{4}$	li.	1093 5 s.
At 4 s. the yard, what		7839 yards?
$\frac{1}{5}$	li.	1567 16 s.
At 2 s. the pound-weight, what		752|7 pound?
$\frac{1}{10}$	li.	752 — 14 s.
At 1 s. the piece, what		775|3 pieces?
$\frac{1}{20}$	li.	387 13 s.

N*Ext followeth in order to be understood, that if the number of shillings be not some even or ali*quot *part of 20, you must then convert the same number of shillings into the* aliquot *parts of 20, and thereof make two or three products, as need shall require; which done, adde them together, and bring them into Pounds. And here, for thy furtherance, I have set down a note of the order of their parts, as they are to be taken.*

s					s		
3	of	2 & 1			13	of	10. 2 & 1
6		4 & 2		&	14		10 & 4
7		5 & 2			15		10 & 5
8		4 & 4	or	5. 2. 1	16		10. 5. 1
9		5 & 4		4. 4. 1	17		10. 5. 2
11		10 & 1			18		10. 4. 4
12		10 & 2			19		10. 5. 4

For 3 s, according to the tenour that you see is expressed in the *Table*, you must first take for 2 shillings the

the $\frac{1}{10}$ of the number that is to be multiplied; then for one shilling you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part: which two summs added together produce the effect desired.

Item, for 6 shillings, according to the note set forth in the *Table*, first for 4 s. I take the $\frac{1}{5}$ of the number that is to be multiplied; then for 2 s. the $\frac{1}{2}$ of the product that came of 4 s: and adde them together.

Or else, as appeareth also in the *Table*, for 5 shillings you may take the $\frac{1}{4}$, and the $\frac{1}{5}$ part of the product that came of 5 shillings, and adde them together.

Item, for 7 s, first for 5 s. take $\frac{1}{4}$ of the product that is to be multiplied; then for 2 s. take the $\frac{1}{10}$ of the number that is to be multiplied: and adde them together, *&c.*

Item, for 8 s, according to reason, and the intent of the *Table*, for the first 4 s. take the $\frac{1}{5}$ of the product, and the same number again for the other 4 s; and adde them together.

Item, for 9 shillings, first for 5 shillings take the $\frac{1}{4}$, then for 4 shillings take the $\frac{1}{5}$; and adde them together.

Otherwise, as you see by the intent of the *Table*, work twice for 9 shillings as was taught even now for 8; and then take the $\frac{1}{4}$ of the last product for the 1 shilling: but 5 and 4 is the shorter.

Item, for 11 s, first dispatch 10 s, for which you must take the $\frac{1}{2}$ of the product; then, lastly, for one shilling take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product; and adde them together.

Item, for 12 shillings, where I will end with the first part of my *Table*, first take the $\frac{1}{2}$ for 10 shillings, and

and then for 2 shillings take the $\frac{1}{5}$ of the summe that came of 10 shillings; take and adde them together; or else, if you please, for 2 shillings you may take the $\frac{1}{10}$ of the whole given number.

To write more of the manner of taking the true parts I omit: The desirous practitioners will (no doubt) conceive it. Also the *Table* is some aid to help the unperfect; whereupon by and by I will set down three or four of these notes in *Examples*, and the rest I will leave to thine own industry and practice, to labour upon.

This is the order most commonly used in practice, when the number of shillings is not an *aliquot* part of a pound. But (loving Reader) after I have touched the even or *aliquot* parts of a pound that falleth out in pence and shillings, I will deliver two new Rules that shall drown this common order quite and clean; wherein shall be comprehended in one line or working both the even and odde parts of shillings under 20, without regard whether it be an *aliquot* or not an *aliquot* part. Which two Rules (when they come in place) I commit to thy friendly judgement in working.

Now follow the Examples upon the notes aforesaid.

At 6 shillings the yard, what	3215 yards?
4 shillings	643
2 shillings	321——10
li.	964--10 s.

Other-

Otherwise. by Multiplication of 6.

3215

6 ſhillings 1929 | 0

li——964——10 ſhillings.

At 7 ſhillings the Ell, what 4563 Ells?

5 ſhillings 1140——15

2 ſhillings 456——6

li. 1597——1 ſhilling.

Otherwiſe by Multiplication of 7.

4563

7 s 3194 | 1

1597——1

At 8 s. the piece, what 7563 pieces?

4 s 1512——12

4 s 1512——12

pounds 3025——4 s.

Otherwiſe by Multiplication of 8.

7563

8 s 6050 | 4

pounds 3025——4 ſhillings.

At 13 s. the piece, what 401 pieces?

10 s 200——10

2 s 40——2

1 s 20——1

pounds 260——13

Other-

Otherwise by Multiplication.

	401
13 s.	1203
	401
	5213
pounds	260——13 s.

These and such like questions of compound numbers, which I have here in this fourth *Rule* for orders sake set down, for that it hath been heretofore a common course of work, I account but superfluous. For in the eighth and ninth *Rules* of this my simple Addition shall appear, that the given price of any even or odde number of shillings, either under or above 20, shall be wrought at one or two workings at the most, how difficult soever the question be.

3 Rule. To reduce pence into pounds at one operation.

Item, *there resteth yet a kinde of practice, how to bring pence into pounds at the first working. Whereupon you must understand that* 240 *pence make one pound, or* 20 s: *In consideration whereof I cut off the last figure or* 0, *and there remaineth but* 24; *of which* 24 8 d. *is the* $\frac{1}{3}$ *part*, 6 d. *is the* $\frac{1}{4}$ *part*, 4 d. *is the* $\frac{1}{6}$ *part, and* 2 *pence is the* $\frac{1}{12}$ *part thereof.*

Whereupon if it were demanded what 1486 yards or pounds of any thing cometh to at 8 pence the yard; in pricking or cutting off the first figure towards your right hand for the 0 that appertaineth to 240, there is remaining of the said summe 148, whereout I take the $\frac{1}{3}$ part, and it cometh to 49 li, and there resteth 1; which 1 I put to the 6 that I prick or cut off, and it maketh 16 pieces of 8 pence; which I double to make into groats, & they make 32; whereof

whereof the $\frac{1}{3}$ maketh 10 s, and there remaineth $\frac{2}{3}$ s, which is 8 d. Whereby it followeth, that the 1486 yards at 8 pence the yard maketh 49 li. 10 s. 8. d, as by the example ſhall appear.

Item, for 6 d. take the $\frac{1}{4}$ part of the number from the prickt figure; and if any unites remain, they are ſo many ſix-pences, whereof taking the $\frac{1}{2}$ they are ſhillings; if there doth remain yet one, it is in value ſix-pence.

Item, for 4 d. take the $\frac{1}{2}$ part of the number from the prickt figure: if any unites do remain, they are ſo many groats, which to convert into ſhillings, take the $\frac{1}{3}$ part; and if any yet remain, they are thirds of ſhillings, each one in value being worth 4 pence.

Item, for 3 pence take the $\frac{1}{4}$ part from the prickt figure: if any unites do remain, they are ſo many pieces of 3 pence, whereof in taking the $\frac{1}{4}$ part it maketh ſhillings: if any thing yet remain, they are the fourth parts of ſhillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth alſo by the *Table*, take the $\frac{1}{2}$ part of the number from the prickt figure: if any thing remain, they are ſo many pieces of two pence, which by taking the $\frac{1}{6}$ part you ſhall turn into ſhillings; and if any unites remain, they are ſo many 6 parts of ſhillings, or pieces of two pence, whether you will.

If one coſt 8 pence, what	1486?
maketh pounds	49—10—8 d.
If one coſt 6 pence, what	7865?
maketh pounds	196--12--6 d.

At

At 4 pence the yard, what	8736 yards?
maketh pounds	145 —— 12 -- 0 d.
If one cost 3 pence, what	9874 worth?
maketh pounds	123 —— 8 -- 6 d.
At 2 d. the Ell, what	7894 Ells worth?
maketh pounds	65 —— 15 —— 8 d.

6 Rule. *BUT if your number of pence be not an* aliquot *or even part of* 24, *then must you bring them into the* aliquot *parts of* 24, *and make thereof divers products, which must be added together, as by the question hereafter following shall appear.*

Item, for 5 d, first take for 3 d; then for 2 d, and adde them together, according to the instruction of the second *Rule*: or else first take for 4 d, then 1 d.

Item, for 7 d, first take for 4 d, then for 3 d, and adde them together.

Item, for 9 d, first take for 6 d, then for 3 d, and adde them together.

Item, for 10 d, first take for 6 d, then for 4 d, and adde them together.

Item, for 11 d, first take for 8 d, then for 3 d, and adde them together: as by these Examples.

Examples.

1. If one yard cost 5 d, what	759 | 6?
4 pence	126 —— 12
1	31 —— 13
maketh pounds	158 —— 5

Other-

Otherwise.

1 ——— 5 ——— 759 | 6
3 pence 94 ——— 19
2 pence 63 ——— 6
maketh pounds 158 ——— 5 s.

2. If one cost 7 d, what 98 | 7 worth?
4 pence 16 —— 9
3 pence 12 —— 6 —— 9
maketh pounds 28 —— 15 —— 9 d.

Otherwise.

1 ——— 7 ——— 98 | 7
6 pence 24 —— 13 —— 6
1 peny 4 —— 2 —— 3
maketh pounds 28 —— 15 —— 9 d.

3. If one cost 9 d, what 98 | 7 worth?
6 pence 24 —— 13 —— 6
3 pence 12 —— 6 —— 9
maketh pounds 37 —— 0 —— 3 d.

Otherwise.

1 ——— 9 ——— 98 | 7
6 pence 24 —— 13 —— 6
3 pence 12 —— 6 —— 9
maketh pounds 37 —— 00 —— 3

4. If one cost 10 pence, what 98 | 7?
6 pence 24 —— 13 —— 6
4 pence 16 —— 9 —— 0
maketh pounds 41 —— 2 —— 6 d.

5. If

5. If one cost 11 pence, what 98|7?

8 pence	32——18--0
3 pence	12——-6--9
maketh pounds	45——-6--9

But if you have any shillings and pence to be multiplied together, then are you to take for the shillings according to the instruction of the third Rule; and for the pence according to the first Rule before mentioned; unless you can spie the advantage thereof, and thereby help your self: as appeareth in this second example, where first I work for 6 d, which is to be rebated out of the given number, and I have 719 li. 11 s, my desire.

At 19 s. 9 d. the yard, what 738 yards?

	738		*Otherwise by Rebating.*
10 s	369—0		738
5 s	184—10		
4 s	147——12	6 d	18———9 s
6 d	18——-9	li.—7—-19——-11 s	
pounds	719————11 s.		

The like again is done by Rebating, as by these two examples appeareth.

At 18 s. the Ell, what 418 Ells?

2 s	41——16
pounds	376——4 s

At 16 s. the Ell, what 517 Ells?

4 s	103——8
pounds	413 — 12 s

And

And now I will touch a little the even parts of a pound that fall yout in pence and ſhillings, whereof for thoſe parts you ſhall take ſuch like parts out of the given number that is to be multiplied, as the price of that given number beareth in proportion to a pound, which alſo for their letter aid is here ſet down.

1 s.	8d.	*is the*	$\frac{1}{12}$	*part of a pound.*
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item, firſt for 1 ſhilling 8 pence, take the $\frac{1}{12}$ part of the given number; and if any thing do remain, they are twelve parts of a pound, each one being in value 1 ſhilling 8 pence.

Item, for 2 ſhillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied; and if any thing do remain, they are eight parts of a pound, each one being in value 2 ſhillings ſix pence.

Item, for 3 ſhillings 4 pence, as appeareth by the *Table*, you muſt take the $\frac{1}{6}$ part of the given number; and if any thing do remain, they are 6 parts of a pound, each one being in value 3 ſhillings 4 pence.

Item, for 6 ſhillings 8 pence, take the $\frac{1}{3}$ part of the number that is to be multiplied; and if any unites do remain, they are thirds of a pound, every one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreviation into the proportionate parts of a pound, as 16 ſhillings 8 pence maketh $\frac{5}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats, by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats.

Then set 60, the groats of a pound, under 50, cutting off the two Cyphers, as is here performed.

And then have you brought 16 shillings 8 pence into the known parts of a pound, which maketh

16	8
3	
5|0	
6|0	

But yet, gentle Reader, for thy farther instruction, I have hereunto annexed in a *Table* how pence and shillings bear proportion to a pound, which I commit to thy friendly benevolence; it will be some aid unto the ungrounded Practitioner. But I count him the best workman that can presently reduce his given price into the known and proportionate parts of a pound.

A Table

A Table of the aliquot parts of a pound or 20 shillings.

S.	D.	l.
0	2	1/120
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	3/20
3	4	1/6
3	9	3/16
4	0	1/5
5	0	1/4
6	0	3/10
6	6	13/40
6	8	1/3
7	0	7/20
7	6	3/8
8	0	2/5

S.	D.	l.
8	4	5/12
8	9	7/16
9	0	9/20
10	0	1/2
11	0	11/20
11	3	9/16
12	0	3/5
12	6	5/8
13	0	13/20
13	4	2/3
13	9	11/16
14	0	7/10
15	0	3/4
16	0	4/5
16	8	5/6
17	0	17/20
17	6	7/8
18	0	9/10
18	4	11/12
18	9	15/16
19	0	19/20

Here follow four examples upon the four Notes delivered.

At 1 s. 8 d. the yard, what maketh pounds	3884 yards? 323—13—4 d.
At 2 s. 6 d. the yard, what maketh pounds	4563 yards? 570—7—6 d.
At 6 s. 8 d. the Ell, what maketh pounds	7562 Ells? 2520—13—4 d.

Now by custome you are able to work by all sorts of summs being delivered in shillings and pence, as one shilling one peny, two shillings two pence, three shillings 3 pence, and so of all other. But I wish you to have some consideration of your questions when they are set down, for there are many subtile abbreviations, and great advantages to be gotten, and easily to be perceived.

As of 3 s. ——— 8 d. of 2 s. and 1 s. — 8 d.

Of 4 s. ——— 2 d. of 3 s. ——— 4 d. and 10 d. which 10 d. is $\frac{1}{4}$ of ——— 3 s. ——— 4 d.

Of 5 s. ——— 8 d. of 4 s. 1 s. ——— 4 d.

Of 5 s. 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean, when you have taken one product, you may oftentimes upon the same take another more briefly then upon the summe which is to be multiplied, &c.

8 Rule. NOW *(gentle Reader) that you have seen the virtue of the even or* aliquot *parts of a pound in shillings alone, and also in the* aliquot *parts of shillings and pence; according to my promise, hereafter followeth*

followeth a brief and easier method for any even number of shillings, either under or above 20, *then ever yet hath been published. Notwithstanding Mr.* Humphrey Baker, *whose travel is worthy commendation, and whom for knowledge sake I reverence, hath in some part touched this first part, though not in this method. The work of the Rule is both pleasant, ready, and brief, as by the variety of the examples delivered thereupon shall appear. And first I will set forth a question, thereby the better to express or teach you the order thereof; which is this:*

If one cost 6 s, what	8574 ?	
1 6 s.	8574	
maketh pounds	2572	4 s.

To the understanding of this example, after you have set down your given number in form of the *Rule of three*, with a line drawn under it, you shall presently set a prick under your first figure 4 towards your right hand, drawing from the prick, as heretofore hath been practised, a little short line; thereto set down the shillings anon: which done, multiply the first figure 4 by 6, the value of your price; (which here you see standeth in sight above the line) it maketh 24, which is one pound four shillings. The one pound keep to carry to the next place, and the four shillings set down at the end of the prescribed line towards your right hand. Thus have you done now with 6 above the line, and also with four in the first place, (for the prick under 4 doth signifie that 4 hath done his office.) Then secondarily, for a general Rule, take but the $\frac{1}{2}$ of the given price, which here is the number that shall now continue the rest of the multiplication, and end the work: whereupon I multiply

Mr. *John Mellis* his first Rule.

Note a general rule.

Here follow four examples upon the four Notes delivered.

At 1 s. 8 d. the yard, what	3884 yards?
maketh pounds	323—13—4 d.
At 2 s. 6 d. the yard, what	4563 yards?
maketh pounds	570—7—6 d.
At 6 s. 8 d. the Ell, what	7562 Ells?
maketh pounds	2520—13—4 d.

Now by custome you are able to work by all sorts of summs being delivered in shillings and pence, as one shilling one peny, two shillings two pence, three shillings 3 pence, and so of all other. But I wish you to have some consideration of your questions when they are set down, for there are many subtile abbreviations, and great advantages to be gotten, and easily to be perceived.

As of 3 s. ——— 8 d. of 2 s. and 1 s. — 8 d.

Of 4 s. ——— 2 d. of 3 s. ——— 4 d. and 10 d. which 10 d. is $\frac{1}{4}$ of ——— 3 s. ——— 4 d.

Of 5 s. ——— 8 d. of 4 s. 1 s. ——— 4 d.

Of 5 s. 10 d. which 10 d. is $\frac{1}{6}$ of 5 s.

And by this mean, when you have taken one product, you may oftentimes upon the same take another more briefly then upon the summe which is to be multiplied, &c.

8 Rule. NOW *(gentle Reader) that you have seen the virtue of the even or* aliquot *parts of a pound in shillings alone, and also in the* aliquot *parts of shillings and pence; according to my promise, hereafter followeth*

followeth a brief and easier method for any even number of shillings, either under or above 20, *then ever yet hath been published. Notwithstanding Mr.* Humphrey Baker, *whose travel is worthy commendation, and whom for knowledge sake I reverence; hath in some part touched this first part, though not in this method. The work of the Rule is both pleasant, ready, and brief, as by the variety of the examples delivered thereupon shall appear. And first I will set forth a question, thereby the better to express or teach you the order thereof; which is this:*

If one cost 6 s, what	8574 ?	
1 6 s.	8574	
maketh pounds	2572	4 s.

To the understanding of this example, after you have set down your given number in form of the *Rule of three*, with a line drawn under it, you shall presently set a prick under your first figure 4 towards your right hand, drawing from the prick, as heretofore hath been practised, a little short line; thereto set down the shillings anon: which done, multiply the first figure 4 by 6, the value of your price; (which here you see standeth in sight above the line) it maketh 24, which is one pound four shillings. The one pound keep to carry to the next place, and the four shillings set down at the end of the prescribed line towards your right hand. Thus have you done now with 6 above the line, and also with four in the first place, (for the prick under 4 doth signifie that 4 hath done his office.) Then secondarily, for a general Rule, take but the $\frac{1}{2}$ of the given price, which here is the number that shall now continue the rest of the multiplication, and end the work: whereupon I multiply

Mr. *John Mellis* his first Rule.

Note a general rule.

3 into 7, ſtanding in the ſecond place, it maketh 21, and with the 1 pound I kept 22; ſet down 2, and keep 2 in mind, working according to the Rule of multiplication, delivering the tenths in mind in their due place: which done, the product from the prick to your left hand repreſenteth the pounds, and the other at the end of the line ſhillings, as appeareth by the examples.

If one yard coſt 2 s. what		7536?	
1	2 s.	7536	
maketh pounds		753	12 s.
If one yard coſt 4 s. what		8792?	
1	4 s.	8792	
maketh pounds		1758	8 s.
If one piece coſt 6 s. what		9537?	
1	6 s.	9537	
maketh pounds		2861	2 s.
If one piece coſt 8 s. what		7509?	
1	8 s.	7509	
maketh pounds		3003	12 s.
If one coſt 12 s. what		5794?	
1	12 s.	5794	
maketh pounds		3476	8 s.
If one coſt 14 s. what		3705?	
1	14 s.	3705	
maketh pounds		4593	10 s.
If one coſt 18 s. what		5703?	
1	18 s.	5703	
maketh pounds		5132	14 s.

If

If one cost	22 s. what	953?	
1	22 s.	953	
maketh pounds		1048	6 s.

Let these suffice (gentle Reader) for an entrance into even numbers. And now I will shew the like rule for any odde or uneven part of a pound,

To help you to the understanding of these other questions that hereafter follow; where in my first example the given number is 6487 at 3 s. the yard: I multiply 3 above the line into 7, it maketh 21. The one shilling is set down, and the 1 pound I keep. Now am I to take the ½ of three, which, because it is an odde number, I cannot.

Therefore I shall keep and continue my multiplication by three still, and work by the ½ of the rest of the given figures or number, to wit, 648. And first the ½ of 8, which is 4, multiplied into 3, maketh 12; thereto joyn the 1 li. in mind, it maketh 13; set down 3, keep one. Then again multiply by two, the ½ of four, it maketh six, and with one in mind it maketh 7. Then lastly, take the ½ of six, which is 3, saying, 3 times 3 is 9, which 9 set down. And so is the question answered, as appeareth by the practice and examples following. Mr. John Mellis his second Rule.

At 3 s. the yard, what		6487?	
1	3 s.	6487	
maketh pounds		973	1 s.
If one yard cost	5 s. what	4269?	
1	5 s.	4269	
maketh pounds		1067	5 s.

At 7 s. the Ell, what		6489?
1	7 s.	6489
maketh pounds		2271 ——— 3 s.
If one Ell cost 9 s, what		2807?
1	9 s.	2807
maketh pounds		1263 ——— 3 s.
At 11 s. the Pistolet, what		8263?
1	11 s.	8263
maketh pounds		4544 ——— 13 s.
If one piece cost 13 s, what		4629?
1	13 s.	4629
maketh pounds		3008 ——— 17 s.

But now note, (gentle Reader) when the given price falleth upon an odde number, as 3, 5, 7, 11, 13, &c. then it is to be presupposed that the given summe to be multiplied must be a summe made of even numbers, 2, 4, 6, 8, 10, &c. else cannot the question be wrought at one line or working.

Providing always, that it may bear an odde figure in the first place towards your right hand; as appeareth in these six examples which last were wrought, and such like, &c. which may bear an odde number for the price, and be done at one line or working very well.

But if the given price be an odde number, and the summe to be multiplied odde numbers also, then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree; which notwithstanding to bring to pass, take this for a general Rule. First, work for the even number contained in that question, or given price, according

A general Rule.

ding as you have learned; and then afterwards for the one odde ſhilling take the $\frac{1}{2}$ of the ſumme given to be multiplied, omitting the firſt prickt place, as was taught for the working of one ſhilling in my firſt Rule of Practice; and adde thoſe two together, and you ſhall have your deſire.

Examples.

At 3 s. the yard, what	7539 yards?
2 s.	753 — 18
1 s.	376 — 19
maketh pounds	1130 — 17 s.
At 7 s. the Ell, what	7539?
5 s.	7539
2 s.	2261 — 14
	376 — 19
maketh pounds	6238 — 13
At 13 s. the yard, what	7534?
10 s.	3767 — 0
2 s.	753 — 8
1 s.	376 — 14
maketh pounds	4897 — 2

And thus have I abridged into theſe two Rules how to bring any number of ſhillings, whatſoever they be, into pounds, with a briefer Method then ever yet hath been publiſhed, which I commend unto thy friendly cenſure and judgment in the uſe and practice thereof. Note this well.

If

If one cost 6 s. 5 d. what	1231?
6 s.	369 —— 6
4 d.	20 — 10 — 4
1 d.	5 — 2 — 7
maketh pounds	394 — 18 — 11
At 14 s. 2 d. what	2825?
14 s.	1977 — 10
2 d.	23 — 10 — 10
maketh pounds	2001 — 0 — 10
At 16 s. 4 d. what	2531?
16 s.	2024 — 16
4 d.	24 — 3 — 8 d.
maketh pounds	2066 — 19 — 8
At 3 s. the Pistolet, what	8325?
maketh pounds	1248 —— 15 s.
At 7 s. the Crown, what	6529?
maketh pounds	2285 —— 3 s.
At 9 s. the piece, what	6567?
maketh pounds	2955 —— 3 s.

These three last questions may seem something harder; yet they are easie enough, if you mark them well: If I should explain them, then are they too easie. Therefore I leave them to whet the minds of the desirous.

10 Rule. ITem, *when any one of the summs which is to be multiplied is composed of many Denominations, and the given number but of one figure alone, then shall you multiply all the Denominations of the other summe*

summe by the same one figure, beginning first with that summe which is least in value toward your right hand, and bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.

At 3 l. 7 s. 4 d. a yard, what are 9 worth?
maketh pounds 30——6s.—0d.

B*UT if in any of the summs that are to be multiplied there be a broken number; first work for the whole, according to the instructions that you have learned; and then take such part of the given price as that broken number beareth in proportion to the price, as in the examples following. After you have wrought for* 3 s. *and for* 6 d, *then are you to take the* $\frac{1}{2}$ *of* 3 s. 6 d. *for the* $\frac{1}{2}$ *yard, and adde that to the summe: So adding all the three products together, which make* 43 l. 2 s. 9 d, *the just price of* 245 $\frac{1}{2}$ *Ells. And thus must you doe of all other.* 11 Rule.

At 3 s. 6 d. the Ell, what	245 $\frac{1}{2}$
3 s.	36—18
6 d.	6—3
$\frac{1}{2}$	1—9
maketh	43—9—5 $\frac{3}{4}$
At 16 s. 4 d. the piece, what	14 $\frac{3}{4}$?
16 s.	11—4—0
4 d.	0—4—8
$\frac{3}{4}$	12—3
maketh pounds	12—0—11

If

If one piece cost 4 li. 3 s. 6 ½ d. what 12 pieces?

4 li.	48		
3 s.	1	16	
6 d.		6	6
½			
maketh pounds	50	2	6

The Proof.

If 12 pieces cost 50 li. 2 s. 6 d. what one piece?
maketh pounds 4 — 3 — 6½

14 Rule. ITem, *touching the manner how to understand the order of this question, and others the like, First, seek how many times* 12 *is contained in* 50, *which is* 4 *times, and so resteth* 2 *pound; which* 2 *pound converted into shillings, and joyned with the other* 2 *shillings, maketh* 42 *shillings, wherein is found* 12 *three times, resteth* 6 s; *which turned into pence, putting thereto the* 6 *pence in the first place, it maketh* 78, *wherein* 12 *is found six times, resteth* 6 d, *which containeth* 12 *but* ½ *a time; put that* ½ *to the* 6 d, *and then the solution is* 4 li. 3 s. 6 ½ d, *as appeareth by the Practice thereof.*

15 Rule. Item, *The like is to be done of any thing that is bought or sold after five score to the hundred, or the Quintall. As for example,*

If 100 pound cost 27 li. 13 s. 4 d, what one pound?

27 li.

27 li.——13 s.——4 d.
2

s.5	53	
	12	
1	10	
5	3	
d.6	4	0
	11	
	10	0. or $\frac{2}{5}$

Maketh 5 s. 6 $\frac{2}{5}$ d.

But to work it more neatly, it is by a little understanding ended thus:

27 li——13 s.——4 d.
20

s.5	53
	12
d.6	40
	100

Maketh 5 s. 6 $\frac{2}{5}$ d.

I have wrought this at length for the aid of the yong learner, becauſe he ſhould underſtand how all the Multiplication is ſet down.

Item, *to the underſtanding of this and ſuch like queſtions, the right-down line is all the guide, which is pulled down cloſe by* 20; *as you ſee in the example, where* 27 li. 13 s. *is reduced all into* s, *and maketh* 553 s.

The 5 *towards the left hand, being ſeparated with the hanging or right-down line, is the juſt number of ſhillings that anſwereth the queſtion. Next,* 53 s. *is multiplied by* 12, *to reduce them to pence; putting to the* 4 d, *it yieldeth for the multiplication of the firſt figure two* 110; *the one beyond the line towards the left hand is* 1 *peny towards the reſt of the price. Then* 53 *alſo multiplied by* 1 *yieldeth* 53; *but the* 5 *behind the line towards the left hand is alſo* 5 *pence more towards the price, which* 1 *and* 5 *I adde together under the line, it maketh* 6 d. *So is there found now, as appeareth by the Titles of ſhillings and pence,* 5 s. 6 *pence.*

Finally, I come now on this ſide the line towards the

the right hand, and under 12 I find first 10, and then 3, which added together make 40; under which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreviated cometh to $\frac{2}{5}$. So the just price of one pound, after 5 score to the hundred, maketh 5 s. 6 $\frac{2}{5}$d. One example more, and so I will leave this Rule.

If 100 cost 10 $\frac{3}{4}$ d. what	9874?			
6 d	246	17		
4 d	164	11	4	
$\frac{2}{4}$ d	20	11	5	
$\frac{1}{4}$ d	10	5	8 $\frac{1}{2}$	

	li.	4	42		5	5 $\frac{1}{2}$
			20			
		8	45			
Maketh	s.		12			
			45 $\frac{1}{2}$	91		
	d.	5	100	100	parts of a peny,	

Also the like may be done of the usual weights here in *England*, (which is 112 for every hundred weight) in case you know the *aliquot* parts of a hundred weight, which are these, 56 li, 28 li, 14 li, and 7 li. For 56 li. is the $\frac{1}{2}$ of 112 li, 28 li. is the $\frac{1}{4}$ of 112 li, 14 li. is the $\frac{1}{8}$, and 7 li. is $\frac{1}{16}$ part.

Therefore for 56 li. take the $\frac{1}{2}$ of the summe of the mony that 112 li. weight is worth.

For 28 li. take the $\frac{1}{4}$ of the summe of mony that 112 pound weight is worth.

For 14 li. take the $\frac{1}{8}$ of the sum that 112 li. is worth.

And for 7 li. the $\frac{1}{16}$ of the summe of mony that 112 li. is worth.

As

As for example; at 17 li. 19 s. the hundred pounds weight, that is to say, the 112 li, what shall 3 quarters and seven pound cost?

1 C——— 17 li.——— 19 s.——— 3 q.——— 7 li.

2 quarterns	8	19	6
1 quartern	4	9	9
7 pounds	1	2	$5\frac{1}{4}$
maketh pounds	14	11	$8\frac{1}{4}$

The Second Chapter treateth of the *Reduction* of divers Measures to others value by Rules of Practice.

NOW *will I shew a few examples of practice in reducing of Measures, as* Ells, Yards, Braces, Pawns of *Genes*, &c. *Much more would I have touched, but that I fear the Book will rise to too great a Volume.* 14 Rule.

In 864 Ells of *Antwerp*, how many yards of *London?*

864	864
432	216
216	648

maketh 648 yards of *London.*

ITem, *in these and such like questions of* Flemmish *measure, to be brought into yards* English, *first take the* $\frac{1}{2}$ *of the given number, as appeareth in the first example*

ample towards your left hand: Then take half of the product, or the ¼ *of the given number, and adde the two products together, and they shall be yards* English *as by the example you may perceive.*

The second example toward your right hand is yet briefer then the first, whose work is this: Take the ¼ *of the delivered number, and that product subtract out of the given number, and the rest sheweth your desire. Of these two waies use which you think best.*

The Proof.

In 648 yards *London*,
How many Ells of *Antwerp*?

	648	
	216	
maketh	864	Ells of *Antwerp*.

15 Rule. ITem, *for the understanding of this work, first take the* ⅓ *part of the yards of* London, *which found adde that* ⅓ *part and the yards together, as appeareth by the Practice, and the product sheweth the Ells of* Antwerp.

Item, in 320 yards of *London*,
How many Ells of *Antwerp*?

maketh	426 ⅔ Ells.		
	320 yards,		Proof.
	106 ⅔		426 ⅔ Ells.
	426 ⅔ Ells	⅔	106 ⅔
			320 yards.

Other Reductions.

16 Rule. ITem, *you shall understand, that forasmuch as six braces of* Millane *make five Ells of* Antwerp, *hereupon, according to the* Rules of Practice, *you may reduce the one into the other, by the like reason afore-said,*

said, in taking the ⅙ part, and then subtract the same, to make Ells of Antwerp. *And again by the contrary, taking the ⅕ part, with adding the given number, to turn the Ells to Braces. As for example,*

In 876 Braces, how many Ells of *Antwerp*?

876	The contrary.
146	730 Ells *Flemmish*.
Ells 730 *Antwerp*.	146 Braces.
	876 Braces.

Ells 730 *Antwerp*.
182 $\frac{1}{2}$
Yards 547 $\frac{1}{2}$ *English*.

Thus appeareth that 876 Braces by Practice make 730 Ells *Flemmish*, which Ells *Flemmish* reduce into *English* Yards.

So again upon the same first question of Braces, I would know how many yards *English* they make, after the rate that 100 Braces are worth 62 $\frac{1}{2}$ yards.

876 Braces.

438
109 $\frac{1}{2}$

I answer, 547 $\frac{1}{2}$ yards.

Item, *To the understanding of this work, and such like, first take the ½ of the given Braces, and after take the ¼ of that half, or the ⅛ of the given number, and adde them together; and the Products are also Yards* English.

Item, *three Ells of* Rochel *make* 5 *Ells at* Lisbon. *So likewise* 3 *Ells at* Lions *make* 5 *Ells at* Antwerp.

 To

To work these and such like, double the Ells of Lions *and the Ells of* Rochel, *and from their Product subtract the* $\frac{11}{21}$, *and the rest shall be the Ells of* Antwerp, *or the Ells of* Lisbon.

I. *Example.*

In 63 Ells of *Lions*, how many Ells of *Antwerp?*

	63
	63
	126
$\frac{1}{2}$	21

Answ. 105 Ells *Ant.*

In 100 Ells of *Rochel*, how many Ells of *Lisbon?*

	100
	200
	$\frac{1}{2}$ 23 $\frac{2}{3}$.

Answ. 166 $\frac{2}{3}$ Ells *Lisb.*

Touching the proof or return of these and such like *questions*, for a general Rule, you shall first take the $\frac{1}{5}$ of the given number, and adde that $\frac{1}{5}$ and the given number together, and the $\frac{1}{2}$ of that product shall be your desire.

II. *Example.*

In 105 Ells of *Antwerp*, how many Ells of *Lions?*

	105
$\frac{1}{5}$)	21
$\frac{1}{2}$)	126

Answ. 63 Ells of *Lions.*

In 166 $\frac{2}{3}$ Ells of *Lisbon*, how many Ells of *Rochel?*

	166 $\frac{2}{3}$
$\frac{1}{5}$)	33 $\frac{1}{3}$
$\frac{1}{2}$)	200

Answ. 100 of *Roch.*

Questions

Questions of Factorage and Interest briefly and truly resolved by the Rule of Practice without Time.

1. *Question.*

AT 5 shillings *per Centum*, what comes 8860 li. 15 s. 4 d. unto?

li. 22. 15. 03. 10
20
s. 303
12
d. 06
4
4 23
100 | 50

Answer. Note that 5 s. is the fourth of 20 s. I take the ¼ part of 8860 li. 15 s. 4 d, which makes 2215 li. 3 s. 10 d. Now the Root is 100, which you should divide by; so cutting the two last figures away of the pounds, you have 22 li.; then multiply 15 li. by 23 s, so adde the 3 unto it, you shall have 303 s; cut away the two last figures, there resteth 03 s. Lastly, there remains 3 s, which I multiply by 12 to bring into pence, and so I finde 0 d. and 46/100 remaining, which being abbreviated make 23/50 parts of a peny. So I finde that there is gained 22 li. 3 s. 0 d. 23/50 parts of a peny.

2. *Quest.* At 10 s. *per centum*, what comes 1448 li. 16 s. 8 d. unto?

li. 7. 24. 8. 4
20
s. 488
12
180
88
d. 10 | 6 | 0 | 3
| 10 | 0 | 5

Answ. Note that 10 s. is the ½ of 20 s. I take the ½ of 1448 li. 16 s. 8 d, which makes 724 li. 8 s. 4 d: cut off the two last figures, and there resteth 7 li; then multiply the 24 li. by 20 s, and adde the 8 s, and it maketh 488 s; cut the

the two last figures off, and there resteth 4 s; then multiply 88 s. by 12 d, and take in 4 d, and there resteth 1060 d; cut off the two last figures, and there resteth 10 d, and $\frac{60}{100}$, which is $\frac{3}{5}$ of a peny. So the whole summe is 7 li. 4 s. 10 $\frac{3}{5}$, which is the answer to the question.

3. *Quest.* At 15 shillings *per centum*, what comes 1008 l. 12 s. 0 d. unto?

Answer. Note 15, that is $\frac{1}{2}$ and $\frac{1}{4}$ of 20; take the $\frac{1}{2}$ of 1008 li. 12 s, there resteth 504 li. 6 s; then take the $\frac{1}{4}$: adde them together, the totall will be 756 li. 9 s; cut off the 2 last figures, resteth 7 li; then multiply by 20 s, and take in your 9 s, it maketh 1129 s: cut off the two last figures, there resteth 11 s: then multiply by 12 d, there cometh 348 d: cut off the last two figures, there resteth 3 d. and $\frac{48}{100}$; which being abbreviated maketh $\frac{12}{25}$ parts of a peny. So shall you finde 7 li. 11 s. 3 d. $\frac{12}{25}$, which is the answer to the question

```
1008 l.12 s.0 d.
 504.6 ------- 0
 252.3 ------- 0
---------------
  7|56.9 ----- 0
   |20
 11|29
   |12
    58
    29
---------------
  3|48|24|12
   100|50|25
```

4. *Quest.* At 1 li. *per centum*, what comes 868 li. 13 s. 4 d. unto?

Answer. Cut away the two last figures, and multiply by 20 and 12, and take in your shillings and pence: and you shall finde 8 li. 13 s. 8 pence $\frac{4}{5}$, as doth appear by this work.

```
       |868 li. 13 s. 4 d.
li. 8  |20
s. 13  |73
       |12
       150
        73
---------------
d. 8   |8 |0  |4|
       |10|0  |5|
```

5. *Quest.*

5. *Quest.* At 2li. *per centum*, what comes 5608 li. 6 s. 8d unto?

Answer. Multiply the whole summe by two lib. thus; then cut off the two last figures of your pounds, as you did before, and you shall finde 112 pound; then multiply by 20 and by 12, taking in your shillings and pence, and you shall find 112 li. 3 s. 4 d: which is either for Factor or Broker, &c.

```
5608 li. 6 s. 8d unto?
            2
---------------------
112|16|13.4
   |20
   |------
  3|33
   |12
   -----
    70
    33
---------------------
    400
```

6. *Question.* At 3 pound *per centum*, what comes 800 li. 18s. 2 d. unto?

Answer. Multiply the sum by 3 li, thus; then cut off the two last figures, and you shall finde 24 pound; then multiply by 20 and by 12, taking in your shillings and pence, and you shall finde 0 s. 6 d. $\frac{27}{50}$ parts of a peny, which is something above a half-peny.

```
800 li. 18s. 2 d. unto?
            3
---------------------
 24|02. 14. 6
   |20
 -----
  0|54
   |12
   ----
   114
    54
---------------------
   6|54|27
    100 50
```

7. *Quest.* At four *per centum*, what comes 298 li. 15s. 9d. unto?

Answer. Multiply by 4 li, thus; cut off the two last figures; multiply by 20 and by 12, taking in your shillings and pence, and you shall finde 11 li. 19 s. 0 d. $\frac{9}{25}$ parts of a peny, which is something above a farthing.

```
298 li. 15s. 9d. unto?
            4
---------------------
11|95.3.0
  |20
  -----
19|03
  |12
  -----
   06
  03
  036 18 9
---------------------
  100 50 25
```


8. *Quest.*

8. *Quest*. At 5 li. $\frac{3}{4}$ *per centum*, what comes

3658 l.	16 s.	8 d.
	unto?	5
18294	03	4
1829	08	4
914	14	2
210|38	05	10
|20		
7|65		
|12		
1|30		
6|6		
7|9|0		
10|0		

Answer. Multiply by 5 li. thus; then take the $\frac{1}{2}$ of the whole summe, and place the figures even; then take the $\frac{1}{2}$ of that $\frac{1}{2}$, and adde all three sums together; cut off the two last figures; then multiply by 20 and by 12, taking in your shillings and pence, and you shall finde 210 l. 7 s. 7 d. $\frac{9}{10}$ parts of a peny; which is the answer to the question.

9. *Quest*. At 6 l. $\frac{1}{2}$ *per centum*, what comes

3684 li.	12 s.	6 d.
	unto?	6
34107	15	0
2842	06	3
369|50	01	3
|20		
05		
01		
15|3		
100|20|		

Answer. Multiply by 6 li, and then take $\frac{1}{2}$ of the whole summe; adde them both together; then multiply by 20 and by 12, taking in your odde shillings and pence, and you shall finde 369 li. 10 s. 0 d. $\frac{3}{20}$ parts of a peny; which is the answer to your question.

10. *Quest*. At 7 li. $\frac{1}{2}$ *per centum*, what comes

3868 li.	13 s.	4 d.
	unto?	7
27080.	13.	4
1934.	06.	8
290|150		0
|20		
300		

Answer. Multiply by 7 li, then take the $\frac{1}{2}$; adde the together; cut off the two last figures; then multiply by 20; you shall finde 290 li. 3 s, the answer to the question.

11. *Quest*.

11. *Quest.* At 8 li. *per centum*, what comes 2560 l. 17 s. 9 d. unto?

Answer. Multiply 8 li; cut off the two last figures; multiply by 20 and by 12; and you shall finde 204 li. 17 s. 5 d. $\frac{1}{25}$ parts of a peny.

2560 l. 17 s. 9 d. unto?
8
204 | 87. 02. 0
| 20
17 | 42
| 12
84
42
5 | 04 | 2 | 1
100 | 50 | 25

Questions of Interest with Time, wrought by Practice.

1. *Question.*

AT 6 *per centum*, what comes unto for 1 month 468 li. 16 s. 8 d.?

Answer. Multiply by 6 li, there cometh 2813 li. 00 s. 0 d; then take for 1 moneth the $\frac{1}{12}$ of the Totall, and you shall finde 234 li. 8 s. 4 d. of the two last figures of the li. Multiply by 20 and by 12, taking in your odd money, and you shall finde 2 li. 6 s. 10 d. $\frac{3}{5}$ parts of a peny, which is the answer to the question.

468 li. 16 s. 8 d.?
2813. 00. 0
li. 2 | 34. 08. 4
| 20
s. 6 | 88
| 12
180
88
d. 10 | 6 | 0 | 3
10 | 0 | 5

2. *Quest.*

2. *Quest.* At 7 li. ½ *per centum*, what comes unto for 2 months

38001 l.	12 s.	8 d?
		7
26604.	08	8
1900.	06	4
28504.	15	0
47|50	15	10
|20		
10|15		
|12		
30		
1 6		
1| 9|0		
10|0		

Answer. Multiply by 7 li, then take ½, adde them two together; then for your two months take the ⅙ of the Totall, multiply by 20 and 12, taking in your odde shillings and pence; and you shall finde 47 pounds 10 shillings 1 peny $\frac{9}{10}$ parts of a peny: which is the answer to the question.

3. *Question.* At 8 pound *per centum*, what comes unto for 3 months

9864 li.	16 s.	4 d?
		8
78918.	10	8
197|29.	12	8
|20		
5|92		
|12		
192		
93		
11|12|	6|3	
100|50|25		

Answer. Multiply by 8 pound, then take for your 3 moneths ¼ of the Totall, multiply by 20 and by 12, adding in your odde shillings and pence; and you shall finde 197 pounds 5 shillings 11 pence $\frac{3}{25}$ parts of a peny, your demand.

4. *Quest.*

4. *Quest.* At 6 pound $\frac{1}{2}$ *per centum*, what comes unto for 4 months

6080 li. 13 s. 0 d?
6
36483 | 18 0
3040 | 06 6
39524 04 6
131|74 14 10
|20
14|94
|12
188
94
11|38 |19
|100|50

Answ. Multiply by 6 pound, then take $\frac{1}{2}$; adde both together: then for your four months take $\frac{1}{3}$ part of the whole, cut away your two last figures, multiply by 20 and by 12, adde in your odde shillings and pence; and you shall finde 131 pounds 14 shillings 11 pence $\frac{19}{50}$ parts of a peny, your demand.

5. *Quest.* At 8. *per centum*, what comes unto for 5 months

3020 li. 00 s. 00 d?
8
24160 00 00
6040 00 00
4026 13 04
100|66 13 04
|20
13|33
|12
|70
|33
4|00

Answer. Multiply by 8 li, then for 5 months take $\frac{1}{4}$ and $\frac{1}{6}$ of the Totall; cut off the 2 last figures of your pounds, multiply by 20 and by 12, adde in your odde shillings and pence; and you shall finde 100 pound 13 shillings 4 pence, your demand.

6. *Quest.*

6. *Question*. At 8 *per centum*, what comes unto for 6 months

Answer. Multiply by 8 li, then for your six months take the $\frac{1}{5}$ of the Totall; cut off the two last figures of your pounds, multiply by 12, taking in your odde shillings and pence; and you shall finde 322 li. 8 s. 5 d. $\frac{19}{25}$ parts of a peny, your desire.

8060 li. 12 s. 0 d?
8
64484. 16 s.
322|42. 08 0
|20
8|48
|12
|96
48
$\frac{76}{100}$ | $\frac{38}{50}$ | $\frac{19}{25}$
5

7. *Quest*. At 8 li. *per centum*, what comes unto for 7 months

Answer. Multiply by 8 li, then for your 7 months take $\frac{1}{3}$ and $\frac{1}{4}$ of the Totall; cut off the two last figures of your pounds; then multiply by 20 and 12, taking in your odde money; and you shall finde 275 li. 2 s. 11 d. $\frac{1}{5}$, your desire.

5896. 00. 0 d?
8
47168. 0. 0
15722. 0. 0
11792. 13. 4
275|14. 13. 4
|20
2|93
|12
190
93
11|2 |0 |1
10|0 |5

8. *Question.* At 8 *per centum*, what comes unto for 8 months

Answer. Multiply by 8 li, then for 8 months take $\frac{2}{3}$ of the totall; cut off the two last figures of your pounds, then multiply by 20 and by 12, adde in your odde money; and you shall finde 116 li. 5. shillings 9 pence $\frac{3}{25}$, your desire.

```
3680 li.  08 s.  0 d ?
                 8
29443.    04     0
 9814.    08     0
 9814.    08     0
116|28    16     0
   |20
  5|76
   |12
   152
    76
9| 12 |6| 3
  10|100|25|
```

9. *Quest.* At 8 li. *per centum*, what comes unto for 9 months

Answer. Multiply by 8 pound, then for your nine months take $\frac{1}{2}$ and $\frac{1}{4}$ of the whole summe; cut off the two last figures of the pounds, then multiply by 20 and by 12, taking in your odde shillings and pence; and you shall finde 221 pounds 1 shilling 11 pence $\frac{7}{25}$, which is something above a farthing.

```
3684 li.  19 s.  0 d ?
                 8
29479.    12.    0
14739.    16.    0
 7369.    18.    0
221|09.   14     0
   |20
  1|94
   |12
   188
    94
 11|28 |14|7
  100|50|25
```

10. *Quest.*

10. *Question.* At $6\frac{1}{4}$ *per centum*, what comes unto for 10 months

Answer. Multiply by 6 li, then take the $\frac{1}{2}$ and $\frac{1}{4}$ of 100 pound, adde all three sums together; then for the 10 months take $\frac{1}{2}$ and $\frac{1}{3}$ of the Totall, adde them together, cut off the two last figures of the pounds, multiply by 20 and by 12, adding in your shillings and pence, cutting off the last figures of your shillings and pence; you shall finde 5 pound 12 shillings 6 pence, your desire.

100. li.	0 s.	0 d ?
		6
600.	0	0
50.	0	0
25.	0	0
675.	0	0
337.	10	0
225.	00.	0
5|62.	10.	0
|20		
12|50		
|12		
100		
50		
6|00		

11. *Quest.* At 8 li. *per centum*, what comes unto for 11 months

Answer. Multiply by 8 pound, then for 11 months take $\frac{2}{3}$ and $\frac{1}{4}$ from the Totall, adde all three sums together; cut off the two last figures of your pounds, multiply by 20 and by 12, adding in your shillings and pence, cutting off the two last figures of your shillings and pence; and you shall finde 65 pounds 0 shillings 7 d. $\frac{17}{25}$ parts of a peny, your desire.

886 li.	16 s.	0 d ?
		8
7094	08	0
2364	16	0
2364	16	0
1773	12	0
65|03	04	0
|20		
0|64		
|12		
128		
64		
7|68|34|17		
100|50|25		

12. *Quest.* At 8 li. *per centum*, what comes unto for 12 months

Answer. Multiply by 8 li, cut off the two last figures of the pounds, multiply by 20 and by 12, adding in your shillings and pence, cut off the two last figures of your shillings, and the two last of your pence; and you shall finde 726 l. 8 s. 11 d. $\frac{17}{25}$ parts of a peny, your desire.

9080 li. 12 s. 2 d?

8

726|44 17 4

|20

8|97

|12

11|68 |34|17

100|50|25

The Third Chapter teacheth of the order and work of the *Rule of three* in broken Numbers after the Trade of *Merchants*, digressing something from Master *Record*, which is comprehended in three Rules.

NOW *that I have somewhat intreated of the* Rules of Practice, *I will give a few instructions, after my simple order, for the working of the* Rule of three *in broken Numbers: wherein I shall need to say the less, because I hope the studious Learner, that hath travelled any thing in the Grounds of* Arts, *is not unfurnished of knowledge capable to understand me.*

But before I deliver any instructions for broken Numbers, I will propose a Question which shall be wrought th[illegible] sundry ways, thereby to shew as it were three degrees of Comparison, how farre the *Rule of three* in broken, for more speed of work, differeth from the whole; which

I ra-

I rather set down for a view, that the studious herein may be more desirous to attain broken, leaving any more to discourse in Dialogue form, but onely to give instructions where need is, and in the rest to put forth the questions with their answers.

My first question is this.

If one yard cost 6 s. 8 d, what are 789 worth at that rate?

The first way.

1 ——— 6 s ——— 8 d ——— 789
12 80

80 63. 120 d.

Here the Product of the summe are pence, according to the nature of the middle number.

11
2370 1
63120 (5280 (263
24444 2220
222

I answer ——— 263 li.

The second way.

1 ——— 6 1/3 s ——— 789
——————————— 20

3 20 ———————

15780 s.

Here the Product of the summs are shillings, according to the nature of the middle number.

1
24786 (5260 (263
3333 2220
li.

The third way.

1 ——— 1/3 ——— 789
3 1 1
789

Here

Here the Product is pounds, according to the title of the second number.

$$\begin{array}{l} \cancel{4} \\ \cancel{189}\ (263 \\ \cancel{333} \end{array}$$

I answer, 263 li.

Now that you have seen the three former virtues of the *Rule of three*, whose Products have first brought forth pence, next shillings, and lastly pounds, I will deliver three notes in order following; and with them a dozen questions, that shall shew the work of the *Rule of three* in broken Numbers or Fractions.

1. The first four shall be sundry questions of a Fraction coming in the second place. Note these three well.
2. The second four shall be of two Fractions coming in the second or third place.
3. The third four of Fractions in all three places.

Notes upon the first Rule, for a Fraction coming in the second place.

My first Question is this;

If one yard cost me 3 shillings 4 pence, what are 756 worth at that price? 1 Rule.

In setting down the question to perform the work, I turn four pence into the part of a shilling, which is $\frac{1}{3}$; and then the question standeth thus: The first, variety.

1———————3 $\frac{1}{3}$———————756

To the ready working of this question, and all such other like, my first note is this, which take for a general Rule, That when any one Fraction shall come

A general Rule. come either in the second or third place, that the Denominator of that Fraction or Fractions must alwayes be brought unto the Number or Numerator of the first place; and thereby multiply the one into the other.

Note this. And this benefit is alwaies gotten by virtue of bringing the Denominator of the second Numbers Fractions unto the first place: The Fraction in the middle number is now released, and the Product that cometh of the Multiplication is of the nature and like the denomination of the whole number in the second place, which here are shillings.

Whereupon now, to work the Question, I bring 3, the Denominator of the Fraction in the second place, unto the first number 1, with a line set under thus, $\frac{1}{}$ and the third under it thus, $\frac{1}{3}$, saying, once 3 is 3, my Divisor: that done, reduce 3 $\frac{1}{3}$, saying, 3 times 3 is 9, and the other 1 over 3 make 10, my second number in the *Rule of three*; by which 10 I do multiply my last number 756, as appeareth by the work thereof, and it yieldeth 7560 shillings, my Dividend.

Then dividing 7560 by 3, my Divisor, it yieldeth in quotient 2520 shillings, which maketh 126 pounds; as appeareth here most plainly, both by the Example and the work.

At 3 s. 4 d. the yard, what 756 yards?

1 — $3\frac{1}{3}$ — 756

3 10 10

7560 s.

1 | 1 |
7560 2520 (126
3333 | 2222

I answer, 126 li.

Yet

Yet otherwise upon the same Question, altering the price now into the proportion it beareth to a pound, for the 3 s. 4 d. is $\frac{1}{6}$ part of a pound; which Example first standeth thus as appeareth on the left hand, and is afterwards wrought as appeareth on the right hand.

1 —— $\frac{1}{6}$ —— 756　　1 ———— $\frac{1}{6}$ ———— 756
　　　　　　　　　　　　　6　　　　1　　　　　　1
　　　　　　　　　　　　　　　　　　　　126 pounds.

As soon as I have carried 6, the *Denominator* of my middle number, unto my first place, as before hath been taught, I pull down 1, the *Numerator* of 6, with a line under 6, thus, $\frac{6}{1}$, and that one in custome I pull down in sight, being the figure that I will multiply my third or last number by, according to the tenour of the *Rule of three*. And because one can neither multiply nor yet divide, (though here it is set down in form of multiplication the rather for your understanding) the Product of the multiplication, according to the declaration of this my first Rule or Note, is converted into the Title of my second number, which here are pounds. Now followeth the division performed in my *Divisor* 6, to make an end of that Question,

The second variety.

13
756 (126.　　which maketh 126 li. as before.
666

And thus much for the variety in working that Question.

And now followeth another.

My second Question.

If one yard of Cotton cost 8 ¼ d. *what* 859?

1———8 ¼———859

4   33   33

2577
2577

28347

(7086 (590 li. (29——10——6 ¾ s. d.

This Question was also wrought like the first, and bringeth forth 29 li. 10s. 6 ¾ d. the price of 859 yards.

My third Question.

If seven pounds of any thing cost 3 li. 10 s. *what come* 987 *pounds to?*

7————3 ½ li.————987
2————7   7

14   6909

6909 (493 $\frac{7}{14}$ | ½

I answer, 493 li. —10 s.

Notes

Notes upon my second Rule for two Fractions coming in the second and third place.

My first Question is this.

IF *one Ell cost* 13 s.—4 d. *what half a quarter or* $\frac{1}{8}$ *of an Ell?*

Answer. First bring 13 s.—4 d. into the parts of a pound, which is $\frac{2}{3}$, and then will the Question stand thus,

1———$\frac{2}{3}$ li.———$\frac{1}{8}$

ITem, for the performance of this work, doe as before was taught in the first Rule: First bring 3, the *Denominator* of the second fraction, unto your first number 1, setting a line under it thus, 1, saying, once 3 is 3: that done, bring 8, the *Denominator* of the third Fraction, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24; which 24 is your *Divisor*. Now have you done with the *Denominator* 8; therefore you shall put a line under thus 3, and the like line also under 8, setting or pulling down under them their own *Numerators*, that is, 2 under 3, and also 1 under 8, as appeareth in the Example: which *Numerators*, for a general rule, are evermore to be pulled down of custom in sight, to multiply the one by the other, according to the tenour of the *Rule of three.* Then I multiply the one by the other, saying, once 2 is 2, which signifieth 2 pound, being of the nature and like denomination of the middle number; which 2 pound is to be reduced into shillings, otherwise it cannot be divided by my first number 24.

Then dividing 40 by 24, the quotient bringeth

forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an Ell worth after that rate, Otherwiſe, although 2 pound could not be divided by 24, yet it might have been abbreviated to $\frac{1}{12}$ pound, which is worth 1 s. 8 d, as before.

li.

1	$\frac{2}{3}$	$\frac{1}{2}$	(1
3	2	1	~~2~~(6
8	20		~~40~~(1 $\frac{2}{3}$ s.
24	40		~~24~~

Second Queſtion.

IF *one pound of any weight coſt* 13 *ſhillings* 4 *pence, what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Anſwer. Reduce the 13 ſhillings 4 pence into the parts of a pound, which is $\frac{2}{3}$, and then will the Queſtion ſtand thus:

li.

1 ——— $\frac{2}{3}$ ——— $\frac{7}{8}$

ITem, for the underſtanding of this, if you mark well the laſt Example, this and the reſt lieth open, and needs ſmall inſtruction. For as you did laſt, ſo again bring the *Denominator* of the ſecond and third Fracti-on unto the firſt figure 1, multiplying the one into the other; which maketh alſo 24, your *Diviſor*.

Note. Then make a line under 3 thus, $\underline{3}$, and a line under 8 thus, $\underline{8}$, and pull down their *Numerators* under each figure, that is 2 under 3, and 7 under 8, which, as I ſaid before for a general rule, I pull down of cuſtome in ſight, to be the two Numbers that of duty ought to be multiplied together: which done, I bring 2, being the leſſer figure, under 7, multi-

multiplying them together, it maketh 14, which are of the nature of the middle number, that is to wit, pounds; which 14 cannot aptly be divided among 4, therefore are reduced into ſhillings, as is plainly to be ſeen in the example: then 280 ſhillings parted among 24 yieldeth for his quotient 11 s. 8 d. your deſire, and the juſt price of $\frac{7}{8}$ of an Ell. Otherwiſe 14, though it could not be divided by 24, might by *Mediation* or *Division* in broken Numbers have been divided or abbreviated to $\frac{7}{12}$, which in effect, being reduced to his known parts, maketh 11 s. 8 d, as before. But my good will and meaning is to aid young beginners; therefore have I reduced the 14 pound into ſhillings, which is the eaſier way.

Now followeth the example.

1 ——	2 ——	7	1
3	3	8	~~2~~
8	2	7	~~4~~(6
24		3	~~2~~8~~0~~ (11 $\frac{2}{3}$ s.
		14	~~2~~~~1~~~~4~~
		20	~~2~~
		280 s. I anſwer, 11 $\frac{2}{3}$ s.	

The third Example.

If 1 yard coſt me 2 s.——6 d. *what* 345 $\frac{1}{4}$ *yards?*

Anſwer. Firſt put 6 d. into the parts of a ſhilling, and then the Queſtion ſtandeth thus:

1 ———— 2 $\frac{1}{2}$ ———— 345 $\frac{1}{4}$.

Item, to the ready underſtanding of this, and all

 ſuch

ſuch like, according as before hath been declared, bring the *Denominators* of the ſecond and third Fractions unto the firſt place, multiplying them the one into the other, all which make 8 for the common *Diviſor*. Then next reduce your ſecond number, ſaying, two times 2 is 4, and 1 is 5, as was taught in the Example aforeſaid. Laſtly, reduce your third number $345\frac{1}{4}$ all into fourths, and they make 1381, which 1381 is to be multiplied by 5, according to the tenour of the *Rule of three*: which done maketh 6905 ſhillings, and divided by 8, your *Diviſor*, yieldeth in Quotient $863\frac{1}{8}$ ſhillings, which maketh in pounds 43 pounds, 3 ſhillings, $1\frac{1}{2}$ d. And ſo much are the $345\frac{1}{4}$ yards worth at that price.

The ſame Queſtion wrought again by two ſhillings 6 pence is now converted into the parts of a pound, and ſtandeth thus:

1 —— $\frac{1}{8}$ —— $345\frac{1}{4}$.

Item, after I have brought here my ſecond and third *Denominator* unto my firſt place, and found 32 to be my *Diviſor*, having thus finiſhed my firſt place with all things unto him belonging, (which is meant of bringing and multiplying the *Denominators* of the ſecond and third Fractions into him) I then goe in hand to ſee what is to doe in my ſecond place, where preſently of cuſtome I pull down my *Numerator* 1 under 8, being the figure in ſight that ſhall multiply my third number.

Then, laſtly, I reduce $345\frac{1}{4}$ all into fourths, as afore was practiſed, which maketh 1381, the which 1381 I am to multiply by 1, my ſecond number: they are nothing increaſed, but by the *Metamorphoſis* of

of my work they are now 1381 pound, being of the nature of the middle number, as I have often shewed you; which divided by 32, my *Divisor*, yieldeth 43 pound and $\frac{5}{32}$, which $\frac{5}{32}$ of a pound reduced into known numbers make 3 shillings $1\frac{1}{2}$ pence, as before.

Example.

```
                                1
1 ——— 1/8 ——— 345 1/4         105                 ______
32     1                      1381    (43            5
                 1381         522                   32
                                3
```

NOW follow four other Questions, which are in all three places broken numbers, or whole and broken together.

Item, First, for the finding out of your *Divisor*, you shall take this for a most certain and general Rule, That you must multiply the *Numerator* of the first number in the Question by the *Denominator* of the second, and that *Product* again by the *Denominator* of the third: and the total thereof shall be your *Divisor*.

Secondly, for a general Rule to find out your *Dividend*, multiply the *Denominator* of the first number by the *Numerator* of the second, and the whole thereof by the *Numerator* of the third: and the total thereof shall evermore be your *Dividend*.

Now for an Example I propose this Question, thereby to make my meaning more plain, and to shew you, as I have done in the rest, the manner and order of the work.

If $\frac{2}{3}$ *of any weight or measure cost* $\frac{5}{6}$ *of a pound or*

20 s, *what are* $\frac{3}{8}$ *of the like weight or measure worth after that rate?*

Example.

$\frac{2}{3}$ —— $\frac{5}{6}$ —— $\frac{3}{8}$

ITem, for the more plain understanding hereof, and all other the like in broken Numbers, first, you shall pull down 2, the *Numerator* of the first Number or Fraction, with a line under, thus, 3 $\frac{2}{3}$; that done, according as you have learned before, bring 6, the *Denominator* of the second Fraction, and set it under two, multiplying the one into the other, which maketh 12: then lastly, bring 8, the *Denominator* of the third Fraction, and set it under 12, multiplying that 12 by 8, which amounteth to 96; or else, for more brief, multiply 6 by 8, saying, six times 8 makes 48, which 48 set under 2, and multiply the one into the other, it maketh 96, as before. And this 96 is the first number in the *Rule of three*: That shall always, for a most general Rule, be your *Divisor*.

Secondly, to work for your *Dividend*, you shall (as it hath been sufficiently declared before) pull down 5, the *Numerator* of your second Fraction, and set it under 6, with a line under, thus, $\underline{6}$.

That done, (as you know) you are to pull down 3, the *Numerator* of the third Fraction, and set it under 8, with a line under it, thus, $\underline{8}$, multiplying the one into the other, according to the tenour of the *Rule of three*, which maketh 15. Then, according to my Note, forget not to bring the *Denominator* of the first Fraction, which is 3, under 15, and multiply them together, which maketh 45, which 45 is your *Dividend*, and are of the nature of

denomi-

denomination of the middle number, as I have taught you before, and therefore are 44 li, which aptly cannot be divided by 96: therefore you shall reduce the 45 pound into shillings, as you see performed in the Example, which amounteth to 900 s; which divided by 96, your *Divisor*, it yieldeth 9 s. and $\frac{16}{20}$ of a shilling, which in lesser terms is $\frac{3}{8}$, which $\frac{3}{8}$ in mony maketh 4 $\frac{1}{2}$ d: and so much will the aforesaid $\frac{3}{8}$ cost, as by the work following shall appear.

The Example.

$\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{8}$	
$\frac{2}{6}$	5	$\frac{3}{5}$	3
12		15	9|6
8		3	900 (9 $\frac{16}{20}$|$\frac{3}{8}$ s.
96		45	96
		20	
		900	

Otherwise, though 45 could not be divided by 96, yet by Division in broken numbers it might have been abbreviated to $\frac{15}{32}$ of a pound, which reduced into known parts will make 9 s. 4 $\frac{1}{4}$ d, as before.

Now my second Example shall be the proof of this Question.

If $\frac{3}{8}$ yards cost $\frac{15}{32}$ of a pound or 20 shillings, what shall $\frac{2}{3}$ cost?

Answer. Work as was taught you before, and you shall have your desire.

Here,

$$\frac{3}{8} \qquad \frac{15}{32} \qquad \frac{2}{3}$$

3	15	2
32	2	
96	30	
3	8	
288	240	

Here, as appeareth by the work, the Multiplication being ended, 240 is to be divided by 288, which to some perchance may seem hard, yet notwithstanding is the work good. Therefore abbreviate 240 by 288, as you see here is practised, and the end of your abbreviation shall come to $\frac{5}{6}$, your desire, $\frac{240}{288}$ $\frac{30}{36}$ $\frac{5}{6}$.

Otherwise, $\frac{240}{288} \Big| \frac{120}{144} \Big| \frac{60}{72} \Big| \frac{30}{36} \Big| \frac{5}{6}$

Otherwise, $\frac{240}{288} \Big| \frac{40}{48} \Big| \frac{5}{6}$

The third Question.

If $\frac{3}{4}$ *Ells cost* 13 s.—4 d, *what* 156 $\frac{1}{2}$ *Ells?*

Answer. To work this Question the shortest way, reduce 13 shillings 4 pence into the parts of a pound, which is $\frac{2}{3}$.

Then, as you did before, after you have set down the Question, the *Numerator* of the first Fraction 3 is pulled down under 4, and the *Denominators* of the other two Fractions multiplied into him, which maketh 18, your *Divisor*.

Then the *Numerator* of the second Fraction is pulled down under 3 in custome now in sight, ready to multiply my third number, which is performed as soon as the last number 156 $\frac{1}{2}$ is reduced into halfs.

Then

Then laſtly, I multiply that product by 4, the *Denominator* of the Fraction, it yieldeth 2504, which I divide by 18, and my quotient is 139 pound, and $\frac{2}{18}$ or $\frac{1}{9}$ of a pound remaining, which is worth 2s.—2$\frac{2}{3}$d. And ſo much will 156 $\frac{1}{2}$ Ells coſt, as by the work following doth appear.

3	2	156 $\frac{1}{2}$	1	
4	3	313	47	
3	2	2	1762	li.
6		626	2504	139 $\frac{1}{9}$
18		4	1888	
		2504	11	

The fourth Queſtion.

If 2 $\frac{1}{2}$ Ells coſt 1 $\frac{2}{3}$ pounds, what cometh 29 $\frac{1}{4}$ Ells to?

Item, to the workmanſhip of this Queſtion, firſt reduce your ſecond number, in ſaying, three times 1 is 3, and two is 5: then bring the multiplication of the *Denominators* of the ſecond and third Fractions, which maketh 12, and multiply that 12 by 5, your firſt *Numerator*, and it maketh 60, which is your *Diviſor*.

Then the *Reduction* of the ſecond number, which is 5, multiplied by 117, the product of the laſt number's reduction, makes 585, which 585 yet reſteth to be multiplied by 2, the *Denominator* of the Fraction in the firſt place, and yieldeth 1170, which divided by your *Diviſor* 60 yieldeth 19 pound, 10 ſhillings, as appeareth by the work thereof.

Thus having now touched the 12 Queſtions whereof I firſt pretended, which with diligence and oft practice

practice I trust are sufficient to aid the desirous unto the working of any broken numbers. I will now treat of divers necessary Rules incident unto traffick, as hereafter followeth.

The Fourth Chapter teacheth of *Loss* and *Gain* in the Trade of Merchandise.

IF one yord cost 6 s—8 d, *and the same is sold again for* 8s——6 d, *the question is, what is gained in one hundred pounds laying out on such commodities.*

Answer. The *Rule of three direct* applied two manner of waies to doe the same. The one is to say, If 6 $\frac{2}{3}$ give 8 $\frac{1}{2}$, what giveth 100? Multiply and divide, and look what your Quotient bringeth forth above your laying out, it is the neat gains, and solution to your Question. If you follow the work, your solution will bring forth 127 li.——10 s, which is 27 li.——10 s. more then your principal. And so much is gained in the 100 pounds laying out.

Item, to work it the other way, which I take the nearest, seek the difference betwixt the just price and the other price, which is one shilling ten pence, then say by the *Rule of three,*

If 6 $\frac{2}{3}$ s. gain 1 $\frac{5}{6}$ s, what shall 100 pound gain?

Multiply and divide, and you shall find 27 li. 10 s. And so much is gained in 100 li. laying out.

You may use which of these two ways you think good.

The

The Proof.

If a yard of cloth be delivered for 8 s. 6 d, *whereupon was gained after the rate of* 27 li. 10 s. *in* 100 *pounds laying out, the question is, what the yard cost at the first hand.*

Answer. Put your gain 27 li.—— 10 s. to 100 pounds, all maketh 127 li.—— 10 s. Then say, if 127 li. 10 s. give but 100 pounds, what giveth $8\frac{1}{2}$ s? Work, and you shall find 6 s. 8 d, the true solution to your question.

Yet another Example or Proof upon the first Question.

If one yard cost 6 s.—— 8 d, *the question is, at what price the same is to be sold again, for to gain* 27 li. 10 s. *in* 100 *pound laying out.*

Answer. Say by the *Rule of three*, if 100 li. gain 127 li. 10 s. what giveth $6\frac{2}{3}$ s? Multiply and divide, and you shall find 8 s. 6 d, your true solution.

If one Ell cost 7 s. 8 d, *and be sold again for* 8 s. 6 d, *the question is, what is gained in* 20 *pound laying out in such commodities.*

Answer. Seek the difference betwixt the just price and the other price, which is ten pence, and then apply the *Rule of three*, as before is taught, saying, if $7\frac{2}{3}$ s. give $\frac{5}{6}$ shillings, what giveth 20 li? Multiply and divide, and you shall find 2 li. $3\frac{11}{23}$ s. And so much is gained in 20 pound laying out.

The Proof also by an example of Loss.

A *Merchant hath bought* Holland *cloth at* 8 s. 6 d. *the Ell, which proveth not to his expectation, whereupon he is content to lose* 2 li. $3\frac{11}{23}$ *in* 20 *pounds laying*

laying out. The question is, what price ought to be made of the Cloth, abating this loss.

Answer. Do as before in Gains hath been taught, putting 2 li. $3\frac{11}{23}$ s. to your 20 pound, all together maketh 22 li.—$3\frac{11}{23}$. Then say by the *Rule of three*, if 22 li. $3\frac{11}{23}$ s. give but 20 li, what shall come of $8\frac{1}{2}$ s? Work, and you shall find 7 s.—8 d, the just price that the Ell ought to be sold for after the rate of this loss.

Thus it appeareth evidently that in company the *Rule* is appliable as well to Gain as Loss.

If $20\frac{1}{4}$ *yards cost* 36 li. 10 s, *how shall I sell the same to gain* $\frac{2}{3}$ *of the Principal, or to make of* 3, 4, *which is all one?*

Answer. By the *Rule of three*, if 3 do give 4, what will $36\frac{1}{2}$ give? Multiply and divide, and you shall find $48\frac{2}{3}$ pounds. Then say again, if $20\frac{1}{4}$ yards do give $48\frac{2}{3}$ pounds, as well principal as gain, what will one yard be worth at that price? Multiply and divide, and you shall find 2 li. $\frac{98}{243}$.

If one Ell of Cloth cost me 8 s. 8 d, *and afterwards I sell* $10\frac{1}{2}$ *Ells thereof for* 5 li. 13 s. 4 d, *I would know whether I win or lose, and how much, upon the* 100 *pounds of mony.*

Ans. See first at 8 s. 8 d. the Ell what $10\frac{1}{2}$ Ells come to, and you shall find 4 li. 11 s, and I sold the same for 5 li.—13 s.—4 d, so that I did gain upon the $10\frac{1}{2}$ Ells 22 s. 4 d. Then if you would know how much is gained in 100 li, I say by the *Rule of three*, if 4 li.—11 s. did gain 22 s.—4 d, what will 100 li. gain? Multiply and divide, and you shall find 24 li.—10 s.—10 d. $\frac{10}{91}$, and so much is gained in the 100 pound of mony.

If $12\frac{1}{2}$ *yards cost me* 11 *pound five shillings, and I sell*

ſell the yard again for 16 *ſhillings, the queſtion is, whether I do win or loſe, and how much, in or upon the pound of mony?*

Anſwer. Look what the $12\frac{1}{2}$ yards come to at 16 s. the yard, and you ſhall find ten pound. But they coſt 11 pound 5 s: So there is loſt upon the whole 1 pound 5 ſhillings. Then to know how much is loſt in the pound, ſay by the *Rule of Three*, if $11\frac{1}{4}$ pound do loſe $1\frac{1}{4}$ pound, what will 1 pound loſe? Multiply and divide, and you ſhall find 2 s. 2 d. $\frac{2}{3}$; and ſo much is loſt in the pound of mony.

If I ſell the 100 *weight of any commodity for* 4 *pound, whereupon I do loſe after ten pound in the* 100 *pound, I demand how much I ſhall loſe or gain in the* 100 *pound, in caſe I had ſold the ſame for* 4 *pound ten ſhillings.*

Anſwer. Say, if 90 pound yield 100, how much will 4 give? Multiply and divide, and you ſhall find $4\frac{4}{9}$. Then ſay again, if $4\frac{4}{9}$ give me $4\frac{1}{2}$, what will 100 come to? Multiply and divide, and you ſhall find 101 pound $\frac{1}{4}$, which is more then 100 pound by 1 li. 5 s; and ſo much is gained in the 100 pound.

A Merchant hath ſold Currans for the ſumme of 430 *pound, and he hath gained therein after* 10 *pound in the* 100 *pound. The queſtion is, to know how much he gained in all.*

Anſwer. Say by the *Rule of Three*, if 100 pound do gain 10 pound, what will 430 pound gain? Multiply and divide, and you ſhall find 43; and ſo much hath he gained in all.

If one yard be worth $28\frac{1}{2}$ s, *for how much ſhall* 10 *yards be ſold to gain after* 8 *pound* 6 *ſhillings* 8 *pence in the* 100 *pound?*

Anſwer.

Answer. First, adde 8 li.——6 s.——8 d. to 100. Then say, if 100 li. do give 108 $\frac{1}{3}$ s. for principal and gain, what will 28 $\frac{1}{2}$ s. principal yield? Multiply and divide, and you shall find 30 $\frac{7}{8}$ s. Then say again by the *Rule of three*, if 1 yard do give 30 $\frac{7}{8}$ s, (which is as well the principal as the gain) what shall ten yards give? Multiply and divide, and you shall find 15 li. 8 s. 9 d. And for the same price shall the ten yards be sold, for to gain after the rate of 8 li. - 6 s. - 8 d. upon the 100.

A Branch or Proof out of this Question.

A Merchant hath sold Cloths for 15 li.—8 s.—9 d, *and he hath gained in the whole the summe of* 1 li.——3 s.——9 d. *The question is, to know how much he hath gained in the* 100 *pound.*

Answer. To know this, first rebate the gains from the price, and there will remain 14 li. 5 s. 0 d. Then say by the *Rule of three direct*, if 14 li. $\frac{1}{4}$ give me 1 li. 3 $\frac{3}{4}$, what will 100 li. give? Multiply and divide, and you shall find 8 li. 6 s. 8 d, the effect desired. The proof is apparent in the Question before.

Yet another Branch or Proof of the first Question.

If ten yards be delivered for 15 li. 8 s. 9 d, *whereupon was gained after the rate of* 8 li. 6 s, 8 d. *upon the* 100 *pound, the question is, what the yard did cost at the first hand.*

Answer. First say by the *Rule of three*, if ten with principal and gain yield 15 li. 8 s. $\frac{3}{4}$ shillings, what shall 1 yield? Multiply and divide, and you shall find

$30\frac{7}{8}$ s. Then say again by the *Rule of three*, if $108\frac{1}{3}$ principal and gain give but 100, what shall $30\frac{7}{8}$ of principal and gain yield? Work, and you shall finde $28\frac{1}{2}$ s. And so much did the yard cost at the first peny.

If one yard cost 36 s, *how much shall* 12 *yards be sold for, to gain after the rate of* 10 li. *in the* 100?

Answer. First say, if 100 give 110 li. principal and gain, what will 36 s. give? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say again by the *Rule of three*, if one yard of principal and gain yield $39\frac{3}{5}$ s, what shall 12 yards gain? Multiply and divide, and you shall finde 23 li. $15\frac{1}{5}$ s, which $\frac{1}{5}$ s. in known number is $2\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gain after the rate of 10 in the 100.

The Proof.

If 12 *yards be sold for* 23 li. 15 s. $2\frac{2}{5}$ d, *whereupon is gained after* 10 li. *in the* 100, *the question is, what the yard cost at the first peny.*

Answer. First say, if 12 give 23 li. $15\frac{1}{5}$ s, what one yard? Multiply and divide, and you shall finde $39\frac{3}{5}$ s. Then say again by the *Rule of three*, if 110 pounds give but 100, what shall $39\frac{3}{5}$ s. give? Work, and you shall finde 36 s, the just price of the yard at the first hand.

Item, *when one Merchant selleth wares to another, and he giveth to the buier* 1 li. 6 s. 8 d. *upon the score or* 20 li, *the question is, how much shall the buier gain upon the* 100 li. *after that rate?*

Answer. First adde 1 li. 6 s. 8 d. unto 20 li, and they are $21\frac{1}{3}$. Then say, if 20 pound give $21\frac{1}{3}$, what shall

shall 100 give? Multiply and divide, and you shall finde 106 $\frac{2}{3}$. So the buier getteth after the rate of 6 $\frac{2}{3}$ li. upon the 100 li.

Gentle Reader, other necessary Questions appertaining to Loss and Gain you shall have in the Eighth Chapter of this Treatise.

The Fifth Chapter treateth of *Loss* and *Gain upon time*, wrought by the *double Rule of Three*, or by the *Rule composed*; which is contained in four special selected Branches or Questions of divers forms, each one of them springing from the first Question, and each one of them also being a Proof to other, &c.

IF *one yard cost me* 2 s. 8 d. *ready money, and after I sell the same again for* 2 s. 10 d. *to be paid for it at the end of* 3 *months, the question is, what I gain upon the* 100 li. *in* 12 *months*.

Answer. First say, if 2 $\frac{2}{3}$ gain $\frac{1}{3}$, what shall 100 li. gain? Multiply and divide, and you shall finde 6 $\frac{1}{4}$ li. Then say again by the *Rule of three*, if three months gain 6 $\frac{1}{4}$ pound, what shall 12 months gain? Work, and you shall finde 25 li. And so much shall I gain in 12 months after that rate.

Item, you may also work it all at one working by the first part of the *Rule of three* composed, saying, if 2 $\frac{2}{3}$ d. in three months do gain $\frac{1}{6}$ of a shilling, (which is 2 d.) what will 100 li. gain in 12 months? Here for thy farther encouragement, the work of this one Example I have put down, to verifie that I affirm in the first part of this *Ground of Arts*, that this Rule,

Rule, and so all others, more rejoyceth in broken then in whole numbers.

s.	months	s.	li.	mo.
$2\frac{2}{3}$	3	$\frac{1}{8}$	100	12
8		1	20	
3 2			2000	
24 1 2000			3	
6 24444 (500 (25			6300	
144 244 220			12	
2			72000	

Where the Multiplication and the Division being ended maketh 25 li, your desire.

If a yard be delivered for 2 s. 10 d. *to be payed at* 3 *months, whereupon was gained after the rate of* 25 li. *in the* 100 *for* 12 *months, the question is now what the yard cost at the first hand.*

Answer. First say, if 12 months gain 25 li, what shall 3 months gain? Work, and you shall finde $6\frac{1}{4}$ li. Then say again the second time, if $106\frac{1}{4}$ li. give but 100, what shall $2\frac{5}{6}$ s. give? Work, and you shall finde 2 s. 8 d; which is the just price that the yard cost at the first hand.

If one yard of Cloth cost me 2 s. 8 d. *ready money, for what term shall I sell the same again for* 2 s. 10 d, *so that I might gain after the rate of* 25 *pound upon the* 100 *pound in* 12 *months?*

Answer. First say, if $2\frac{2}{3}$ gain $\frac{1}{6}$, what shall 100 li. gain? Multiply and divide, and you shall finde $6\frac{1}{4}$ li. Then say again for the second work, if 25 pound be come of twelve months, what shall come of $6\frac{1}{4}$? Work, and you shall finde 3 months, the just term of time that the Cloth ought to be delivered

at 2 s. 10 d. to gain 25 li. upon the 100 li. in twelve months.

If one yard cost me 2 s. 8 d. *ready money, for what price shall I sell the same again to be paid at the end of* 3 *months, so that I may gain after the rate of* 25 *pound in the* 100 *pound for* 12 *months?*

Answer. First say, if 12 gain 25 li, what shall three months gain? Multiply and divide, and you shall finde $6\frac{1}{4}$ li. Then say for the second work, if 100 li. give $106\frac{1}{4}$, what giveth $2\frac{2}{3}$ s? Work, and you shall finde 2 s. 10 d. And for that price must the yard be sold, to gain after 25 pound in the 100 pound for 12 months.

Many other of these *Questions* I might here have delivered, but for fear the Book would rise to too thick a volume, and so make the price so much the dearer, whereby it might not be so portable to my Country-men as I wish it. But these 4 I have of purpose framed in this order, having relation one to another, assuring you that what Question soever may be proposed within the compass of this Rule, you shall finde by one of these 4 to make a Solution. And moreover divers others are yet to be delivered, where the Creditor giveth divers days of payment, which can never be well wrought, nor yet understood, unless you can first finde by Art the just times that all those payments, how different soever they be, ought to be paid at once. Whereupon first I think good here to give some instructions unto such a *Rule*; for it is the onely aid for the finishing of such Questions as hereafter shall follow.

The

The Sixth Chapter treateth of Rules of *Payment*, which is a right neceſſary Rule, and one of the chiefeſt handmaids that attendeth upon Buying and Selling, *&c.*

Example.

A *Merchant doth owe a ſumme of money, whereof the* $\frac{1}{3}$ *is to be paid at ſix months, and the* $\frac{1}{2}$ *at eight months, and the reſt at a year. If he would pay all at one payment, the queſtion is, what time ought to be given him?*

Anſw. I have omitted the quantity of the ſumme, for you ſhall underſtand the Rule is appliable and yieldeth a true ſolution to what ſumme ſoever ſhall be propoſed: But now for order ſake in teaching, I do imagine the ſumme to be 60 pound, whereupon the manner of this work is to multiply the proportionate part of the money by the time, as in company. Then 20 being the firſt payment, and the $\frac{1}{3}$ of 60, this $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment, maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh two months: next 30, which is the $\frac{1}{2}$ multiplied by his term 8, yields 4 months: then the reſt, which is 10 li, muſt needs be abbreviated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 months produceth $\frac{12}{6}$, and maketh two months. All which added together, as appeareth in the Operati-

$\frac{1}{3}$ by $\frac{6}{1}$	2 Months.
$\frac{6}{3}$	
$\frac{1}{2}$ by $\frac{8}{1}$	4 Months.
$\frac{8}{2}$	
$\frac{1}{6}$ by $\frac{12}{1}$	2 Months.
$\frac{12}{6}$	

on, maketh 8 months, which is the just time that all those payments ought to be paid at once.

A Merchant hath 800 li. *to pay, the* $\frac{1}{6}$ *thereof ready money, the* $\frac{1}{4}$ *at two months, the* $\frac{1}{2}$ *at* 4 *months, and the rest at a year. The question is, if he would pay all at one payment, what time ought to be given him.*

Answer. The ready money is never multiplied: then $\frac{1}{4}$ multiplied by two months, as you did before, maketh $\frac{1}{2}$; then $\frac{1}{2}$ by 4 produceth 2 months, as appeareth here in the operation. But now for the rest of the money, you cannot multiply it untill you have sought what proportion it beareth to 800 pounds. Therefore you must subtract the ready mony, the $\frac{1}{4}$ and $\frac{1}{2}$ out of the principal. The rest will be 66 $\frac{1}{3}$ li, which you must look what part it beareth to the principal, which you shall finde to be $\frac{1}{12}$: the same you must also multiply by his time 12 months, and it yieldeth 1 month. So all make 3 $\frac{1}{2}$ months, as appeareth in the operation.

$\frac{1}{4}$	$\frac{2}{1}$		
	$\frac{1}{4}$		$0\frac{1}{2}$
$\frac{1}{2}$	$\frac{4}{1}$		
	$\frac{4}{2}$		2
$\frac{1}{12}$	$\frac{12}{1}$		
	$\frac{1}{1}$	$\frac{2}{2}$	1

A Merchant is to pay 1200 li. *in three terms, that is to wit,* 400 li. *at two weeks, and* 600 li. *at four months, lastly,* 200 li. *at five months. The question is, in what time they ought to be paid at once.*

Answer. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times, as before, and you shall have $\frac{2}{3}$ weeks, more 8 weeks, and lastly 3 $\frac{1}{3}$ weeks, which together maketh 12 weeks, or 3 months, your desire.

A Merchant is to pay 600 *pound in three terms, whereof* 100 *pound is paid present, more* 300 *pound at twenty days, and the rest at five months, accounting thirty*

thirty days to a month. The question is, what time ought these payments to be paid at once?

Answer. Work, and you shall finde 2 months.

The Seventh Chapter treateth of *Buying* and *Selling* in the Trade of Merchandize, wherein is taken part ready money, and divers days of Payment given for the rest, and what is wone or lost in the 100 pound forbearance for twelve months, more or less, according to the quantity of money, or proportion of time, *&c.*

A *Merchant hath bought Sattins which cost* 8 s. *the yard ready money, and he selleth the same again to another man for* 10 s. *the yard, but he giveth* 2 *days for the payment, that is to say,* 3 *months for the one half, and* 5 *months for the other half. The question is, to know how much the seller doth gain upon the* 100 li. *in* 12 *months after that rate.*

Ans. Seek first, by the Rules of Payment, at what time those two payments ought to be paid at once, and you shall finde 4 months, at which time the second Merchant ought to have paid the whole entire payment: and therefore say by the first part of the *Rule of* 3 composed, if 8 s. in 4 months do gain 2 s, what will 100 li. gain in 12 months?

	s.	m.	s.	li.	m.
	8	4	2	100	12
	4			20	
	32			2000	
	1			2	
	16			4000	
	48000	1 s.		12	
	322	(1800 (75		4800	
	3	220			

 Multiply

Multiply and divide, and you shall finde 75 li, as appeareth in the example. And so much doth the first Merchant gain upon the 100 li. in 12 months.

A Merchant hath sold 50 Cloths at $9\frac{1}{2}$ *the piece, to be paid the* $\frac{1}{2}$ *at four months, the* $\frac{1}{3}$ *at five months, and the* $\frac{1}{6}$ *at 7 months, and the sellers minde is to take no more but after 8 li. in the 100 for 12 months. The question is now, what the first Merchant gaineth in the sale of these Cloths after that rate.*

Answ. First, look what the 50 Cloths come to at that price, and you shall finde 475 li. Then secondly, according to your direction in the Rules of Payment, seek at what time all the payments are to be performed at once, and you shall finde $4\frac{5}{6}$ months. Then thirdly say, by the first part of the *Rule of three composed,* if 100 li. in 12 months gain 8 li, what will 475 li. gain in $4\frac{5}{6}$ months? Work, and you shall finde 15 li. and $\frac{11}{36}$ of a pound; which is the neat gains that the first Merchant hath after the rate aforesaid.

A Merchant hath bought Holland *at* 7 s. 3 d. *the Ell ready money, and he selleth the same again for* 8 s. 4 d. *the Ell, to be paid* $\frac{1}{4}$ *part in ready money, more* $\frac{1}{3}$ *part at 2 months, and the rest at 4 months. The question is now, to know how much the first Merchant doth gain upon the* 100 li. *in* 12 *months after that rate.*

Answer. According to the direction delivered you in the Rules of payment, the ready money is not to be multiplied. Then working for the other two payments, to finde out the true proportion at what time they ought to be paid at once, you shall finde for $\frac{1}{3}$ at two months, $\frac{2}{3}$ of a month; and the rest of the mony, which is $\frac{5}{12}$, multiplied by his term 4 months, yieldeth $1\frac{2}{3}$ months: both which added together make $2\frac{1}{3}$ months, the

the just time that both the payments ought to be performed at once. And therefore say by the first part of the *Rule of three composed*, if 74 $\frac{1}{4}$ in 2 $\frac{1}{3}$ months do gain $\frac{12}{48}$ of a li, what shall 100 li. gain in 12 months after that rate? Work, and you shall finde 78 $\frac{172}{805}$ li. And so much doth he gain upon 100 pounds in 12 months.

A Merchant hath bought 30 Cloths at 6 li. the piece ready money: Afterward he selleth 10 of them for 7 li. the piece for 3 months term, and the other 20 he selleth for 8 li. the piece for 4 months term. The question is now, what he gaineth upon 100 pounds in 12 months.

Ans. First, find the value of the 30 Cloths, which amount to 180 li, 2dly, Seek what the 10 pieces come to at 7 li, and what the 20 pieces come to at 80 li: the one comes to 70, and the other to 60; both which together make 230, which is 50 li. more then they cost. 3ly, As I have taught you in the Rule of Payment, proportionate the first and 2d prices unto the proportion they bear unto 230, the product of their 2 prices, and you shall find $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then 4ly, multiply those parts by their times, and you shall have $\frac{21}{23}$ and $\frac{64}{23}$, both which together make 3 whole months and $\frac{16}{23}$ of a month; which is the just time that both those payments ought to be paid at once. Then say by the first part of the *Rule of 3 composed*, if 180 li. in 3 $\frac{16}{23}$ months do gain 50 li, what shall 100 gain in 12 months? Multiply and divide, & you shall find 90 $\frac{10}{51}$ l: and so much doth he gain upon 100 li. in 12 months.

A Merchant hath bought Cinnamon which cost him 9 s. the li, ready money. The question is now, at what price he ought to sell the 100 weight, to wit, 112 li, to be paid the $\frac{1}{4}$ at 2 months, and the residue at the end of 3 months, so that he may gain after the rate of 10 l. upon 100 li. for 12 months.

Answ.

Ans. Seek first by the Rule of Payment at what term both the payments ought to be paid at once, where the $\frac{1}{4}$ multiplied by his term 2 months maketh $\frac{2}{4}$ months: likewise the next payment, which is $\frac{3}{4}$, multiplied by his term three months, maketh $2\frac{1}{4}$ months: both which added together make $2\frac{3}{4}$ months, which is the time that both the payments ought to be paid at once. Then say by the *Rule of three*, if 12 months do give me ten pounds, what will $2\frac{3}{4}$ months give? Multiply and divide, and you shall finde $2\frac{7}{24}$ pounds. Then say again by the *Rule of three*, if one pound cost me 9 s, what will 112 pounds cost? Multiply and divide, and you shall find 50 li. 8 s. Then say once again, if 100 pound do give $102\frac{7}{24}$, what will $50\frac{2}{5}$ pounds give? Multiply and divide, and you shall finde 51 li. 11 s. $1\frac{1}{5}$ d. And for that price ought I to sell 112 pounds of Cinnamon, to be paid at the two severall payments aforesaid, to gain thereby after the rate of ten pounds upon the hundred pound in twelve months.

Brief Rules for our Hundred weight here at London, *which is after* 112 *pound for the* 160.

ITem, *he that multiplieth the pence that one pound weight is worth by* 7, *and divideth the Product by* 15, *shall finde how many pounds in money* 112 *pound weight is worth.*

And contrariwise, he that multiplieth the pounds that 112 *pounds weight is worth by* 15, *and divideth the Product by* 7, *shall finde how many pence in money the one pound weight is worth.*

Example.

At 10 *pence the pound weight, what is* 112 li. *weight worth?*

Answer.

Answer. Multiply 10 by 7, and thereof cometh 70; the which divide by 15, and you find $4\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li. 13 s. 4 d. after the rate of 10 pence the pound aforesaid.

At 6 pounds the 112 pound weight, what is one pound worth?

Answer. Multiply 6 by 15, and thereof cometh 90; the which divide by 7, and you shall finde $12\frac{6}{7}$ d. So much is one pound worth when the 112 pounds did cost 6 pound.

The Eighth Chapter treateth of *Tares* and *Allowances* of Merchandize sold by weight, and of Losses and Gains therein, *&c.*

At 16 *pound the* 100 *Suttle, what shall* 895 *pound Suttle be worth, in giving* 4 *pound weight upon every* 100 *for Treat?*

Answer. Adde 4 unto 100, and you shall have 104. Then say by the *Rule of three*, if 104 be worth 16 pounds, what are 895 pounds worth? Multiply and divide, and you shall finde 137 li. 13 s. $10\frac{2}{13}$ d: and so much shall the 895 pound weight be worth.

Item, *at* 3 s. 4 d. *the pound weight, what shall* $754\frac{1}{2}$ *pound be worth, in giving* 4 *pound weight upon every hundred for Treat.*

Answer. See first by the *Rule of three* what the 106 pound is worth, saying, if one cost $3\frac{1}{3}$ s, what 100? Multiply and divide, and you shall finde $16\frac{2}{3}$ pounds. Then adde 4 unto one 100, and they are

104.

104. Then ſay again by the *Rule of three*, if 104 be ſold for 16 $\frac{2}{3}$ pounds, for how much ſhall 754 $\frac{1}{2}$ be ſold? Multiply and divide, and you ſhall finde 120 li. 18 s. 3 $\frac{3}{13}$ d. And for ſo much ſhall the 754 $\frac{1}{2}$ pound be ſold at 3 s. 4 d. the pound, in giving 4 upon the 100.

Other neceſſary brief Rules there are for the finding of Treats, or caſting up Cheſts of Sugar, &c. which, for that it is a myſtery, I omit. If any lack inſtruction that way, they ſhall finde me ready to pleaſure them.

Item, *if* 100 *pounds be worth* 36 s. 8 d, *what ſhall* 860 *pounds be worth, in rebating* 4 *pound upon every* 100 *for tare and cloff?*

Anſwer. Multiply 860 by 4, and thereof cometh 3440; the which divide by 100, and you ſhall have 34 $\frac{2}{5}$ pounds: abate 34 $\frac{2}{5}$ from 860, and there will remain 825 $\frac{3}{5}$ pounds. Then ſay by the *Rule of three*, if 100 li. coſt 36 $\frac{2}{3}$ s, what will 825 $\frac{3}{5}$ coſt after that rate? Multiply and divide, and you ſhall finde 15 li. 2 s. 8 $\frac{16}{25}$ d. And ſo much ſhall the 860 coſt, in rebating 4 li. upon every 100 for tare and cloff.

Item, *whether doth he loſe more that giveth* 4 li. *upon the* 100, *or he that rebateth* 4 li. *upon the* 100?

Anſwer. Firſt note, that he that giveth 4 pound on the 100, giveth 104 for 100; and he which rebateth 4 pounds upon the 100, giveth the 100 for 96. Therefore ſay by the *Rule of three*, if 104 be delivered for 100, for how much ſhall the 100 be delivered? Multiply and divide, and you ſhall finde 96 $\frac{2}{13}$. And he which rebateth 4 in the 100, maketh but 96 pounds of 100, ſo that he loſeth 4 pounds in the 100; and the other, which giveth 4 pounds unto the 100, loſeth but 3 $\frac{11}{13}$ pounds upon the 100. Thus you

you may see that he which abateth 4 pounds in the 100 loseth more by $\frac{11}{13}$ pound in the 100 pounds, then the other which gave 4 pounds upon the 100 for tare and closse.

If 100 *pound of any thing cost me* 23 s. 4 d, *the question is, how I shall sell the pound, to gain after the rate of ten pounds upon the* 100 *pound.*

Answer. Say by the *Rule of three*, if 100 pounds give 100 pounds, what shall 23 $\frac{1}{3}$ s. give? Multiply and divide, and you shall finde 1 $\frac{17}{60}$ pounds. Then say again, if 100 pound be worth 1 $\frac{17}{60}$ pounds, what is one pound worth? Multiply and divide, and you shall finde 3 d. $\frac{2}{25}$. And so much is the pound worth in gaining ten pounds upon the 100.

Item, *A Grocer hath bought C. weight of commodity for* 6 li. 10 s. *The question is now, to know how many* li. *thereof he shall sell for* 33s.4d. *to gain* 20 s. *in C. weight.*

Answer. Adde 20 s. unto 6 li. 10 s, and they make 7 li. 10 s. Then say, if 7$\frac{1}{2}$ pound yield me 112 pound, what shall 1 $\frac{2}{3}$ pounds yield? Multiply and divide, and you shall finde 24 $\frac{8}{9}$ li. And so many pound ought he to sell to gain 20 s. in his C. weight.

Item, *If one pound weight cost* 3 s. 4 d, *and I sell the same again for* 4 s, *what is gained in a hundred pound of money laid out in that commodity?*

Answer. You may say, if 3 $\frac{1}{3}$ s. give 4, what will 100 pound gain? But then when you have found, you must subtract 100 pounds out of the Product, the rest is your neat gain. Or else, to produce the neat gain in your work at the first, subtract the just price out of the over-price, as I taught before in the first beginning of Loss and Gain, and your conclusion shall be all one. Multiply and divide by which of

the

the two ways you think good, and you shall find that he gaineth 20 pounds in the 100 pound.

Item, *If the pound weight which cost* 4 s. *be sold again for* 3 s. 4 d, *I demand what is lost in the* 100 *pounds of money.*

Answer. Say, if 4 s. lose $\frac{2}{3}$ s, what shall 100 lose? Multiply and divide, and you shall finde 16 li. 13 s. 4 d. And so much is lost upon the 100 li. of money.

Item, *If C. weight of any commodity cost* 45 li, *and the buier repenting would lose* 5 *pounds in the* 100 *of money, I demand how the pounds may be sold, his loss to be neither more nor less then after the rate aforesaid of five by the hundred.*

Answer. By the *Rule of three*, if 100 lose 5, what shall 45 lose? Work, and you shall finde 2 $\frac{1}{4}$ pound, which rebated from the principal 45, resteth 42 li. 15 s. Lastly, say, if 112 yieldeth but 42 li. 15 s, what one pound? Multiply and divide, and you shall finde 7 s. 7 d. $\frac{17}{28}$. And so much is the pound worth after that loss.

A Grocer hath bought three pieces of Raisins weighing 175 $\frac{1}{2}$ *pounds*, 182 $\frac{1}{4}$ *pounds*, 191 *pounds, Tare for each frail* 2 $\frac{1}{4}$ *pounds*, *at* 25 $\frac{1}{2}$ s. *the C. weight. The question is, what they amount to in mony.*

I answer, 6 li——3 s.——4 $\frac{23}{28}$ d.

A Grocer hath bought three sacks of Almonds weighing 267 $\frac{1}{2}$ *pound, tare two pound*, 257 $\frac{1}{2}$ *pound, tare* 2 $\frac{1}{2}$ *pound*, 252 *pound, tare* 3 *pound, at* 2 s. 10 $\frac{1}{2}$ d. *the pound; what amount they to in money?*

I answer, 110 li.——12 s.——3 $\frac{1}{4}$ d.

The

The Ninth Chapter treateth of *Lengths* and *Breadths* of *Arras* and other Cloths, with other Questions incident unto Length and Breadth.

I*F a piece of* Arras *be* 7 *Ells and* $\frac{3}{4}$ *long, and* 5 *Ells and* $\frac{2}{3}$ *broad, how many Ells square doth the same piece contain?*

Answer. Multiply the length by the breadth, that is to say, $7\frac{3}{4}$ by $5\frac{2}{3}$, and thereof will come $43\frac{11}{12}$ Ells. So many Ells square doth the same piece contain.

Item, *more, a piece of* Arras *doth contain* 22 *Ells square, and if the same were in length* $3\frac{1}{2}$ *Ells, I demand how many Ells in breadth the same piece doth contain.*

Answer. Divide 22 Ells by $3\frac{1}{2}$, and thereof cometh $6\frac{2}{7}$. So many Ells doth the same contain in breadth.

Item, *more, a Merchant hath* $3\frac{1}{4}$ *Ells of* Arras *at* $1\frac{2}{3}$ *Ells broad, which he will change with another man for a piece of* Arras *that is* $\frac{7}{8}$ *Ells square. The question is, how many Ells of that squareness ought the first Merchant to have?*

Answer. Multiply the first Merchant's piece his length by the breadth, and you shall finde it containeth $5\frac{5}{12}$ Ells, which $\frac{5}{12}$ Ells you shall divide by $\frac{7}{8}$, and you shall finde $6\frac{4}{21}$ Ells. And so many Ells of that squareness ought the latter Merchant to give the first.

Item, *A Student hath bought* $3\frac{1}{2}$ *yards of broad Cloth at* 7 *quarters broad, to make a Gown, and should line the same throughout with Lamb, at a foot square each skin. The question is now, how many skins he ought to have.*

Answer.

Answer. Seek first the number of yards square that his Cloth containeth, which to doe, multiply $3\frac{1}{2}$, his length, by $1\frac{3}{4}$, his breadth, and you shall find $6\frac{1}{8}$ yard square: then say by the *Rule of three*, if one yard square give 9 foot, what shall $6\frac{1}{8}$? Work, and you shall finde $55\frac{1}{8}$ skins.

Item, *more, a Lawyer hath a rich piece of Seeling come home, which is* 24 *foot and* 3 *inches long, and* 7 *foot and* $2\frac{1}{2}$ *inches high: the Joyner is to be paid by the yard square: the question is, how many yards this containeth.*

Answer. Multiply his length by his breadth, that is to wit, $24\frac{1}{4}$ foot by $7\frac{5}{24}$ foot, and you shall finde $174\frac{77}{96}$ foot square; which 174 you shall divide by 9, (for so many foot make a yard square) and you shall finde 19 yards 3 foot and $\frac{77}{96}$ of a foot. And so many yards doth this piece hold.

Item, *I bought a piece of* Holland *cloth containing* 36 *Ells* $\frac{1}{3}$ Flemmish. *The question is, how many Ells* English *it makes.*

Answer. You must note, that five Ells *Flemmish* do make but three Ells *English*.

Therefore say by the *Rule of three*, if five Ells *Flemmish* make but three Ells of *English*, how many Ells *English* will $36\frac{1}{3}$ Ells *Flemmish* make? Multiply and divide, and you shall finde $21\frac{4}{5}$. And so many *English* doth $36\frac{1}{3}$ Ells *Flemmish* contain. The like is to be done of others.

Item, *more, I have bought* 342 *Ells* Flemmish *of* Arras *work, at two Ells broad* Flemmish, *and I would line the same with Ell-broad Canvas of* English *measure. The question is, how many Ells* English *will serve my turn.*

Answer.

Answer. Forasmuch as three Ells *English* are worth five Ells *Flemmish*, therefore put three Ells *Flemmish* into his square, in multiplying three by himself, which maketh nine. Likewise multiply the *English* Ell, which is five quarters, every way into himself squarely, and you shall find 25. Then multiply 342, which is the length of the piece, by 2, which is the breadth, and thereof cometh 684. Then say by the *Rule of three*, as before, if 25 Ells square of *Flemmish* measure be worth nine Ells square of *English* measure, what are 684 of *Flemmish* measure? Multiply and divide, and you shall finde 246 $\frac{6}{25}$ Ells *English*.

The same is also wrought by the backer *Rule of three*, in seeking the squares contained in the *Flemmish* Ell of two Ells broad, (which are 18) and also in seeking the squares contained in the *English* Ell, (which are 25 :) Then say by the *Rule of three* backward, if 18 quarters require 342 Ells, what shall 25 quarters give? Multiply and divide by the *Rule of three* reverse, and you shall finde as before 246 $\frac{6}{25}$ Ells *English*.

Item, *more, at three shillings four pence the* Flemmish *Ell, what is the* English *Ell worth after the rate?*

Answer. Say, if three quarters give 3 $\frac{1}{3}$ s, what giveth five quarters? Multiply and divide, and you shall finde 5 s. 6 $\frac{2}{3}$ d.

Item, *more, at* 8 s. 4 $\frac{2}{3}$ d. *the* Flemmish *Ell square, what is the* English *Ell worth after that rate?*

Answer. According to the reason of the last Question, consider that a *Flemmish* Ell square is equall to nine quarters of a Yard *English*, and an *English* Ell square is equall to 25 quarters of a Yard. Therefore

fore say by the *Rule of Three*, if 9 quarters give $1\frac{7}{3}$ s. what 25 quarters? Work, and finde 23 s. $1\frac{7}{9}$ pence. And so is the *English* Ell worth.

Item, *more, at* 6 s. 8 d. *the Ell square, what shall a piece of Cloth cost that is* $7\frac{1}{2}$ *Ells long, and* $3\frac{1}{4}$ *Ells broad?*

Answer. Multiply the breadth by the length, and you shall find $24\frac{3}{8}$ Ells square cost $6\frac{2}{3}$ s; what $24\frac{3}{8}$ Ells ? Multiply and divide, & you shall find 8 pounds, 2 s. 6 pence, and so much the same piece of cloth cost.

Item, *more, a Mercer sold three pieces of Silk, to wit,* $24\frac{1}{4}$, $13\frac{2}{3}$, *and* 25 *yards, at* $9\frac{3}{4}$ s. *the yard, and was glad to receive in part of payment again a cloth containing* $34\frac{1}{3}$ *yards at* $7\frac{2}{3}$ *shillings the yard. The question is now, what the Debtor is in the Creditors debt: Work, and you shall find he oweth the Mercer* 17 li. 8 s. 11 d.

The Tenth Chapter treateth of *reducing of Pawns of* Geans *into* English *yards.*

NOte that 100 *Pawns do make* 26 *yards, whereupon three Pawns* $\frac{11}{13}$ *do make one yard, and one Pawn after the rate and proportion is* $\frac{13}{50}$ *of a yard.*

In 4563 *Pawns of* Geans, *how many yards* English?

Answer. Say by the *Rule of Three*, if a hundred Pawns do make 26 yards, what will 4563 Pawns make? Multiply and divide, and you shall find 1186 yards $\frac{19}{50}$. So many yards do 4563 Pawns make.

Otherwise take some other number at your pleasure,

ſure, as ten pawns, which is the $\frac{1}{10}$ part of 100, then, to find his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{3}{5}$, and then alſo by the *Rule of Three*, if ten pawns give $2\frac{3}{5}$ yards, what will 4563 pawns give? Work, and you ſhall find $1186\frac{39}{50}$ yards, as before.

More, at 2 s. 6 d. *the Pawns of* Geans, *what will the* Engliſh *yard be worth after the rate?*

Anſwer. Say by the *Rule of Three*, if $\frac{13}{50}$ of a yard coſt $2\frac{1}{2}$ s, what one yard? Multiply and divide, and you ſhall find 9 s. $7\frac{5}{13}$ d.

More, if $346\frac{1}{2}$ *Pawns coſt* 30 li. 13 s. 4 d. *ſterling, what is that the* Engliſh *yard after the rate?*

Anſwer. Say by the *Rule of three*, if $346\frac{1}{2}$ Pawns coſt $30\frac{2}{3}$ pounds, what are $3\frac{11}{13}$ pawns worth, (for ſo many pawns make a yard?) Multiply and divide, and you ſhall find $\frac{9200}{27027}$ parts of a pound, which in known numbers is worth 6 s. 9 d. $\frac{6273}{9009}$.

The Eleventh Chapter treateth of *Rules of Loan and Intereſt*, with certain neceſſary queſtions and proofs incident thereunto, &c.

ITem, *I lent my friend* 326 *pounds for* $5\frac{1}{2}$ *months ſimply without any Intereſt, upon condition to have the like courteſie again when I need. But when I came to borrow, he could ſpare me but* 149 li. 8 s. 4 d. *The queſtion is now, how long time I ought to have the uſe thereof, to countervail my friendſhip before-time ſhewed him.*

Anſwer. Say by the backer *Rule of Three*, if 326 pounds give $5\frac{1}{2}$ months, what time will $149\frac{5}{12}$ pounds give?

give? Multiply and divide, and you shall find twelve months; and so long time ought I to use his mony.

The Proof.

Item, *I lent my friend* 149 li. 8 s. 4 d. *for twelve months. The question is now, how much mony he ought to lend me again for* 5 $\frac{1}{2}$ *months to recompense my friendship shewed him.*

Answer. Say by the Backer or Reverse *Rule of three*, If twelve months give 149 $\frac{5}{12}$, what shall 5 $\frac{1}{2}$ months give? Work, and you shall find 326 pounds; and so much ought he to lend me to requite my gentleness or good turn.

Two other branches yet more for proof out of the same Question.

Item, *I lent my friend* 149 li. 8 s. 4 d. *for* 12 *months, to have the like friendship again when I need; and coming to borrow of him, he very courteously took me* 326 *pounds, (for that he could well then spare the same.) The question is now, how long I ought to occupy it, not usurping friendship, but in his due time to restore it again.*

Answer. Say by the *Rule of three* reverse, If 149 $\frac{5}{12}$ pounds give twelve months, what shall 326 pounds give? Multiply and divide, and you shall find that at 5 $\frac{1}{2}$ months term I ought to restore it again.

Proof.

Item, *I lent my friend* 326 *pounds for* 5 $\frac{1}{2}$ *months. The question is now, how many pounds he ought to lend me for* 12 *months, to recompense this pleasure again.*

Answer. Work by the *Rule of three* reverse, as you have

have done before, and you ſhall finde 149 li. ——8 s. ——4 d.

Again, four other ſelected queſtions of *Loan and Intereſt*, all out of one branch, and each one alſo a neceſſary queſtion, and a particular proof to other.

ITem, *I lent my friend* 430 *pounds at Intereſt for three months, to receive after the rate of* 8 *pounds in the* 100 *pounds for* 12 *months. The queſtion is, what the Intereſt cometh to.*

You may, if you pleaſe, work it at two workings by the *Rule of three* direct, in ſaying, If 12 months give 8 pounds, what giveth three months? Multiply and divide, and it giveth 2 pound.

Then for the ſecond work ſay, If a hundred pound yield 2 pounds, what yieldeth 430 li? Multiply and divide, and you ſhall finde 8 li. 12 s; and ſo much comes the loan of 430 li. to for 3 months after the rate of 8 pound in the 100 li. for 12 months.

Otherwiſe wrought thus by the *Rule of three* at twice alſo.

If 100 pounds give 8 pounds, what giveth 430 li? Multiply and divide, and you ſhall finde 34 li. $\frac{2}{5}$. Then again for the ſecond work ſay, If 12 months give 34 pounds $\frac{2}{5}$, what giveth three months? Work, and finde 8 li. 12 s, as before.

Otherwiſe yet at one working by the firſt part of the rule of five numbers forward, in ſaying, If 100 pounds in twelve months gain eight pounds, what ſhall 430 pounds gain in three months? Mul-

tiply the first by the second for your Divisor, and the other three, the one into the other, for the Dividend, and you shall finde 8 pounds 12 shillings, as aforesaid.

Proof.

Item, *A friend of mine received of me 8 pounds 12 shillings for the Interest and use of 430 pounds for three months term. The question is now, what he took in the 100 pound for 12 months after that rate.*

Answer. For most brief, say by the first part or rule of five numbers forward, If 430 pounds in three months did pay 8 pound 12 shillings, what doth 100 pound in 12 months take after the rate? Work, and you shall finde 8 pounds, and so much he took upon the 100 pounds for 12 months.

A third Question and proof also by the Backer Rule of five Numbers.

Item, *I lent my friend 430 pounds, to receive for the Interest thereof after the rate of 8 pounds in the 100 for 12 months. The question is now, how long time my friend ought to give the use thereof, that it may be returned with 8 pounds 12 shillings gain.*

You may work it, if you please, by the *Rule of three direct* at twice, in saying, If 100 pounds yield 8 pounds, what yieldeth 430 pound? Multiply and divide, and finde 34 pound and $\frac{2}{5}$.

Then again for the second work say, If 34 $\frac{2}{5}$ pounds give 12 months, what giveth 8 $\frac{3}{5}$ pounds? Multiply and divide, aud you shall finde 3 months, and so long time ought my friend to use it to return with 8 li. 12 s. gain.

Other-

Otherwiſe at one working by the backer Rule of 5 numbers, in ſaying, If 100 li. in 12 months do gain 8 li, how much time ſhall 430 li. be a gaining of 8 li. 12 s? Multiply the firſt and the ſecond into the laſt for your Dividend, and the third and fourth multiply together for your Diviſor, and then divide, and you ſhall find three months, the juſt time that my friend ought to uſe it to return it with 8 li. 12 s. gain.

A fourth derived Queſtion out of this Branch, which is a proof of this laſt, and alſo of the other two going before.

ITem, *How much money ought a Merchant to deliver after* 8 *li. in the* 100. *for twelve months, that in three months he may gain* 8 *pounds twelve ſhillings?*

Anſwer. You may alſo, if you pleaſe, work it by the *Golden Rule of three* at twice: firſt ſaying, If three months give $8\frac{3}{5}$ pound, what gain 12 months? You ſhall finde $34\frac{2}{5}$. Then ſay again, If 8 pounds be come of 100 pounds, what ſhall come of 34 pounds 8 ſhillings? Work, and you ſhall finde the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliver.

But moſt briefly it is anſwered by the Backer Rule of five numbers, where I argue thus, ſaying, If 100 pounds be 12 months a gaining of 8 pound, then but for three months term onely to take 8 pounds 12 s. muſt needs be a good round ſum. To work it, ſet your numbers thus, 100——12——8——3——$8\frac{3}{5}$, multiplying the firſt into the ſecond, and alſo by 43 the

the product of the fifth, for your Dividend, and the third and fourth together with 5, the Denominator of your fraction, for your Divisor: then divide, and you shall finde as before 430 pounds, the true solution to your question.

The Twelfth Chapter treateth of the *making of Factors*, which is taken in two sorts.

THE first is, when the estimation of the Factor is taken upon the sending of the Merchant; as if the estimation of his person be $\frac{1}{4}$, it is understood that he shall have $\frac{1}{4}$ of the gain, the Merchant the other $\frac{3}{4}$.

The other sort is, when the estimation of his making is out of the sending of the Merchant; as if the order and agreement between them were such, that the Merchant shall put in 800 li, and the Factor for his making shall have $\frac{1}{4}$, neverthelesse he shall have but $\frac{1}{5}$ of the gain or profit, for the $\frac{1}{4}$ of 800 is 200, (for the estimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 li. is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under such condition, that the said Factor shall have the $\frac{1}{4}$, and after certain time they find in profit 124 li. 6 s. 8 d. I demand how much the Merchant shall have hereof, and how much ought the Factor to have.

Answer. *When the estimation of the Factor is out of the sending of the Merchant, it maketh*

li.

li. s. d.
99——9——4 } *for the* { *Merchant,*
24——17——4 } *for the* { *Factor.*

But if that his estimation be at the sending of the Merchant, then it maketh but

li.——s.——d.
93——5——0 } *for the* { *Merchant.*
31——1——3 } *for the* { *Factor.*

For the Merchant is then to have $\frac{3}{4}$, *and the Factor* $\frac{1}{4}$.

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 li. to have the $\frac{1}{5}$ part of the profit. I demand now for how much his person is esteemed, when the same is counted upon the sending of the Merchant.

Answer. *According to the tenour and order before prescribed in the first Rule, that is, if his estimate be* $\frac{1}{4}$, *he shall have the* $\frac{1}{4}$ *of the gain. Therefore say by the* Rule of three direct, *If* $\frac{1}{4}$ *taken put in 400 pound, what is the estimate or putting in of* $\frac{1}{5}$ *taking? Multiply and divide, and you shall finde 320 pounds, and at so much is the person of the Factor estimated.*

Otherwise.

To finde the estimation of the person of the Factor, you shall consider, that seeing it was agreed between them that the Factor should take the $\frac{1}{5}$, then the Merchant shall have the residue, which is $\frac{4}{5}$: wherefore the gain of the Merchant unto that of the Factor is in such proportion as 5 unto 4. Then if you will know the estimation of the person of

of the Factor, say, If 5 give 4, what will 400 give? Multiply and divide, and you shall find 320 pound. And so much is the person of the Factor esteemed to be worth.

Other conditions then these aforesaid may also be between Merchants and Factors, without respect either of sending, or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stock which the Merchant layeth in, as the gain of the said Factor is unto the gain of the Merchant. As thus, If a Merchant do deliver into the hands of his Factor 400 pound, and he to have half the profit, the person of the said Factor shall be esteemed to be worth 400 pound: and if the Factor do take but $\frac{1}{3}$ of the gain, he should have but $\frac{1}{2}$ so much of the gain as the Merchant taketh, which must have $\frac{2}{3}$, wherefore the person of the Factor is esteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to say, two hundred pound.

And if the Factor did take the $\frac{2}{5}$ of the gain, then the Merchant shall take the residue, which are $\frac{3}{5}$; wherefore the gain of the Merchant unto the Factor is then in such proportion as 3 unto 2: whereupon if you will then know the estimation of the person of the Factor, say, If 3 give 2, what shall 400 give? Work, and you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the Factor esteemed to be worth.

And if the Merchant should deliver unto his Factor 400 pound, and the Factor would lay in 80 and his person, to the end he might have the $\frac{1}{2}$ of the gain, I demand, how much shall his person be esteemed?

Answer. *Abate* 80 *from* 400, *and there will remain* 320. *And at so much shall his person be esteemed.*

A Mer-

A Merchant hath delivered unto his Factor 900 pounds to govern in the Trade of Merchandize, upon condition that he shall have the $\frac{1}{3}$ of the gain, if any thing be gained, and also to bear the $\frac{1}{3}$ of the losse, if any thing be lost. Now I demand how much his person was esteemed at.

Answer. *Seeing that the Factor taketh the $\frac{1}{3}$ of the gain, his person ought to be esteemed as much as $\frac{1}{2}$ of the stock which the Merchant layeth in, that is to say, the $\frac{1}{2}$ of 900 pound, which is 450. The reason is, because $\frac{1}{3}$ of the gain that the Factor taketh is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gain that the Merchant taketh, and so the Factor his person is esteemed to be worth 450 pounds.*

A Merchant hath delivered unto his Factor 600 pounds, and the Factor layeth in 250 pounds and his person. Now because he layeth in 250 pounds and his person, it is agreed between them that he shall take the $\frac{2}{5}$ of the gain. I demand for how much his person was esteemed.

Answer. *Forasmuch as the Factor taketh $\frac{2}{5}$ of the gain, he taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laid in. Then subtract 250. which the Factor did lay in, from 400 pound which should have been his whole stock, and there remaineth 150 pound for the estimation of his person.*

More, a Merchant hath delivered unto his Factor 800 li, upon condition that the Factor shall have the gain of 160 li. as though he laid in so much ready mony: I demand what portion of the gain the Factor shall take.

Answer. *See what part the 160 (which the Factor laid in) is of 960, which is the whole stock of their company,*

company, and you shall finde $\frac{1}{6}$; *and such part of the gain shall the Factor take.*

But in case that in making their Covenants it were so agreed betwen them, that the Factor should have the gain of 160 *pound of the whole stock which the Merchant layeth in, that is to say, of the* 800 *pound, then should the Factor take* $\frac{1}{5}$ *of the gains, for* 160 *is* $\frac{1}{5}$ *of* 800 *pound.*

The Thirteenth Chapter treateth of *Rules of Barter and exchanging Merchandize*, which is distinct into seven Rules, with divers other necessary questions incident thereunto.

The first Rule.

TWO Merchants willing to change their Merchandize the one with the other, the one hath 24 broad Cloaths at 10 li. 10 s. the piece, the other hath Mace at 12 shillings the pound. The question is, how many pounds of Mace he ought to give for his Cloth, to save himself harmless, and be no loser.

Answer. *Seek first by the* Rule of three *what the* 24 *Cloaths cost at* 10 *pound* 10 *shillings the piece, and you shall finde* 252 *pound. Then to finde the quantity of Mace, say again by the* Rule of three, *If* 12 *shillings buy one pound, what shall* 252 *pound buy me? Work, and you shall finde* 420 *pound of Mace: and so many pound ought he to give for his Cloaths.*

The Proof.

Two barter. The one hath 420 pounds of Mace at

at 12 s. the pound, to barter or change broad Cloaths at 10 pounds 10 shillings the piece. The question is, how many broad Cloaths he ought to give for all his Mace.

Answer, *First say, If one cost 12 shillings, what 420? You shall find 5040 shillings. Then say again, If 10 $\frac{1}{2}$ pounds give one cloth, what shall 5040 shillings give? Work, and you shall find 24 Cloaths, your desire.*

The second Rule.

Two change Merchandize for Merchandize. The one hath Pepper at two shillings 4 pence the pound, to sell for ready mony, but in barter he will have no less then three shillings the pound; and the other hath Holland at five shillings six pence the Ell ready mony. The question is now, at what price he ought to deliver the Ell in the barter to save himself harmless.

Answer. *Say by the* Rule of three direct, *If 2 $\frac{1}{3}$ ready mony give 3 shillings in barter, what shall 5 $\frac{1}{2}$ give in barter? You shall find 7 $\frac{1}{14}$ shillings, and at that price ought the second Merchant to sell his Holland in barter.*

The Proof.

Two barter. The one hath Holland at 5 s. 6 pence the Ell, to sell for ready mony, and in barter he will have 7 $\frac{1}{14}$ shillings; the other hath Pepper at 2 s. 4 pence the pound, to sell for ready mony. The question is now, how he ought to sell in barter.

Answer. *Say by the* Rule of three direct, *If 5 $\frac{1}{2}$ ready mony give 7 $\frac{1}{14}$ shillings in barter, what ought 2 $\frac{1}{3}$ to take in barter? Multiply and divide, and you shall find 3 shillings, your desire.*

The

The third Rule.

Two barter: The one hath cloth of *Arras* at 30 s. the Ell ready money, but in barter he will have 35 $\frac{1}{2}$ s. and the other hath White-wines, which he delivered in barter for 16 pound the Tun. The question is now, what his Wines cost the Tun in ready mony.

Answer. *Say by the* Rule of Three direct, *If* 35 $\frac{1}{2}$ *shillings in barter give but* 30 *shillings ready mony, what did* 16 *pound in barter cost? Work, and you shall find* 13 *pound* 10 *shillings* $\frac{30}{71}$. *And so much cost his Wines for a Tun ready mony.*

The Proof.

Two barter Merchandize for Merchandize: The one hath White-wines at 13 pounds 10 s. $\frac{30}{71}$ s. the Tun to sell for ready mony, but in barter he delivered it for 16 li; the other, to make his match good and save himself harmless, delivereth *Arras* at 35 $\frac{1}{2}$ s. the Ell. The question is now, what an Ell of his *Arras* cost in ready mony.

Answer. *Say by the* Rule of Three direct, *If* 16 *pounds in barter give but* 13 *pounds* 10 $\frac{30}{71}$ *shillings in ready mony, what shall* 35 $\frac{1}{2}$ *shillings yield in barter? Work, and you shall find* 30 *shillings, your desire.*

The fourth Rule.

Two barter: The one hath Kerseys at 14 pounds the piece ready mony, but in barter he will have 18 pounds, and yet he will have the $\frac{1}{3}$ part of his over-price in ready mony; and the other hath Ginger at eight groats the pound to sell for ready mony. The question is, how he ought to deliver the Ginger by the pound in barter to save himself harmless, and make the barter equall.

Answer.

Answer. *Item*, for the working of this question, and such other like, you must understand, if the party over-selling his wares require to have also some portion in ready mony, as $\frac{1}{2}\frac{2}{3}\frac{3}{4}$, &c. then shall you first rebate the same demanded part, whatsoever it be, from the over-price, and also from the just price. And those two numbers that shall remain after the subtraction is made shall be the two first numbers in the *Rule of Three*. And the just price of the same Merchandize shall be the third number, which by the operation of the *Rule of Three direct* shall yield you a true solution how and at what price you shall over-sell that your merchandize, to save your self harmless, and make the barter equall.

Example.

Take the $\frac{1}{3}$ of eighteen, which is the over-price of his Cloth, which $\frac{1}{3}$ of eighteen is six, which you must subtract from 18, there rest 12. And also abate it from 14, which is the just price of the Cloth, and there remaineth 8, which 8 and 12 are the two first numbers in the *Rule of Three*. Then take eight groats, or $2\frac{2}{3}$ shillings, for the third number. Then say by the *Rule of Three direct*, If eight pounds give 12 pounds, what shall $2\frac{2}{3}$s. give? Multiply and divide, and you shall find 4 shillings. And for so much shall the second Merchant sell his Ginger or his commodity in barter, to balance the same equall.

14	18
6	6
8	12

The Proof.

Two barter: The one hath fine Kerseys at 14 pounds the piece ready mony, but in barter he will have 18 pounds,

pounds, and yet he will have the $\frac{1}{3}$ part of his over-price in ready mony; and the other hath Ginger, which he, having cunning enough to make the barter equall, delivered in barter for 4 shillings the pound. The question is now, what his Ginger cost him in ready mony.

Answer. After you have made the subtraction, abating 6, the $\frac{1}{3}$ part of 18, both from 18 and 14, (as before was taught you) then will there remain 8 and 12 for your two first numbers in the *Rule of three.* Then say, If 12 give 8, what shall come of 4 the over-price of the pound of Ginger? Multiply and divide, and you shall find 2 s. 8 pence, your desire.

Two Merchants barter Merchandize for Merchandize. The one hath Devonshire *whites, at 7 pound 13 shillings 4 pence the piece ready mony, but in barter he doth them away for 8 pound 3 shillings 4 pence, and yet he will have the $\frac{1}{3}$ part of his price in ready mony; and the other hath Cottons at three pounds the piece ready mony. The question is now, at what price he ought to sell or exchange his Cottons in barter to save himself harmless, and make the barter equall.*

li.	s.	d.	li.	s.	d.
7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$5\frac{1}{3}$
4	18	$10\frac{2}{3}$	5	8	$10\frac{2}{3}$

Answer. First seek the $\frac{1}{3}$ part of 8 li. 3 s. 4 d, which is 2 li. 14 s. 5 $\frac{1}{3}$ d, which rebated from 8 li. 3 s. 4 d, there resteth, as appeareth by the Example abovesaid, 5 li. 8 s. 10 $\frac{2}{3}$ d, which is $\frac{2}{3}$ parts of 8 li. 3 s. 4 d; also rebated from 7 li. 13 s. 4 d, there resteth 4 li. 18 s. 10 $\frac{2}{3}$ d, the two first numbers in the *Rule of three*, and the three pounds, which is the neat price

of

of the piece of Cotton, is the third number. Then say by the *Rule of three direct*, as was taught before, if 4 li. 18 s. 10 $\frac{2}{3}$ d. give 5 li. 8 s. 10 $\frac{2}{3}$ d, what shall three pounds give? Multiply and divide, and you shall find three pounds 6 $\frac{6}{89}$ s, the just price that he ought to deliver his Cottons in barter at.

The fifth Rule.

Two Merchants will change Merchandize for Merchandize. The one hath Kerseys at 40 s. the piece, to sell them for ready money; and in barter he will sell them for 56 s. 8 d; and he will gain after ten pound upon the 100 pound; and yet he will have the $\frac{1}{2}$ of his over-price in ready mony. The other hath Flax at 3 d. the pound ready mony. The question is now, how he shall sell the pound of his Flax in barter.

Answer. See first at 10 pound upon the 100 pounds what the 56 $\frac{2}{3}$ s. cometh to, in saying (by the *Rule of three direct*,) if 100 pounds give 110 pounds, what 56 $\frac{2}{3}$ s? Multiply and divide, and you shall find 3 pound 2 shillings 4 pence, of which the $\frac{1}{2}$ that he demandeth in ready mony is 1 pound 11 shillings 2 pence; the same 31 s. 2 d. abated from 40 shillings, and also from 56 s. 8 d, there will remain 8 s. 10 d. and 25 s. 6 d, for the two first numbers in the *Rule of three*, and 3 pence the price of the pound of Flax for the third number. Then multiply and divide, and you shall find 8 $\frac{31}{53}$ d. And for so much shall he sell the pound of Flax in barter.

The sixth Rule.

Two are willing to exchange Merchandize. The one hath *Norwich* Grograms at 25 s. the piece ready

 mony;

mony; and in barter he will have 30 s; and he will have the $\frac{1}{4}$ part of his over-price in ready mony. The other hath *Norwich* Stockins at 40 s. the dozen to sell for ready mony. But inasmuch as the first Merchants Grograms are no better, he would deliver them so to balance the Barter, that he may gain 10 pounds in the 100 pounds. The question is now, how he shall sell his Hose the dozen in barter, according to his request.

Answer. Say, if 100 give 110 li, what shall 40 s. give, which is the just price of the dozen of Stockins? Multiply and divide, and you shall find 44 s. Then take the $\frac{1}{4}$ of the 30 s, which is 7 s. 6 d, and subtract it from 25 s, and also from 30 s, and there will remain 17 s. 6 d. and 22 s. 6 d, for the two first numbers in the *Rule of Three*, and 44 shillings, which is the just price, (with his gain in the dozen of Stockins) for the third number. Then multiply and divide, and you shall find 56 s. 6 $\frac{6}{9}$ d. And for so much he is to sell his dozen of Stockins in barter.

The seventh Rule.

Two Merchants will change their Merchandize one with the other. The one hath 720 Ells of *Cambrick*, at 5 s. the Ell to sell for ready mony; but in barter he requireth 6 s. 8 d: and yet notwithstanding he loseth by it after 10 pounds in the 100 pounds; whereupon he requireth one half of his over-price in ready mony. And the other Merchant, having skill enough to make the Barter equal, delivered *English* Saffrons at 30 s. the pound. The question is now, what his Saffrons cost the pound in ready mony.

Ans. You must first seek what is lost upon the 100 li: which to doe, you may say, (if you please) if 100 pound

pound lose 10, what shall 6 $\frac{2}{3}$ lose? Work, and you shall find $\frac{2}{3}$ s, (or 8 d,) which must be rebated from 6 s. 8 d, so resteth 6 s. still. Or you may say, if 100 pound give me but 90 pounds, what shall 6 s. 8 d. give? Work this way either, and you shall finde also, as before, directly in your Quotient 6 s, your desire. Then are you next to cast up what the 720 Ells of *Cambrick* come to at 6 s. 8 d. the Ell, and you shall find 240 pounds; the $\frac{1}{2}$ whereof the *Cambrick* Merchant will have in ready money, (which is 120 pounds.) Next, you must cast what the *Cambrick* cometh to after his loss in the 100 pounds, which, as you found, is but 6 s. an Ell, and you shall finde 216 pounds. Now must you subtract his ready money (which is 120 pounds in all) out of 240 pounds, and also out of 216 pounds, and there will remain 120 pounds and 96 pounds for your two first numbers in the *Rule of three*, and 30 shillings is the over-price of your Saffron for the third number. Then multiply and divide, and you shall finde 24 shillings. And so much did his Saffron cost in ready money.

Two Merchants barter: the one hath fifty Cloths to put away for ready money at 11 *pounds the Cloth, and in barter putteth them away for* 12 *pounds, taking* Holland *Cloth at* 20 d. *the* Flemmish *Ell, which was worth no more but* 18 d. *The question is now, what* Holland *payeth for the Cloth, and what he winneth or loseth by the bargain.*

Answer. Fifty Cloths at 11 pounds the Cloth come to 550 pounds, and put away at 12 pounds the piece make 600 pound. Then to finde what *Holland* payeth for the Cloth, say by the *Rule of three direct*, if 20 d. buy one Ell, what 600 pounds?

Work, and you shall finde 7200 Ells. Now to finde the estate of his gain or losse, you must seek what his 7200 Ells come to at 18 d. the Ell. Work by the *Rule of proportion direct*, and you shall finde 540 pounds, which is not so much as his Cloths were worth in ready mony by ten pounds. And so much lost the first Merchant by his Exchange.

A *Venetian* hath in *London* 100 pieces of Silk, to put away for ready money at 3 li. the piece; but in barter he delivered them for 4 li. the piece, taking Wools of a Fellmonger at 7 li. 10 s. the C. weight, which was worth no more but 6 li. the C. ready money. The question is now, what Wools payeth for the Silks, and which of them winneth or loseth by the Barter.

Ans. A hundred pieces of Silk at 3 li. is in all 300 li, and at 4 li. is 400 li. Then to finde what Wools pay for the Silk, say by the *Rule of three direct*, if $7\frac{1}{2}$ li. buy me C. weight, what 400? Work, and finde $53\frac{1}{3}$ C. weight of Wool. Now to finde the estate of their gain and losse, cast up his Wools at 6 li. the C, (for so much they were worth ready money) and you shall finde 320 pound, which is 20 pound more then the Silks were to be sold at for ready money: whereby the *Venetian* gained 20 pounds by the Barter.

A Merchant hath $53\frac{1}{2}$ weight of Wool, at 6 pounds the C. to sell for ready money; but in barter he will have 7 pounds 10 s: and another doth barter with him for Silks, which are worth three pounds a piece ready money. The question is now, how he ought to deliver his Silks the piece in barter, and how many payeth for the Wools.

Answer. Say by the *Rule of proportion*, (or by the *Rule of three direct*) if 6 pounds for C. weight ready money

money yield me 7 li. 10 s, what will 3 li. yield, which is the juſt price of a piece of Silk in barter, to make the Truck equal? Work, and find 3 li. 15 s, the price of a piece of Silk in barter. Then ſay, if 3 li. 15 s. require one piece of Silk, how many pieces of Silk are bought with 400 pound, which is the value of $53\frac{1}{3}$ C. weight of wool at 7 li. 10 s. Work by the *Rule of three direct*, and you ſhall find 106 pieces of Silk, and $\frac{2}{3}$ of a piece. And ſo many of Silk pay for the Wool, and neither party hath advantage of other.

Two men will change Merchandize the one with the other. The one of them hath Beer at 6 s. 8 d. the Barrel, to ſell for ready money; but in barter he will ſell the Barrell for 8 s; and yet he will gain moreover after 10 pound upon the 100 pounds. And the other hath white *Spaniſh* Wool at 20 s. the Rove, to ſell for ready money. The queſtion is now, how he ſhall deliver the Rove of Woool in barter to ſave himſelf harmleſs.

Anſwer. Say, if $6\frac{2}{3}$ s, which is the juſt price of the barrell of Beer, be ſold in barter for 8 ſhillings, for how much ſhall 20 ſhillings (which is the juſt price of the Rove of Wool) be ſold in barter? Work by the *Rule of three direct*, and you ſhall finde 24 s. Then becauſe the firſt Merchant will gain after 10 pounds upon the 100 pounds, he maketh his 100 pounds 110 pounds. And therefore ſay by the *Rule of three*, if the ſecond Merchant of 110 pounds do make but 100 pounds, how much ſhall he make of 24 s? Multiply and divide, and you ſhall finde 21 s. 9 d. $\frac{9}{11}$ of a peny. And for ſo much ſhall he ſell the Rove of Wool to be delivered in barter, to the end the firſt Merchant may give 10 in the 100.

Two Merchants will change their Commodities the

one with the other. The one of them hath white Paper at 4 s. the Ream, to sell for ready money; and in barter he will doe it away for 5 s; and yet he will gain moreover after the rate of 10 pounds upon the 100 pounds. And the other hath Mace at 14 s. 6 d. the pound weight, to sell in barter. Now I demand what the pound did cost in ready money.

Answer. Say, if 5 s. (which is the over-price of the Paper in barter) be come of 4 s. the just price, of how much shall come $14\frac{1}{2}$ shillings, which is the surprise of the pound of Mace in barter? Multiply and divide, and you shall finde $11\frac{3}{5}$ s. Then because the first Merchant of Paper will gain after 10 upon the 100, say, if 100 do give 110, what shall $11\frac{3}{5}$ shillings give? Work, and you shall finde 12 s. $9\frac{3}{25}$ d. And so much did the pound of Mace cost in ready money.

The Fourteenth Chapter treateth of *Exchanging of money* from one place to the other.

EXchange is no other thing, then to take or receive Money in one City, to render or pay the value thereof in another City, or else to give Money in one place, and receive the value thereof in another, at term of certain days, months, or Fairs, according to the diversity of the place.

But this practice chiefly consisteth in the knowledge of the Money or Coins in divers places, of which, for thy benefit, (after a few Examples given to the Introduction of this work) I will set down certain notes of the

the diverſity of the common and uſuall Coins in moſt places in Chriſtendome for Traffick.

And firſt I will begin at *Antwerp*, where they uſe to make their accounts by *Deniers de groſſe*, that is to ſay, pence *Flemmiſh*, whereof 12 do make 1 s. *Flemmiſh*, and 20 s. do make one pound *de groſſe*.

Item, A Merchant delivered at *Antwerp* 400 pound *Flemmiſh*, to receive in *London* 20 s. ſterling for every 23 s.——4 d. *Flemmiſh*. The queſtion is now, how much ſterling mony is to be received at *London* for 400 pounds *Flemmiſh*.

Anſwer. Say by the *Rule of three*, if $23\frac{1}{3}$ *Flemmiſh* give 20 s. ſterling, what 400 pounds *Flemmiſh*? Work, and you ſhall find 342 li.———17s.———$1\frac{5}{7}$ pence. And ſo much ſterling ſhall I receive in *London* for the ſaid 400 pounds *Flemmiſh*.

Otherwiſe alſo wrought by Rules of Practice, in taking the $\frac{1}{7}$ of the *Flemmiſh* mony delivered, and abating the ſame from the principal, the reſt is *Engliſh* mony, as before.

400 li.	0 s.	0 d.
57	2	$10\frac{2}{7}$
342	17	$1\frac{5}{7}$ ſterling.

A Merchant at *London* delivered 200 li. ſterling for *Antwerp*, at 23 s.——5 d. *Flemmiſh* the pound ſterling. The queſtion is, how much he muſt receive at *Antwerp*.

Anſwer. Say by the *Rule of three*, if 1 pound ſterling give 23 s. 5 d. *Flemmiſh*, what 200 li. ſterling? Work, and thou ſhalt find 234 li.——3 s.——4 d. So many pounds *Flemmiſh* ſhall he receive at *Antwerp* for the ſaid 200 pounds ſterling.

Otherwise by Practice.

	1——3——5——200	
3 s. 4 d.		33——6——8
1 d.		——16——8
maketh sterling		——234 li.——3 s.——4 d.

In *London* 20 pound sterling is delivered by Exchange for *Antwerp* at 23 s. 9 d. *Flemmish* the pound sterling. The question is, at what rate the *Flemmish* mony ought to be returned to gain 4 pounds upon the 100 pound sterling at *London*.

Answer. First say by the *Rule of three direct*, if 1 pound sterling give 23 $\frac{3}{4}$ *Flemmish*, what 200 pounds sterling? Multiply and divide, and you shall find 237 pounds 10 shillings. The which to return to gain 8 pounds sterling in *London*, say by the *backer Rule*, if 200 pounds sterling require in exchange 23 s. 9 d. *Flemmish*, what the exchange to make 208 li. sterling? Work by the Rule, and find 22 s. 10 $\frac{1}{26}$ d. *Flemmish*, the effect in the question required.

If I take up mony at *Antwerp* after 19 s. 4 d. *Flemmish*, to pay for the same at *London* 20 s. sterling, and when the day of payment is come, I am forced to return the same mony again in *London*, to pay my Bill of Exchange, so that for 20 shillings which I take up here at *London*, I must pay 19 s. 6 d. at *Antwerp*; I demand whether I do win or lose, and how much, in or upon the 100 pounds of mony.

Answer. Say by the *Rule of three*, if 19 $\frac{1}{2}$ give 19 $\frac{1}{3}$, what will 100 pounds give? Multiply and divide, and you shall find 99 li. 2 $\frac{106}{117}$ s; which being abated from 100 pounds, there will remain 17 s. $\frac{11}{117}$. And so much I do lose upon the 100 pounds of mony.

If

If I take up at *London* 20 ſhillings ſterling to pay at *Antwerp* 22 s. 4 d, and when the day of payment is come, my Factor is conſtrained to take up mony again at *Antwerp*, wherewith to pay the aforeſaid ſumme, and there he doth receive 23 s. 4 d. *Flemmiſh*, for the which I muſt pay 20 s. at *London*; the queſtion is now, whether I do win or loſe, and how much, upon the 100 li. of mony after that rate.

Anſwer. Say by the *Rule of proportion*, if $22\frac{1}{3}$ s. give $23\frac{1}{3}$ s, what will 100 pounds give? Multiply and divide, and you ſhall find 104 pounds 9 ſhillings $\frac{37}{67}$; from the which abate 100 pounds, and there will remain 4 pounds 9 ſhillings $\frac{37}{67}$. And ſo much is there gained upon the 100 pounds of mony.

In *Antwerp* is delivered 200 pounds *Flemmiſh* by exchange for *London*, at 20 ſhillings ſterling for every 23 ſhillings 4 pence *Flemmiſh*. The queſtion is, at what rate the ſame is to be returned, to gain 10 pounds upon the 100 pounds *Flemmiſh* in *Antwerp*.

Anſwer. Firſt, ſay by the *Rule of three*, if $23\frac{1}{3}$ *Flemmiſh* give 20 s, what ſhall 200 pounds gain? Work, and you ſhall find 171 pounds 8 s. $6\frac{6}{7}$ d. Then ſay again by the *Rule of three direct*, if 171 pounds 8 s. $6\frac{6}{7}$ d. ſterling give me 210 pounds *Flemmiſh*, what ſhall 20 s. ſterling give? Work, and you ſhall find 24 s. 6 d. *Flemmiſh*. And at the ſame rate ought the ſame to be returned at *Antwerp*, to gain 10 pounds upon the 100 *Flemmiſh*.

A Merchant of *Antwerp* delivereth 234 li. 3 s. 4 d. *Flem.* to receive at *London* 200 li. ſterling. The queſtion is now, how the exchange goeth after this rate.

Anſwer. Say by the *Rule of three direct*, if 200 give 20, what $234\frac{1}{6}$? Multiply and divide, and you

ſhall

ſhall find 23 s.—5 d. And for ſo much goeth the exchange.

Item, the exchange from *London* into *France* is not like as it is to *Flanders*, but it is delivered by the *French* Crown, which is worth 50 Soulx *Turnois* the piece.

Whereupon alſo you muſt note, that in *France* they make their accounts by Franks, Soulx, and Deniers *Turnois*, whereof 12 Deniers make one Soulx *Turnois*, and 20 Soulx make one pound *Turnois*, which they call a Liver or Frank. But the Merchants to make their Accounts do uſe the *French* Crown, which is current among them for 51 Soulx *Turnois*. But by exchange it is otherwiſe, for it is delivered but for 50 Soulx *Turnois* the Crown, or as the taker up of the mony can agree with the deliverer. And note that this △ Character repreſenteth the Crown by exchange, and is ever 50 Soulx *Turnois* or *French* mony.

A Merchant delivereth at *London* 240 pounds ſterling, after five ſhillings ſix pence the Crown, to receive at *Paris* 50 Soulx *Turnois* for every Crown. I demand how much *Turnois* or *French* mony payeth the Bills for the ſaid 240 pounds ſterling.

Anſwer. Say by the *Rule of three*, if $5\frac{1}{2}$ s. ſterling give me 50 s. *Turnois*, what ſhall 240 pounds ſterling give? Reduce the pounds into ſhillings, then multiply and divide, and you ſhall find 2181 Livers, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ *Turnois*. And ſo much pay the Bills at *Paris* for the 240 pounds ſterling.

A Merchant delivereth at *Roan*, or elſewhere in *France*, 1430 pounds or Franks, the which Frank or pound is 20 Soulx, or a pound *Turnois*, to receive in

London

London 6 s. 4 d. ſterling for every △ of 50 Soulx *Turnois*. The queſtion is, how much ſterling money I ought to receive at *London* for my 1430 pound *Turnois*.

Anſwer. Say, if 2 $\frac{1}{2}$ pounds give me 6 $\frac{1}{3}$ s, what will 1430 give me? Work, and you ſhall find 3622 $\frac{2}{3}$ ſhillings ſterling, which maketh 181 li. 2 s. 8 d. And ſo much money is to be received at *London* for the ſaid 1430 Livers *Turnois*, after 6 s. 4 d. for every △ of 50 Soulx.

In *London* is delivered 200 pound ſterling by exchange for *Paris*, at 5 s. 9 d. the △ of 50 Soulx *Turnois*. The queſtion is, at what price the ſaid △ is to be returned, to gain 6 pounds upon the 100 pounds ſterling at *London*.

Anſwer. Firſt, ſay by the *Rule of three direct*, if 5 $\frac{3}{4}$ s. ſterling give 50 Soulx *Turnois*, what ſhall 200 pound ſterling give? Work, and you ſhall finde 1739 Franks or Livers, 2 $\frac{14}{23}$ Soulx. Then the ſame to return, and gain 6 pounds upon the hundred pounds in *London*, ſay by the *Rule of three direct*, if 1739 Franks, 2 $\frac{14}{23}$ Soulx yield 212 pound, what the △ of 50 Soulx? Work, and finde 6 s. 1 $\frac{7}{50}$ d, the effect required in the queſtion.

A Merchant delivered in *London* 160 li. ſterling, to receive in *Biſcay* for every 5 s. 6 d. one Ducat of 374 Marvides. The queſtion is, how many Marvides ought I to receive at *Biſcay*?

Anſwer. Say, if 5 $\frac{1}{2}$ s. ſterling give 374 Marvides, what ſhall 160 pounds ſterling give? Multiply and divide, and you ſhall finde 217600 Marvides. And ſo many I ought to receive at *Biſcay* for my 160 pounds ſterling.

A Mer-

A Merchant delivered in *Baion* 40000 Marvides, to receive in *London* 5 s. 8 d. sterling for every Ducat of 374 Marvides. The question is now, how much sterling money payeth the Bills of Exchange for the said 40000 Marvides.

Answer. Say, if 374 Marvides make one Ducat, what 40000 Marvides? Multiply and divide, and finde $106\frac{178}{187}$.

Then say again, if 1 Ducat give $5\frac{2}{3}$ s, what giveth $106\frac{178}{187}$ Ducats? Work, and finde 30 l. 6 s. $\frac{34}{561}$. Otherwise it is wrought more brief at one working, as in the last question before, in considering that 5 s. 8 d. containeth one Ducat, or 374 Marvides. Therefore say by the *Rule of three*, if 374 Marvides give $5\frac{2}{3}$ s, what 40000 Marvides? Work, and you shall also find in your Quotient 30 l. 6 s. $\frac{34}{561}$ s. And so many pounds sterling is to be received for the 40000 Marvides.

In *London* 200 pounds delivered by exchange for *Vigo*, 374 Marvides the Ducat of 5 s. 10 d. sterling, maketh $256457\frac{1}{7}$ Marvides; the which to return, and gain 10 li. upon the 100 pounds in *London*, say by the *Rule of three direct*, if 220 li. require $256457\frac{1}{7}$ Marvides, what 5 s. 10 d? Work, and finde 340 Marvides, the price of every Ducat in return; which is the effect in the question required.

These may seem sufficient for Instructions.

NOtwithstanding, for thy farther aid and benefit, hereafter follow six speciall and most brief *Rules of Practice*, for *English*, *French* and *Flemmish* money.

How

teacheth

1 How to turn *Flemmish* to *English* sterling.
2 How to turn *English* sterling to *Flemmish*.
3 How to turn *Flemmish* to *French*.
4 How to turn *French* into *Flemmish*.
5 How to turn Sterling into *French*.
6 How to turn *French* into Sterling.

The Fifteenth Chapter treateth of the said six Rules of brevity, and of *Valuation* of *English*, *Flemmish* and *French money*, and how each of them may easily be brought to others value.

How briefly to reduce pounds, shillings and pence Flemmish, *into pounds, shillings and pence* English *sterling*.

It is to be noted, that 7 pounds *Flemmish* make but 6 pound sterling, 7 s. *Flemmish* make 6 s. sterling, and 7 d. *Flemmish* 6 d. sterling; so that 7 yieldeth but 6. Wherein is evident that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into *English* money. Wherefore to know how much 233 li. 13 s. 4 d. *Flemmish* maketh *English*, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction is the summe required. So that 233 li. 13 s. 4 d. *Flemmish* maketh 200 li. 5 s. 8 $\frac{4}{7}$ d. sterling. Rule 1.

Example.

Example.			Another Example.		
li.	s.	d.	li	s.	d.
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{3}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	5	$8\frac{4}{7}$ Ster.	266	11	$5\frac{1}{7}$

Rule 2. *To reduce pounds, shillings and pence sterling, into pounds, shillings and pence* Flemmish.

Note that a pound sterling maketh 1 li. 3 s. 4 d. *Flemmish*, that is, $1\frac{1}{6}$ li, 1 s. sterling maketh $1\frac{1}{6}$ s. *Flemmish*, and 1 d. sterling maketh $1\frac{1}{6}$ d. *Flemmish*. So that there is gained (if it may be so called) $\frac{1}{6}$ of the summe being thus reduced to *Flemmish*; for of $1\frac{1}{6}$ is made $\frac{7}{6}$, which is one whole and $\frac{1}{6}$. Then to know how much 237 li. 7 s. 6 d. sterling maketh *Flemmish*, subtract from your sterling the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, and it maketh 276 li. 18 s. 4 d; which is the summe required.

Example.			Another Example.		
li.	s.	d.	li.	s.	d.
237	7	6 Ster.	337		
$\frac{1}{6}$ 39	11	3	$\frac{1}{6}$ 56	3	4
276	18	9 *Flem.*	393	3	4

Rule 3. *To reduce pounds, shillings and pence* Flemmish, *into pounds, shillings and pence* French.

Ye shall note, that the equality of *Flemmish* and *French* money is this, that is to say, the pound *Flemmish* maketh 7 pound $\frac{1}{5}$ *French* or *Turnois*, 1 s. *Flemmish* maketh $7\frac{1}{5}$ s. *French*, and a groat *Flemmish* maketh $7\frac{1}{5}$ d. *French*.

Where-

Wherefore to know how much 143 li. 4 s. 9 d. *Flemmish* maketh *French*, ye must multiply the whole number twice by 6, beginning at pence, and so forward, and the Product of your second multiplication divide by 5; so the work is finished. Or multiply the said summe by 7, and take out of it $\frac{1}{5}$, adding it to the Product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li. 4 s. 9 d. *Flemmish* maketh 1031 li. 6 s. 2 $\frac{1}{5}$ d. *French* or *Turnois*.

Example.

li.	s.	d.	
143	4	9	*Flem.*
		6	
859	8	6	
		6	
5156	11	0	*Fren.*
$\frac{1}{5}$ 1031	6	2$\frac{2}{5}$	*Fren.*

The same otherwise.

li.	s.	d.
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
1031	6	2 $\frac{2}{5}$

Another Example.

143 l. *Flem.*
6

858
6

$\frac{1}{5}$ 5148 *French.*

1029 li. $\frac{3}{5}$ or 12 s. *Fren.*

Or thus.

143
7

1001
$\frac{1}{5}$ 28 — 12

1029 li. — 12

To

Rule 4. *To reduce pounds, shillings and pence* French, *into pounds, shillings and pence* Flemmish.

Multiply 233 li.—8 s.—4 d. *French* by 5, and divide the Product twice by 6, that is, the said number by 6, and the Product or Quotient again by 6; and the Quotient of this second Division is the thing required. So that 233 li.—8 s.—4 d. *French* make 32 li. —8 s.—4 $\frac{5}{9}$ d. *Flemmish.*

Example.				Another Example.		
li.	s.	d.		li.	s.	d.
233	8	4	*Fren.*	753	*French.*	
		5		5		
1167	1	8		3765		
$\frac{1}{6}$ 194	10	3 $\frac{1}{3}$		$\frac{1}{6}$ 627	10	
$\frac{1}{6}$ 32	8	4 $\frac{5}{9}$	*Flem.*	$\frac{1}{6}$ 104	11	8 *Flem.*

Rule 5. *To reduce pounds, shillings and pence Sterling, into pounds, shillings and pence* French *or* Turnois.

The pound Sterling maketh 8 li. 8 s. *French,* that is to say, 8 $\frac{2}{5}$ pounds, the shillings make 8 $\frac{2}{5}$ shillings, and the peny 8 $\frac{2}{5}$ d. *French.* Wherefore to know what 231 li. 13 s. 4 d. Sterling maketh *French,* ye must multiply the whole summe by 42, that is by 7, and the Product of it by 6, and divide this second Product by 5; and that is the summe required.

Otherwise, multiply the summe Sterling by 8, and adde twice to the Product $\frac{1}{5}$, and it shall produce the summe required. So that both ways 231 li. 13 s. 4 d. Sterling maketh 1946 pound *French,* as here under followeth.

Example.

Example.

li. s. d.
231---13---4 Ster.
6
____
1390---0---0
7
____
$\frac{1}{5}$ 9730---0---0
____
$\frac{1}{5}$ 1946---0---0 *French.*

The same otherwise.

li. s. d.
231---13---4 Ster.
8
____
1853---6---8
46---6---8
46---6---8
____
1946---0---0 *French.*

Another Example.

753 Ster.
6
____
4518
7
____
$\frac{1}{5}$ 31626
____
$\frac{1}{5}$ 6325---4 *Fren.*

The same.

753 Ster.
8
____
6024
$\frac{1}{2}$ 150---12
____
$\frac{1}{2}$ 150---12
____
6325---4 *Fren.*

To reduce pounds, shillings and pence French, *into pounds, shillings and pence sterling.* Rule 6.

To know how much 1256 li. 12 s. 6 d. *French* maketh in sterling mony; multiply the summe by 5, and divide the *Product* by 7 and 6 at twice, and the last *Quotient* shall be the thing required; that is to say, 1256 li. 12 s. 6 d. maketh 149 pounds 11 s. 11 $\frac{4}{7}$ d. sterling.

Example.	Another Example.
li. s. d.	li. s. d.
1256—12—6 *French.*	2531—0—0 *Fren.*
5	5
6283—2—6	12655
$\frac{1}{8}$ 1047—3—9	$\frac{1}{8}$ 2109—3—4
$\frac{1}{7}$ 149—11—11 $\frac{4}{7}$ Ster.	$\frac{1}{7}$ 301—6—2 $\frac{2}{7}$ Ster.

Note, that when any mony is given by exchange at *London* for *Roan* at 71 $\frac{1}{2}$ d, or rather 71 $\frac{1}{7}$, for the Crown of 50 s. *French*, there is neither gain nor loss, for it is one mony for another, accounting 8 li. 8 s. *French* for one pound sterling. So the giver loseth the time of payment, which is about 15 days, and he that taketh it hath the gain of the same.

They of *Roan*, that put forth or take mony by exchange for *London*, ought to have like consideration.

Item, when any man giveth at *London* 64 pence $\frac{1}{3}$, or rather 65 $\frac{1}{7}$ d, to have at one of the Fairs of *Lions* a Crown *de Marc*, he that so giveth the mony loseth the time, and he that taketh it gaineth the same: for 62 pence $\frac{1}{7}$ is equal in value to 45 s. *French*. He that putteth or taketh mony at *Lions* for *London* ought to consider the same.

Item, when any man delivers in *Antwerp* 75 pence, to receive at *Lions* a Crown *de Marc*, he that putteth it forth loseth the time, and he that taketh it gaineth the same: for 75 groats *Flemmish* is equal in value to 45 s. *French*.

Thus for this time I make an end of the practice of Exchange,

Exchange, *and the instructions thereunto belonging: and, according to my promise, yet farther to gratifie such as are desirous to know the common* Coyns *used for traffick among Merchants in these Cities following, a brief declaration of their Moneys and the reckonings and account of them I will here set down.*

The Sixteenth Chapter containeth a Declaration of the *valuation* and *diversity* of *Coins* of most places of Christendome for traffick, and the manner of *Exchange* in those places from one City or Town to another: which to know is right necessary for Merchants, by means whereof they do find the gain or loss upon the Exchange.

ITem, forasmuch as the greatest diversity of mony of exchange is at *Lions*, therefore I will begin duly of the Mony of that place.

At *Lions* they use Franks, Soulx, and Deniers *Tournois*. A Frank maketh 20 Soulx, and 1 Soulx 12 Deniers. But the Merchants, to keep their Books of Accounts, do use *French* Crowns of the mark at 45 Soulx the piece, and do divide it into 20 Soulx, one Soulx is 12 Deniers.

Item, a Mark of Gold maketh 65 ▲ of the mark, which serveth for exchange; which is divided into 8 ounces, the ounce into 24 pence or Deniers, the Denier into 24 grains, and so the summe or whole by imagination or guesse.

▲ This mark standeth for a Crown.

Also at *Lions* there are four Fairs in a year, at the which they do commonly exchange, which are

from three months to three months.

At *Geans* they use the Soulx: one Ducat maketh 3 pound.

At *Naples* they use Ducats, Taries, and Grains: the Ducat maketh five Taries, and one Tary twenty Grains: but they take six Ducats, which make 30 Taries, for the ounce.

A Ducat maketh ten Carlins, and a Carlin ten Grains; so that two Carlins make a Tary, and 100 Grains make a Ducat.

At *Rome* they use the Ducats of the Chamber: one Ducat is worth 12 Guillis, and one Guillis 10 Soulx.

At *Venice* they use Ducats currant at 124 Soulx a piece, or 24 Deniers, & one Denier makes 32 Picolis.

At *Palerm* and *Messina* they write after Ounce, Tary and Grains; and one Ounce is worth 6 Ducats of 30 Taries, and one Tary is 20 Grains, and one Grain 6 Picolis, one Ducat is also worth 24 Carlins.

At *Millane* they use li. s. d. of Ducat Imperials, and △ of exchange is worth 4 li.

At *Lucques*, *Florence* and *Ancona*, they use the △ of Gold: in Gold the *French* Crown is worth 7 li, but at *Bologne* 3 li. 10 s.

At *Barcelone* they use the Soulx: the Ducat of exchange is worth 22 Soulx.

At *Valence* and *Saragosse* they use the Liver, Soulx, and Denier: the *French* Crown of exchange is worth 20 Soulx, and one Soulx is 12 Deniers.

At the Fairs of *Castile* they use Marvides: the Ducat is worth 375 Marvides.

At *Lisbone* they use the Rayes: one Ducat of exchange is worth 400 Rayes.

At *Noremberg*, *Franckford* and *Augst* in *Germany*,

many, they use the Krentzers, whereof 60 make a Floren.

At *Antwerp* they use li. s. d. *de gros*, and they exchange into the Denier *de gros*, to wit, our *English* peny.

At *London* they use the li. the s. and d. sterling, and they exchange in pence sterling.

The Exchange of Lions *at sundry places.*

Item, at *Lions* there is Exchange in three sorts, at the Cities and Towns following.

First, they deliver at *Lions* one Mark, to have or receive at *Naples* almost 41 ½ Ducats, at *Venice* 70 Ducats currant, at *Rome* 63 Ducats of the Chamber, at *Lucques* and *Florence* 65 △ of Gold, at *Millane* 82 △.

And contrariwise, at the said Cities aforesaid they do give so much of mony to have a mark of *Lions*.

Secondly, they give at *Lisbon* one △ of mark of 45 Soulx *Tournois* apiece, to have at *Geans* almost 68 Soulx, at *Palerm* and *Messine* almost 24 Carlins, at *Barcelone* 22 Soulx, at *Valence* or *Saragosse* 20 Soulx, at the Fair at *Castile* 350 Marvides, at *Lisbon* 360 Rays, in *Antwerp* 57 Deniers *de gros*, and at *London* 70 d. sterling.

And contrariwise, they give in the said Cities almost as much of their mony to have a *French* Crown of the mark at *Lions*.

Thirdly, they do give at *Lions* a △ of the Sun, to have almost 93 Krentzers at *Franckford*, *Ausburg*, *Noremberg*, or other Cities in *Almain*.

Also at *Lions* onely they do pay, they change the ⅔ in Gold, and ⅓ in mony, or else all in mony, in giving 1 ½ for the hundred.

Changes at Naples, *and other Towns.*

Item, at *Naples* they give or deliver almost 112 Ducats, to receive at *Rome* 100 Ducats of the Chamber at the old value.

Through *Lucques* and *Florence* they deliver 100 Ducats Carlins, to receive there almost 86 △ of Gold.

Through *Palerm* and *Messine*, one Ducat of 5 Tary, to receive there almost 164 grains.

Through *Millane*, one Ducat, to receive there almost 90 Soulx.

Through *Geans*, one Ducat, to receive there almost 65 Soulx; the whole summe to be paid within ten days after the sight of the Bill of exchange.

Also at *Naples* they deliver one Ducat, to receive in *Antwerp* almost 67 d. or Deniers *de gros* within two months. At *London* almost 60 d. sterling in three months. At *Barcelone* almost 20 Soulx within two months.

At *Valence* almost 18 Soulx within two months. At *Lisbon* 333 Rayes within three months. And at the Fair at *Castile* almost 340 Maryides.

Change of Venice *to other places.*

At *Venice* they deliver 100 Ducats currant, to receive in *Almain* almost 140 Florens at 60 Krentzers the piece.

At *Lucques* and *Florence* almost 108 △ of Gold in ten days.

Likewise at *Venice* they deliver a Ducat currant, to receive at *Palerm* and *Messine* almost 21 Carlins: at *Millane* almost 93 Soulx; at *Geans* almost 69 Soulx, the whole at ten days end.

Of

Of the Pair or Pari.

As touching the exchange, it is necessary to understand or know the *Pair*, which the *Italians* call *Pari*, which is no other thing then to make the mony of the change of one City or Town to or with the mony of another, by means whereof they do find the gains or loss upon the exchange.

Example.

Item, having received Letters of credit of one of *Antwerp*, that the Δ of the Sun is there worth 7 Soulx, the question is, what the same is worth at *London*, when the *Pair* of exchange goeth for 23 shillings.

Answer. Say, if 23 give but 20, what giveth 7? Work, and find 8s. 1 $\frac{1}{3}$ d. And so much is the Δ of the Sun worth at *London*.

The Seventeenth Chapter containeth also a Declaration of the diversity of the *Weights and Measures* of most places of Christendone for traffick. At the end of which discourse are two *Tables*, the one for *Weight*, and the other for *Measure*, proportionate and reduced to an equality of our *English* Measure and Weight; by the aid whereof the ingenious may easily by the *Rule of three* convert the one into the other at pleasure, &c.

AT *London*, and so all *England* through, are used 2 kinds of Weights and Measures, as the *Troy*-weight and the *Haberdupoise*. From the *Troy*-weight is derived the proportion and quantity of all kind of dry and liquid Measures, as Pecks, Bushels, Quarters, &c. wherewith is

bought and sold all kind of Grain and other Commodities mete by the Bushell; and in liquid, Ale, Beer, Wine, Oyl, Butter, Honey, &c. Upon these grounds and Statutes is Bread made and sold by the *Troy*-weight; and so is Gold, Silver, Pearl, precious Stones, and Jewels. The least quantity of this *Troy*-weight is a grain: twenty four of these grains make a peny-weight, twenty peny-weights an ounce, and twelve ounces a pound: two pounds or pints of this weight make a quart. And so ascending into bigger quantities, are produced the Measures whereby are sold our other natural sustenance, *viz.* Ale or Beer, with all other necessary Commodities, as Butter, Honey, Herrings, Eels, Soap, &c. All which last before rehearsed, though their Measures (wherein they are contained) be framed and derived from the *Troy*-weight, yet are they in traffick with divers Commodities, as Lead, Tin, Flax, Wax, with all other Commodities both of this Realm and of other forein Countries whatsoever, bought and sold by the *Haberdupoise* weight, after sixteen ounces to the pound, and 112 pound to the hundred weight. And to every hundred is allowed but 12 pound weight at the common beam. From hence is also derived the weight of *Suffolk* Cheese, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pound. And also the Barrel of *Suffolk* Butter is, or should be, of like weight with the weight of Cheese, *viz.* 256 pounds. More, 14 of these pounds make a Stone, and 26 Stone containeth a Sack of *English* Wool. Forein Wool, to wit, *French*, *Spanish*, and *Estrich*, is also sold by the pound, or C. weight, but most commonly by the Rove,

See farther of these Weights and Measures in Reduction, beginning pag. 111.

Rove, 25 pound to a Rove. Other Commodities of Tale are bought and sold by the C, fivescore to the C, except headed ware, to wit, Cattel, Nails, and Fish, which are sold after sixscore to the C. There are also two other sorts of measures, to wit, the Ell and the Yard. By the Ell is usually mete Linen cloath, as Canvas, &c. And by the Yard Silks, Woollen cloaths, &c.

Antwerp.

At *Antwerp* are also two sorts of weights, their Gold and Silver weight, and their common weight. Gold and Silver is weighed by the mark, the mark is 8 ounces, the ounce 20 esterlings, and the esterling 32 of our grains. The Goldsmiths divide that into smaller, but not the Merchants. The proof of Gold is made by Karects, whereof 24 make a mark of fine Gold: the Karect is 24 grains. The proof of the mony is made by Deniers: 12 Deniers is 1 s. fine, that is, a mark of fine silver: the Denier also is divided into 24 grains, and the grain into four quarters.

Item, 100 marks in *Antwerp Troy*-weight maketh at *Lions* 103 marks, 2 $\frac{1}{2}$ ounces, and 20 grains, 23 d; at *Noremberg* 103 marks, 2 $\frac{1}{2}$ ounces, 2 Quints, 3 Deniers: at *Franckford* 105 marks; at *Ausburg* 104 marks, 3 ounces, 1 Quint; at *Venice* 103 marks, 1 ounce, 7 Deniers, 18 grains; at *London* 66 pounds.

The Mark of gold or silver at *Antwerp Troy*-weight, which is 8 ounces, maketh 7 $\frac{1}{2}$ ounces common weight, with which all other Merchandize is weighed. So that the *Troy*-weight is greater then the common weight by 6 $\frac{1}{4}$ in the C. By this weight of *Troy* they also weigh Musk, Amber, Pearl, &c.

All Silks are bought at *Antwerp* by the *Bourges* Ell, which

which is greater then the common measure, by which they retail by 2 in the hundred. Their common Ell is $\frac{3}{4}$ of our Yard, and $\frac{3}{5}$ of our Ell.

Lions.

At *Lions* are used three sorts of weight, whereof the first is the common Town-weight, with which they weigh all kind of Spicery, and divers other Merchandize. The second is called *Geneva* weight, which is 8 in the C. greater then the common weight, with which they weigh Silks, &c. The third is *French* weight, called commonly the Mark-weight, and 100 pounds thereof maketh 106 $\frac{1}{4}$ li. *Geneva*, and 114 $\frac{3}{4}$ of their common weight: with which *French* weight are weighed all things that pay Custome or Toll.

At *Lions* are also used two sorts of Ells or Aulnes. The one wherewith they measure grosse cloaths, as Canvass and such like. The other is called the *French* Ell or Aulne, with which they measure all other kind of Merchandize, whereof 7 common Town-Ells make 11 ordinary *French* Ells.

Roan.

At *Roan* 6 $\frac{1}{2}$ Muids of Salt, being the measure of the place, make a C. at *Armuiden* in *Zeland*; and the C. of *Bronage* measure of *Armuiden* maketh at *Roan* 11 Muides. 30 Muides make a Last of Corn, and 16 a Last of Oats. 100 pound weight there maketh at *London* 114 $\frac{1}{2}$, and 190 $\frac{1}{4}$ at *Antwerp*. And 200 Ells make at *London* 115 $\frac{1}{4}$.

Noremberg.

A 100 pound weight at *Noremberg* maketh at *London*

London 111 $\frac{3}{4}$, at *Antwerp* 107$\frac{1}{5}$: and 100 Ells at *Noremberg* make at *London* 75$\frac{1}{5}$, at *Antwerp* 95$\frac{1}{5}$, &c.

Lisbon.

The C. weight at *Lisbon* maketh 4 Roves, every Rove 32 pounds: so that their C. weight is 128 pounds, and their pound containeth 14 ounces, and 100 pounds of their weight maketh at *London* 113$\frac{1}{8}$.

Their Silk, Cloath of Gold and Woollen, is measured with a measure which they call a Cubit, containing about $\frac{3}{4}$ of a Varre of *Castile*. Howbeit their common measure is called a Varre, which maketh five Palms, and containeth 1$\frac{1}{8}$ of a Varre of *Castile*. Our Ell of *London* is equal with the Varre of *Lisbon*.

All kind of Merchandize brought from *Flanders*, *Roan*, or *Britain*, payeth at *Lisbon*, as a Duty or Custome to the King, 20 in the C; which they call the tenth in Merchandize, and the other tenth in mony.

Note also, that all kind of Merchandize coming to *Lisbon* by land payeth less in custome then that that cometh by water.

Sivil.

The Rove of *Sivil* is 30 pound; 4 Roves make their C. weight, which is 120 li. The 100 pounds of *Sivil* maketh at *London* 102 pounds. Their other common measure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be two sorts of weight, the one called *la Grosse*, the other *la Sottile*. With the grosse is weighed

weighed all kind of great wares, and with the small all kind of Spicery and such like. 96 pounds of gross weight there make at *London* 100 pound; and 100 pounds of Spicery there, without any tare or allowance, make at *London* 94, and with tare 65.

Their own common measures are Braces, whereof 100 make at *London* 55 ½ Ells, at *Antwerp* 92 ½, &c.

Florence.

At *Florence* the 100 l. weight maketh at *Aquila*, for Saffron, 110, and 145 pounds of *Florence* make at *Roan* but 100 pounds. The weight of *Florence* and that of *Lucques* is all one.

Their other measures are Braces, whereof 100 make at *Antwerp*, *Bourges* measure, 81 ⅗ Ells; 100 Braces there make at *London* 49 Ells, &c.

Lucques.

The *Lucques* Sattens are commonly sold at *Lions* by weight, and 133 ⅓ pound make at *Lions* 100 pound; so that 1 pound ⅓ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 make at *London* 50 Ells, at *Antwerp* 83 ⅓ Ells, &c.

Aquila.

At *Aquila* their 100 pounds make at *London* 71 ¼: their 136 ⅔ pounds of Saffron make at *Geneva* but 100, and 11 li. of *Geneva* maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and a small.

small. The C. weight or great weight containeth four Roves, the Rove 36 li: so the C. great weight is 144 li. And the C weight small containeth but 120 pounds, and is also parted into four Roves, which is 30 pounds to a Rove. By the small is sold the Scarlet grain, with all other kind of Spicery; and by the great is sold Wool, with all such like gross wares. The 1 $\frac{1}{3}$ pounds of Silk at *Valentia* maketh at *Lions* one pound *Geneva* weight. The Charge of great Merchandize at *Valentia* containeth 432 pounds, and in small wares 360 pounds.

The weight here and at *Barcelone* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Dantzick.

At *Dantzick* or *Spruceland* the rule is, that whosoever buieth any Merchandize there, buieth it by the Ship-pound, which is 320 li. 20 Lispounds make a Ship-pound, and the Lispound containeth 16 pound; which Ship-pound of *Dantzick* maketh at *Antwerp* 266 $\frac{2}{3}$ li. Their 100 li. weight maketh at *London* 86, &c.

Their other common measures are Ells, whereof 100 make at *London* 72 $\frac{1}{4}$, and at *Antwerp* 120 $\frac{1}{2}$ Ells.

Tolouse.

At *Tolouse* 6 Cabes of Woad make a Charge, two Cisterns of corn-measure and all kind of grain make a Charge; the Cistern weigheth 160 li. weight of that place. Their 100 in weight maketh at *London* but 91 $\frac{1}{2}$ pound.

Geans.

Geans.

At *Genua*, or *Geans*, 100 li. of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerp* 68$\frac{3}{5}$. 100 li. weight at *Genua* maketh at *Venice*, to wit Suttle, 106 li.

Their other common Measures are Palms, where of 100 make at *London* 20$\frac{3}{4}$ Ells, and at *Antwerp* 34[illegible]

The rest are supplied in two Tables, which here after follow, whereby the ingenious may gather [illegible] desire.

The Table of the agreement of the Weights of di vers Countries the one with the other, being reduced to an equality as followeth.

112 pounds weight at London make at		112 pounds weight at London make at	
Antwerp	107 $\frac{1}{5}$	*Venice* grosse weight	105 [illegible]
Frankford	099	*Venice* suttle weight	166 [illegible]
Colen and *Ausburgh*	102 $\frac{1}{2}$	*Aquina*	157 [illegible]
Noremberg	100 [illegible]	*Vienna*	089 [illegible]
Roan	098	*Presslaw*	130 [illegible]
Paris	102 $\frac{1}{2}$	*Leipsig*	101 [illegible]
Lions	118 [illegible]	*Dantzick*	129 [illegible]
Diep	100 $\frac{1}{4}$	*Lubeck*	097 [illegible]
Geneva	090 [illegible]	*Barcelone*	144 [illegible]
Tolouse	122 [illegible]	*Lisbone*	099
Rochell	124 [illegible]	*Geans*	157 [illegible]
Marseilles	124 $\frac{3}{4}$		
Sivil, &c.	109 $\frac{1}{4}$		

The other Table of agreement of Measures of di vers Countries reduced unto an equality, by the aid whereof you may with the use of the *Rule of three* convert either more or less of any one Measure unto the other.

Antwerp

60 Ells or 75 Yards at *London* make at			
	Antwerp	100	*Ells.*
	Neremberg	104 $\frac{1}{2}$	
	Frankford	125	
	Leipsig	125	
	Preslaw	125	
	Dantzick	183	
	Vienna in *Austria*	87	*Aulnes.*
	Lions in *France*	60 $\frac{3}{4}$	
	Paris in *France*	57	
	Roan in *Normandy*	52	
	Lisbon	60	*Varres.*
	Sivil in *Spain*	81	
	Castile in *Spain*	81	
	Mathera Isles	62	
	Venice	108	*Braces.*
	Lucques	120	
	Florence	122 $\frac{1}{2}$	
	Millain	138	
	Rome	90	*Canes.*
	Geans	288 $\frac{6}{13}$	*Palms.*

The Eighteenth Chapter treateth of *Sports* and *Pastimes* done by number.

IF you would know the number that any man doth think on imagine in his mind, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, ask him whether it be even or odde: if he say odde, bid him take one to it, to make it even, and for that one keep one in your mind. Now after he hath taken one to it, to

weighed all kind of great wares, and with the small all kind of Spicery and such like. 96 pounds of gross weight there make at *London* 100 pound; and 100 pounds of Spicery there, without any tare or allowance, make at *London* 94, and with tare 65.

Their own common measures are Braces, whereof 100 make at *London* $55\frac{1}{2}$ Ells, at *Antwerp* $92\frac{1}{2}$, &c.

Florence.

At *Florence* the 100 l. weight maketh at *Aquila*, for Saffron, 110, and 145 pounds of *Florence* make at *Roan* but 100 pounds. The weight of *Florence* and that of *Lucques* is all one.

Their other measures are Braces, whereof 100 make at *Antwerp*, *Bourges* measure, $81\frac{2}{3}$ Ells; 100 Braces there make at *London* 49 Ells, &c.

Lucques.

The *Lucques* Sattens are commonly sold at *Lions* by weight, and $133\frac{1}{3}$ pound make at *Lions* 100 pound; so that 1 pound $\frac{1}{3}$ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 make at *London* 50 Ells, at *Antwerp* $83\frac{1}{3}$ Ells, &c.

Aquila.

At *Aquila* their 100 pounds make at *London* $71\frac{1}{4}$: their $136\frac{2}{3}$ pounds of Saffron make at *Geneva* but 100, and 11 li. of *Geneva* maketh 15 li. at *Aquila*.

Valentia.

At *Valentia* be two sorts of weights, a great and a small,

small. The C. weight or great weight containeth four Roves, the Rove 36 li: so the C. great weight is 144 li. And the C weight small containeth but 120 pounds, and is also parted into four Roves, which is 30 pounds to a Rove. By the small is sold the Scarlet grain, with all other kind of Spicery; and by the great is sold Wool, with all such like gross wares. The 1 $\frac{1}{3}$ pounds of Silk at *Valentia* maketh at *Lions* one pound *Geneva* weight. The Charge of great Merchandize at *Valentia* containeth 432 pounds, and in small wares 360 pounds.

The weight here and at *Barcelone* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Dantzick.

At *Dantzick* or *Spruceland* the rule is, that whosoever buieth any Merchandize there, buieth it by the Ship-pound, which is 320 li. 20 Lispounds make a Ship-pound, and the Lispound containeth 16 pound; which Ship-pound of *Dantzick* maketh at *Antwerp* 266 $\frac{2}{3}$ li. Their 100 li. weight maketh at *London* 86, &c.

Their other common measures are Ells, whereof 100 make at *London* 72 $\frac{1}{4}$, and at *Antwerp* 120 $\frac{1}{2}$ Ells.

Tolouse.

At *Tolouse* 6 Cabes of Woad make a Charge, two Cisterns of corn-measure and all kind of grain make a Charge; the Cistern weigheth 160 li. weight of that place. Their 100 in weight maketh at *London* but 91 $\frac{1}{2}$ pound.

Geans.

Geans.

At *Genua*, or *Geans*, 100 li. of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerp* 68 $\frac{3}{5}$. 100 li. weight at *Genua* maketh at *Venice*, to wit Suttle, 106 li.

Their other common Measures are Palms, whereof 100 make at *London* 20 $\frac{3}{5}$ Ells, and at *Antwerp* 34 $\frac{1}{2}$.

The rest are supplied in two Tables, which hereafter follow, whereby the ingenious may gather his desire.

The Table of the agreement of the Weights of divers Countries the one with the other, being reduced to an equality as followeth.

112 pounds weight at London make at		112 pounds weight at London make at	
Antwerp	107 $\frac{1}{5}$	*Venice* grosse weight	105 $\frac{3}{[illegible]}$
Frankford	099		
Colen and *Ausburgh*	102 $\frac{1}{4}$	*Venice* suttle weight	166 [illegible]
Noremberg	100 $\frac{1}{3}$		
Roan	098	*Aquina*	157 [illegible]
Paris	102 $\frac{1}{2}$	*Vienna*	089 [illegible]
Lions	118 $\frac{1}{4}$	*Prestaw*	130 [illegible]
Diep	100 $\frac{1}{4}$	*Leipsig*	101 [illegible]
Geneva	090 $\frac{1}{3}$	*Dantzick*	129 [illegible]
Tolouse	122 $\frac{1}{4}$	*Lubeck*	097 [illegible]
Rochell	124 $\frac{1}{4}$	*Barcelone*	144 $\frac{1}{[illegible]}$
Marseilles	124 $\frac{3}{4}$	*Lisbone*	099
Sivil, &c.	109 $\frac{1}{4}$	*Geans*	157 $\frac{1}{5}$

The other Table of agreement of Measures of divers Countries reduced unto an equality, by the aid whereof you may with the use of the *Rule of three* convert either more or less of any one Measure unto the other.

Antwerp

60 Ells or 75 Yards at *London* make at	*Antwerp*	100	*Ells.*
	Noremberg	$104\frac{1}{2}$	
	Frankford	125	
	Leipsig	125	
	Preslaw	125	
	Dantzick	183	
	Vienna in *Austria*	87	*Aulnes.*
	Lions in *France*	$60\frac{3}{4}$	
	Paris in *France*	57	
	Roan in *Normandy*	52	
	Lisbon	60	*Varres.*
	Sivil in *Spain*	81	
	Castile in *Spain*	81	
	Methera Isles	62	
	Venice	108	*Braces.*
	Lucques	120	
	Florence	$122\frac{1}{2}$	
	Millain	138	
	Rome	90	*Canes.*
	Geans	$288\frac{6}{23}$	*Palms.*

The Eighteenth Chapter treateth of *Sports* and *Pastimes* done by number.

IF you would know the number that any man doth think on imagine in his mind, as though you could divine, bid them triple it, or put twice so much more to it as it is, which done, ask him whether it be even or odde: if he say odde, bid him take one to it, to make it even, and for that one keep one in your mind. Now after he hath taken one to it,

to

to make it even, bid him give away half, and keep the other half for himself: which when he hath done, bid him triple that half; and again, after he hath tripled it, ask him whether it be even or odde: if he say odde, then bid him take one to make it even again, and for that last one keep two in your mind. Now after he hath made his number even, bid him cast away the one half, and keep the other still; from which half that he keepeth cause him subtilly to put away or give you nine out of his number as oft as he can, and for each 9 that he giveth you keep 4 in mind, and thereunto joyn the 3 which I bade you keep, and you shall have your desire.

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it even; which first 1 given is for you to keep in mind. Then the half of his 22 being cast away, he reserveth still 11, which after you have bid him triple, it maketh 33. Then in giving of him one again to make it even, upon that last 1 reserve 2 in your mind; then his half of 34 maketh 17, from whence he can give you 9 but once. Therefore that yielding to you 4, and the 3 that you keep, maketh 7, your desire.

Another kind of Divination, to tell your friend how many pence or single pieces, reckoning them one with another, he hath in his purse, or should think in his mind.

Which to doe, first bid him double the pieces he hath in his purse, or the number he thinketh, (if he participate his number or secrecy unto some one friend that sitteth by him that can but multiply, and adde never

so

so little, if their number be great, then shall they work as you bid them so much the surer.)

Now after he hath doubled his number, bid him adde thereunto 5 more: which done, bid him multiply that his number by 5 also: which done, bid him tell you the just summe of his last multiplication; which summe the giver thinketh is nothing available, because it is so great above his pretended imagination; yet thereby shall you presently, with the help of Subtraction, tell his proposed number.

The Rule is this.

Imagine he thought 17, double 17, and it maketh 34, whereunto if you adde 5, it maketh 39, which multiplied by 5, as here is practised, it yieldeth 195, which 195 is the summe delivered you in the work: then for a general Rule, you shall evermore cut off the last figure toward your right hand with a dash of your pen, as here is performed, as a figure nothing available unto your work, and then rebate 2 from your first figure, after 5 is cut off, and the rest shall evermore be your desire; as by this example doth appear.

17
2
———
34
5
———
39
5
———
195
2
———
17

Another of a Ring.

If in any company you are disposed to make them merry by manner of divining, in delivering a Ring unto any one of them, which after you have delivered unto them, that you will absent your self from them, and they to devise after you are gone which of them shall have the keeping thereof, and that you at your return will tell them what

person hath it, upon what hand, upon what finger, and what joynt: To doe this, cause the persons to sit down all in a row, and to keep likewise an order of their fingers. Now, after ye are gone out from them to some other place, say unto one of the lookers on, that he double the number of him that hath the Ring, and unto the double bid him adde 5; and then cause him to multiply the Addition by 5, and unto the product bid him adde the number of the finger of the person that hath the Ring. And lastly, to end the work, beyond that number, towards his right hand, let him set down a figure signifying upon which of the joynts he hath the Ring, as if it be upon the second joynt, let him put down 2. Then demand of him what number he keepeth, from the which you shall abate 250, and you shall have three figures remaining at the least. The first towards your left hand shall signifie the number of the person which hath the Ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the joynt.

Example.

Imagine the seventh person is determined to keep the Ring upon the fifth finger, and the third joynt: first double 7, *it maketh* 14; *thereto adde* 5, *it maketh* 19; *which multiplied by* 5 *yieldeth* 95; *unto which* 95 *adde the number of the finger, and it maketh* 100; *and beyond* 100, *towards the right hand, I set down* 3, *the number of the joint; all maketh* 1003, *which is the number that is to be delivered you; from which abating* 250, *there resteth* 753; *which prefigureth unto you the seventh person, the fifth finger, and the third joynt.*

But

But note, that when you have made your subtraction, if there do remain 0 in the place of tens, that is to say in the second place, you must then abate 1 from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth finger; as if there should remain 803, you must say, that the seventh person upon his tenth finger, and upon his third joynt, hath the Ring.

Another of three Dice.

If a man do cast three Dice, you may know the points of every one of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiply by 5, and unto the product adde the points of one of the other Dice, and behind the number towards the right hand to put the figure which signifieth the points of the last Die, and then ask what number he keepeth, from which abate 250, and there will remain three figures, which do note unto you the points of every Die.

Another of things hidden.

If three divers things are to be hidden of three divers persons, and you to divine which of the three persons hath the three divers things, doe thus: Imagine the three things to be represented, *A*, *B*, *C*. Then secondly, keep well in your minde which of the persons you mean to be the first, second, and third. Then take 24 Counters or Stones, and your three things, and give *A* to the party whom you imagine to be your first man, and therewithall give him one of your 24 Counters in his hand, and *B* unto your second man,

 and

and therewithall 2 Counters, and *C* unto your third man, and therewithall 3 Counters, and leave the rest, which are 18, still among them: which done, separate your self from them, and afterwards bid them change the things among them as they shall think good: which done, after they are agreed, bid him that hath such a thing as before you have represented by *A*, for every Counter that he hath in his hand to take up as many more, and him that hath *B*, for every one in his hand to take up 2, and him that hath *C*, for every one in his hand to take up 4, and the rest of them to leave still upon the board. These 3 things and the three persons being fully printed in your minde, come to the Table, and you shall evermore finde one of these 6 numbers, 1, 2, 3, 5, 6, or 7. If therefore one remain still upon the board, then have they made no exchanges but keep them still as they were delivered unto them. So that the first man hath *A*, the second *B*, and the third man *C*. But if 2 remain, then the first man hath *B*, your second man *A*, and your third man *C*. The rest of the work and the order thereof are here apparent by the Table following.

	1	*A*			1	*B*
1	2	*B*		5	2	*C*
	3	*C*			3	*A*
	1	*B*			1	*C*
2	2	*A*		6	2	*A*
	3	*C*			3	*B*
	1	*A*			1	*C*
3	2	*C*		7	2	*B*
	3	*B*			3	*A*

Another

Another divination of a number upon the casting of two Dice.

First, let the Caster cast both the Dice, and mark well the number: then let him take up one of them, it maketh no matter which, and look what number it hath in the bottom, and adde all together: then cast the Die again, and keep in his minde what all together maketh: then let the Dice stand, and bring seven with you, and thereunto adde the rest of the pits that you see upon the upper side of the Dice, and so many did the Caster cast in all.

FINIS.

An Appendix concerning the Resolution of the *Square* and *Cube* in Numbers, to the finding of their sides: By *Ro. Hartwell.*

A *Figurate Number* is a number made by the multiplication of one number or more by another. A figurate number what.

The *sides* of a *figurate Number* are the numbers by whose multiplication it is made. The sides of a figurate number what.

A Figurate number is two-fold, a
Plain.
Solid.

And it is
Of one Multiplication,
Or consequently of many,
as a
Plain.
Solid.

And in each
both Æquilater.
and Inæquilater.

A plain figurate number.

A figurate number made of one multiplication by two sides or numbers multiplied together, is called *a plain figurate number*.

For every number made by the mutuall multiplication of two numbers may be called *a Plain*, because it bringeth forth a right-angled parallelogramme, according to his unites disposed in length and breadth, the sides whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5, is called *a Plain*, and the sides thereof are 4 and 5, as here;

```
* * * * *
* * * * *
* * * * *
* * * * *
```

Because the unites thereof disposed in length and breadth, as the sides do express, do bring forth an *Inæquilater* Parallelogramme, for that the numbers or sides are inequal.

By like reason 36, made by multiplication of 6 by 6, is called an *Æquilater* Plain, for the sides thereof 6 and 6 are equal.

Moreover, one and the same plain number may have many sides, as the plain number 24 hath sides 4 and 6, 3 and 8, 2 and 12. For it is produced from the mutuall multiplication of these numbers. Whereupon for the invention of the sides, to wit in *æquilater* Plains, it is needfull to give one of the sides, by which the Plain it self divided, the other side is made known. As the Plain 48 being divided by the side 8, the quotient 6 is the remaining side. Notwithstanding another resolution and inquisition doth happen in the sides of the *Æquilater* Plains.

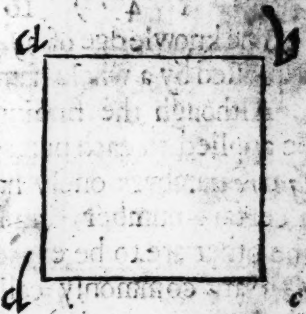
An Æquilater Plain or Quadrat what.

An *Æquilater Plain* is a number made by two equal sides, or by any number multiplied by it self. It is vulgarly called *a Square* or *Quadrat*; by the

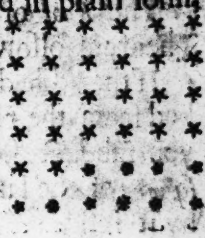
Arabians

Arabians Zensus: it is commonly expressed by this note z, by us q.

A *Quadrat* or *Square* in Geometry is called *a right-lined plain figure*, made by four equal right lines, and so many right Angles; and every one of the lines is called the *side of the Quadrat*, as in this figure *a b c d*, whose *side* is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude hereof, that number is called a *Quadrate* which is made by the multiplication of two equal numbers, or of one in it self: in which manner 36 is made, by 6 multiplied in it self, or by the mutual multiplication of 6 and 6. For if 36 unites be placed in plain form, it bringeth forth a perfect Geometrical Quadrat, having in every side six unites, as here.



The side or root of a number what.

The number whereof the *Quadrat* is produced by multiplication in it self is called the *side* or *root* of the *Quadrat*.

Concerning the *Extraction* of the *quadrat* or *square Root*.

THerefore to finde the *quadrat Root*, or the *side* of any quadrat number, is to search a number, which brought or multiplied in it self maketh the number propounded: concerning the finding whereof, as it is requisite that the *sides* (being lesser then 10) of the

the *squares* under an hundred should be gathered by the Table of Multiplication; so the *sides* of the greater *squares* are to be sought out by Art. First, the *squares* whose *sides* are simple numbers are here set down as you see.

The roots.	1	2	3	4	5	6	7	8	9
The squares.	1	4	9	16	25	36	49	64	81

The knowledge of a *square* is by finding out his *side* expressed by a whole number.

Although the finding out of the *side* of a *square* be applied to each number given, as to a *square*, yet *square* numbers onely have a *side* to be expressed by a certain number of unites, or by rational numbers; the other are to be expressed but onely in power. The *sides* are commonly called *Roots* by a Metaphoricall phrase.

The *Root* or *side* of a *square* is to be found by the Theorem following.

If the odde degrees of a *square* number being marked from the right toward the left hand with points, you subduct from the number given the particular *square* of the last period, setting the *side* thereof alone by it self;

Then going on, if you divide the remainder (if there be any) with the figure going before it, by the double of the *side* set alone by it self,

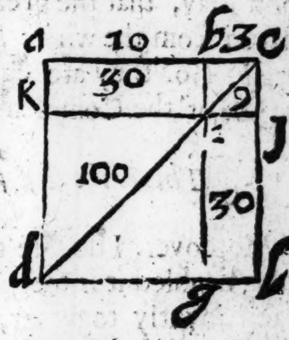
And multiply the Quotient found out (being placed by the *side* which was first set alone by it self, and also before the doubled number on the right hand) by both the numbers (namely, by the double number and the figure set by it self) being counted as one Divisor, subducting the products from the given number, and then renew this last work of division so many

times

times as there are pricks remaining, the *side* of the *square* shall be found out.

This artificial device is taken out of the 4 *Prob.* 2. l. *of* Euclide; *where by demonstration it is proved, that if a right line be cut into two Segments, howsoever the* square *of the whole line is equal to the* squares *of the Segments, and the two right-angled figures made of the Segments: as in the figure annexed, the two Diagonals,* k g *and* b f, *are the* squares *of the Segments* a b, *and* b c; *also the Complements* b k *and* f g *are the right-angled figures made by multiplying the line* a b *by* b c.

To extract the square root.



The self-same parts are to be found in any *square* number. As for example, let the number be 169, whose *side* is 13. This *side* being divided into two pieces, 10 and 3, multiplying each piece by it self once, namely 10 by 10, and 3 by 3, then multiplying one by another, as 10 by 3, and 3 by 10, so shall you have 4 plain numbers, whereof 2 are *square*, as here you see.

The first example.

Therefore as the *square* 169 is made by adding together of these 4 plain numbers, so by subducting them severally it is resolved.

10	3
10	3
100	
30	
30	
9	
169	

First therefore, I mark each odde place with points, because the particular *squares* are to be found in the odde places. Then for so much as the unite standing under the first point next the left hand, & representing the last period, is both a *square* and the side of a *square*;

square; that figure therefore being set alone in the Quotient, and being subducted from the unite standing over the point, there remaineth nothing.

This unite set alone by it self in the Quotient shall signifie 10, when another figure is set by it representing the side of some other particular *square*. Whereupon I say, that the greater Diagonal *k g* is now subducted from the whole *square*, and the *side* of it *k i*, or *a b*, (for they are equal one to another) and also the *side* of the Complement is found out.

This is the first step to this Resolution.

Moreover, I double the figure found out, because being doubled it is the *side* of both the Complements taken joyntly together, namely *k i* and *g i*. Then setting 2, the doubled number, under 6, I divide 6 (which in this place is as much as 60, and representeth both the Complements) by 2, the Quotient is 3, representing the other *side* remaining of the Complement, namely *i f* or *b e*, which number I set in the Quotient, and count it for the Segment remaining of the right line given. Wherefore because this number 3 is the side of the remaining Diagonall, that is to say, of the lesser *square b f*, therefore being set by the Divisor on the right hand, and multiplied by it self and also by the Divisor, it bringeth forth three plain numbers, namely, the *square b f*, and the two Complements *a i*, and *i l*, which being subducted from the numbers standing over them, there remaineth nothing.

The

The example is thus.

169 (12 *Which is all one* 169
1x3 *as if you had put* 100 *The greater Diagonall.*
3 *down the numbers found out in* 60 *The Complements twofold.*
69 *this manner,* 9 *The lesser Diagonall.*

169

The subtilty of this invention is illustrated by many examples.

Let the *square* given be 1764. This number, being marked with two points, telleth us that the *side* thereof is to be written with two figures. The second example.

First therefore, beginning at the point on the left hand, I seek the *side* of the last period, namely 17. But for so much as it is no *square* number, I take 4, the *side* of the next lesser *square*, which I set alone by it self in the Quotient, and then multiply it by it self; the Product is 16, which being subducted from 17, there resteth 1. Moreover, I double the *side* found out, the product is 8. I place this doubled number under 6, and by it divide 16 standing above it; the Quotient is 2, which must be set by 4. This quotient 2 must be set before the Divisor 8, on the right hand under the point, and then it must be multiplied both by it self, and into 8; the Product is 164, which being subducted from the figures standing over them, there remaineth nothing: whereby I gather that the number given is a just *square*.

The

The example standeth thus.

$$\begin{array}{l} \cancel{1} \\ \cancel{1}\cancel{7}\cancel{6}\cancel{4}\ (42 \\ 168\cancel{2} \\ \quad \cancel{2} \\ \hline \ 164 \\ \hline 1764 \end{array}$$

The Collection.

The same manner of working is to be followed in greater square *numbers given, saving that the former part of the work is to be used but once, but the latter part is to be followed so many times as there are points remaining excepting the last.*

The third example.

As in 5, 47, 56, I say, that the *side* of the *square* next unto 5 is 2; therefore 2 being set in the Quotient, and multiplied by it self, makes 4, and taken from 5, the remainder is 1. Moreover, I double the Quotient, the Product is 4, which I set under the next figure toward the right hand, and thereby divide 14, the Quotient is 3; which 3 being set both in the Quotient, and also before the Divisor towards the right hand, I multiply both the numbers by it, the Product is 129: this being subducted from 147 standing above it, the remainder is 18. But because there is yet one point remaining with which I have not meddled, I therefore again double all the whole Quotient; for in this case I must take 23 for the *side* of one former *square*,

Note.

and generally in great numbers, when I light upon more particular *squares* then two, I must esteem them but as two, and take the *sides* which are first found out but as the *sides* of one onely *square*. Therefore twice 23 is 46: by this I divide 185, the number to be set in the Quotient is four; which number

number also must be set before the Divisor on the right hand: then must 464 be multiplied by 4, the Product is 1856; this Product being subtracted from the numbers standing over it, there remaineth nothing. The example standeth thus.

$\cancel{1}\cancel{1}8$
$\dot{\cancel{5}}\cancel{4}\dot{\cancel{7}}\cancel{5}\dot{6}$ (234.
$\cancel{4}\cancel{4}\cancel{3}8\cancel{4}$
129

4
1856
54756 *The Collection.*

See also the Example following.

$\dot{1}0\dot{9}4\dot{2}8\dot{6}4$ (3308.

Therefore out of this invention is this Consectarie,

The number whose side cannot be expressed by whole numbers is not a square number. 4 Example of a surd number.

Such are all prime numbers, and (the *squares* themselves excepted) all other compound numbers. For if in them you desire to find out the *square side*, you shall labour in vain, because they are not *squares*; for to the whole numbers arising in the Quotient there will be some Fraction adjoyned, whereby it cometh to pass that the number of the *side* is not to be expressed by a true number, and it is commonly called a *surd number*.

Notwithstanding, if you adjoyn to the *side* found out the number remaining, taking his denomination from the double of the *side* augmented by an unite,

you

you shall finde the next *side* that may be like to the *side* of a *square*.

As if from 40 you take the nearest *square*, to wit 36, the remainder is 4: here therefore the *side* sought for of the *square* exceedeth not the *side* found out by an unite, but either by one or more parts of some whole number: wherefore I double 6, the *side* found out, and adde an unite to it being doubled, the total is 13: this number I set under 4 the remainder, and say that the *side* of 40 demanded as near as may be is $6\frac{4}{13}$: the Denominator of the Fraction being added to the greatest *square* in the number given, namely unto 36, maketh the next greatest *square* above it, namely 49, whose *side* is 7. But this *surd side*, to wit, $6\frac{4}{13}$, multiplied by it self, maketh $39\frac{133}{169}$, which are not just equal to 40, the given number.

Judge the like concerning the rest which are not *squares*.

Thus much concerning plain figurate Numbers; but especially such as are square numbers.

Concerning *solid figurate Numbers*.

A solid figurate number.

A Solid figurate Number *is made of two multiplications by three numbers or sides multiplied together, admitting length, breadth, and thicknesse*.

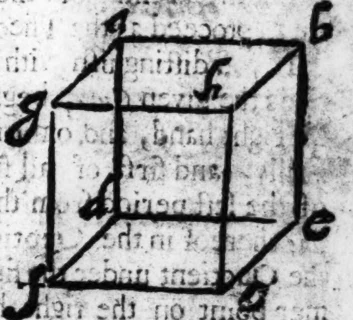
Therefore every number made by the mutual multiplication of three numbers may be called a *solid*, because it bringeth forth a right-angled Parallelipipedon, disposed according to his unites in length, breadth and thicknesse, the *sides* whereof are the three multiplying numbers. As the number 30 made by the mutuall

tuall multiplication of 2,3 and 5, is called an *Inaquilater solid number*, and the *sides* thereof are 2,3 and 5; because the unites thereof disposed by a certain distance one from another in length, breadth and depth, as the *sides* do express, do bring forth in resemblance an *Inaquilater Parallelipipedon*, for that the numbers of *sides* are inequal.

By like reason 216 made by multiplication of 6 by 6, and the Product thereof by 6, is called an *Æquilater solid*, for the *sides* thereof, 6,6 and 6, are equal.

An Æquilater *is a number made by three equal* sides, *or by any number multiplied by it self, and that Product again by the foresaid number. And it is called an* Æquilater *and* Æquiangled Parallelipipedon *or* Cube, *and is commonly represented by us thus*, C. An Æquilater solid or Cube.

A *Cube* in Geometry is a right-angled Parallelipedon, having six equall surfaces, and 8 solid angles, and 12 *sides*; as this figure *a b c d e f g h*, whose *side* is *a b* or *a d*, also *b c* or *c d*, either *c e* or *e f*, likewise *c h* or *b g*, also *g f* or *d f*, or *d a* and *g a*.



The number whereof the Cube *is produced by multiplication in it self twice, is called the* side *or* root *of the* Cube; *which being found out in whole numbers, the* Cube *is known.* The side or root of the Cube.

Con-

Concerning the Extraction of the *Cubick Root.*

THerefore every *Cube* in numbers hath such a *side* as may be expressed in whole numbers, but in magnitudes it is not always so, as indeed in magnitudes there are many things not to be expressed in whole number. Now forasmuch as the *side* of any *Cube* under 1000 is a simple figure, it is necessary, before we undertake to finde out the *side* of any great number, to know what *Cube* is made of each simple figure, and what is the *side* of any *Cube* lesser then 1000, as I have here set them down.

Roots.	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	216	342	512	729

But in searching out the *sides* of greater *Cubes*, we are to proceed as the Theorem following teacheth us.

If you distinguish with points as it were into periods the given *Cube*, beginning at the first figure on the right hand, and omitting each two figures continually, and first of all subduct the particular *Cube* of the last period from the given number, setting the *side* thereof in the Quotient, and then set triple of the Quotient under the figure next following the former point on the right hand, and the *square* of the Quotient being tripled beneath it one degree more towards the left hand, afterward divide the number above written by the triple of the *square*, setting the Quotient by it self, and then multiply the Divisor by the Quotient found out, and the triple *square* by the

square

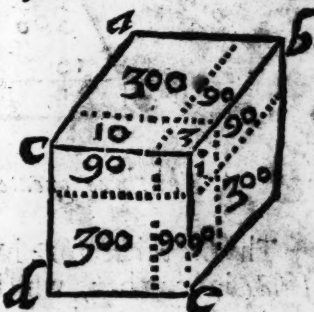
square of the Quotient, and the Quotient cubically, subducting the Products (so orderly added together, that each figure may answer the numbers whereof it was multiplied) from the number given, and renew this last manner of division so many times as there are points remaining, the *side* of the *Cube* shall be found out.

This artificial device is drawn out of that Theorem which *Ramus* made, imitating that of *Euclide* concerning *square* numbers, in this manner:

The extraction of the Cubick side or root.

If a right line be cut into two Segments, the Cube *of the whole line shall be equal to the* Cubes *of the Segments, and the two* solid *figures comprehended three times under the* square *of his Segment and the Segment remaining.*

As the line *c i*, which is 13, is cut into two Segments, 10 and 3, therefore the *Cube* of the whole line, namely 2197, is equal unto the *Cubes* of the Segments, namely unto 100 and 27, also to the two-fold *Solids* or *Parallelipipedons* thrice taken, whereof three have like solidity, the solidity of each of the three lesser is 90, being made of the *Square* of the Segment 3, that is to say of 9 multiplied by the other Segment 10. These three Parallelipipedons joyntly taken together make 270. But the three greater Parallelipipedons, each containing 300, being made of 100, the *Square* of the greater Segment 10, multiplied by the lesser Segment 3, and they being taken joyntly together, make 900.



The

A

The Cube of the greater segment 10.

The Cube of the lesser segment 3.

B

C

The three lesser Parallelipipedons.

D

The three greater Parallelipipedons.

The *Cube* therefore hath eight particular *solids* in number, which are made of the parts of the number given, namely of 10 and 3, in this manner. First, let there be four plain numbers made, each part being multiplied by it self, and one by another.

10	3
10	3
	9
	30
	30
	100

If

If again I multiply the Plains by the same parts, there will arise 8 *Solids*, as you see here.

9	9
30	30
30	30
100	100
3	10
27	90
90	300
90	300
300	1000

All these being added together are equall to the Cube *of the whole, to wit* 2197.

Therefore the same way that is kept in making this *Cube* is also to be followed in resolving the *Cube*.

As for example, I mark the *Cube* given with points in this manner, 2197. The first example to extract the Cubick root.

Then I subduct the particular *Cube* of the number set under the last point: but for so much as that number is no *Cube*, I take the nearest to it, namely, an unite, which also I set in the Quotient. This unite in the number given is 100, but in the Quotient it is but 10: the unite subducted from 2, the remainder is 1, which must be written over the number given. So that the greater *Cube A* is to be supposed to be subducted from the number given.

This is the first step of this work.

1
2197 (1

After I triple the Quotient found out, (that is to say, I multiply it by 3:) this triple representeth the three *sides* (joyntly taken together) of the three lesser *solids* marked with *C*: I place the tripled number under 9. Again, I multiply the Quotient square-wise, and triple the Product, which

maketh likewise 3. This Product resembleth the three *square sides* (taken joyntly together) of the three greater *solids*, marked with *D*. I place the *Product* a degree lower towards the left hand, underneath 1. With it I divide 11, which written above it, the Quotient is 3. This Segment or Quotient 3, being multiplied by 3 the *Divisor*, maketh 9, which in respect of the place wherein it standeth is 900, and representeth the three greater *solids* marked with *D*, taken joyntly together. Furthermore, the same Quotient being multiplied square-wise maketh 9, and multiplied afterward by the triple number standing under 9, it maketh 27, which in respect of the place wherein it standeth is 270, and representeth the 3 lesser *solids* marked with *C*. Last of all, the same Quotient multiplied cubically breedeth the lesser *Cube B*. These 3 Products therefore being added together, and the total subducted from the numbers standing over it, there remaineth nothing, which importeth the given number is a *Cube*.

The example is as you see.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus :	900 *The 3 greater Parallelipipedons.*
27	270 *The lesser Parallelipipedons.*
27	27 *The lesser Cube.*
2197	2197

The matter may be explained by many examples.

The second example of the Cubick root.

Let the side of the given *Cube* 16387064 be sought out, contrive it therefore (as it were) into certain periods

periods with points. Then first of all search out the *side* of the *Cube* next to the left hand. But forasmuch as 16 is no *Cube*, take 2 the *side* of the next *Cube* under it, that is to say of 8, and set it in the Quotient, and subduct 8 the *Cube* thereof from 16, there remaineth 8. The first work is not to be renewed throughout the whole number, but the rules following must be repeated as often as there are points remaining.

The first step to find out the Root 8
is in this manner. 16387064 (2
8

Moreover, triple the Quotient now found out, and the Product is 6, which is to be placed under 8, namely under the figure following the next prick towards the right hand. Then multiply the Quotient by this tripled number, (or, which is all to one purpose, square the Quotient, and then triple the Product) it maketh 12; set that number in a lower place one degree nearer the left hand, and make it the Divisor: divide 83 by 12, observing this rule in chusing your Quotient, that it be no greater then that the numbers afterward produced by multiplication may not exceed the numbers standing over it. So that here you shall take 1 in 8 but 5 times. Afterward by this number 5 multiply the Divisor 12, and by the *square* of 5 multiply the tripled number 6, and last of all multiply 5 cubically: so shall you produce three numbers, namely 60, 150, 125, to be described in such sort as you see. These numbers added together, and subducted from 8387, the remainder is 762.

The second step to find out the Root *in this manner.*

8762
16387064 (25
6
12
60
150
125
7625

And because there is yet one point remaining, this last manner of Division must be wrought again.

First, therefore, I triple the Quotient, the Product is 75, which must be so placed, that the first figure thereof, namely 5, may stand under 6, the second under the 0. Again, multiply the Quotient by this tripled number, (or, which is all one, square the Quotient, and triple the Product) it maketh 1875, which must be the *Divisor*, whose first figure, namely 5, must be placed under 7, the last figure of the tripled number. Then seeing that 1 may be contained in 7 many times, but I can take it but 4 times; I set 4 in the Quotient, and multiply the *Divisor* by 4, the Product is 7500. Afterward I square 4, it maketh 16, which I multiply by the tripled number 75, the Product is 1200. Last of all, I multiply 4 cubically, it maketh 64. These Products added all together make 762064, which number being subducted from the *Cube* given, there remaineth nothing, whereby I gather that the number given is exactly cubicall.

The

The third step to find out the side *is in this manner.*

762
1887064 (254
75
1875
7500
1200
64
762064

Behold also the Example following.

614125000 (850

The third example of the Cubick root.

Another manner of working.

Hitherto the Princely high-way to find out the *side* of the *Cube* hath been declared.

But there are moreover certain other waies also bending thereto, and leaning to the same principles, whereof this is one.

Having found out in the Table of simple *Cubes* the first figure representing the *side* of the *Cube* contained in the number standing under the first point on the left hand, set it in the Quotient, and subduct the particular *Cube* of that figure as you did before: then square that figure, and triple that square: the Product shall be the Divisor; the first figure whereof shall be set under that figure which is on the right-hand next of all to the point (now examined) before going.

The second form.

See how many times the Divisor is contained in the number written over it, and multiply the Divisor in the Quotient, and subduct the Product from the Dividend. Yet here you must take heed that you take not a greater Quotient then that the Products made after-

afterward thereby may be ſubducted from the number given.

The ſubduction being done, triple the firſt figure which was ſet in the Quotient, and adde to the triple the laſt number which was ſet in the Quotient on the right hand of the Product.

This totall multiplied by the *ſquare* of the figure laſt found out, ſet down the Product ſo that the firſt figure thereof toward the right hand may ſtand under the point next before going on the ſame hand, and finally ſubduct the ſame from the number given.

The fourth example of the Cubick root.

As in 804357, the particular *Cube*, namely 729, being taken from the number ſtanding under the laſt period upon the left hand, there remaineth 75357; the ſide of that particular *Cube* being 9, I ſet in the Quotient. Then I ſquare that *ſide*, it maketh 81, and triple the *ſquare*, the Product 243 is my Diviſor, which I ſet under the given number; ſo that 3 may ſtand under 3: with this Diviſor divide the number ſtanding over it, you ſhall find 2 to be contained in 7 three times. Therefore I ſet 3 in the Quotient, and multiply the Diviſor by it, the Product is 729, which being ſubducted from 753, the remainder is 24.

The Induction is thus.

2

~~7~~~~5~~~~4~~

1~~0~~~~4~~~~3~~57 (93

243

~~7~~~~2~~~~9~~

Moreover I triple 9, the product is 27, by which on the right hand I ſet 3 the Quotient laſt found out, the total is 273.

This

This Number I multiply by 9, the *square* of the Quotient last found out, the Product shall be 2457; which being subducted from the superiour number, there remaineth nothing.

The Induction is thus.

~~2~~~~4~~
8~~0~~~~4~~~~3~~~~5~~~~7~~ (93
732
9
2457

Another manner.

THE *self-same work may be dispatched another way, a little differing from the former, in this manner.*

The figure in the Quotient being found out by subducting the particular *Cube*, and also the second figure in the Quotient being found by division, let the totall Quotient be tripled, and let the tripled number be multiplied by the former figure in the Quotient. Then let the Product be multiplied again by the latter figure found out, and let a Cypher be set on the right hand of that Product. Last of all, let the *Cube* of the latter figure found out be added to this Product, and let the total summe be subducted from the number given. As in 373248. The third form.

The

The first induction is in this manner.

30
3̇7̸3248̇ (7

343

The fifth example.

Moreover, I square the *side* found out, it maketh 49, and triple the square, the Product is 147, which shall be the Divisor; by this I divide 302, the number written over it, the Quotient is 2. Now I triple the total Quotient 72, it maketh 216, and multiply this triple by 7, the former figure in the Quotient, the Product is 1512. I multiply this Product also by 2, the latter figure of the Quotient, and set a Cypher on the right hand of it, so as it maketh 30240. Unto this number last of all I adde 8, the *Cube* of the latter figure found out, the totall is 30248; which being subducted from this figure above it, there remaineth nothing.

The second induction is thus.

30
3̇7̸3248̇ (72

147

30248

All the points of the number given being examined, if any thing remain, it signifieth the number given is no *Cube*: wherefore the true *side* of it cannot be exactly given in numbers. Yet if it please you to sift out the nearest *side* that may be by the first kind of reduction of mixt numbers, you shall reduce the

number

number given unto a cubicall fraction of a greater denomination, and afterward seek out the cubicall *side* of that fraction.

For example sake, because 120 is no *Cube*, therefore let it be reduced into sixty cubicall parts, after this manner. Multiply 60 cubically in it self, it maketh 216000; by this being taken for the Denominator of the fraction multiply 120 the number given, the Product is 25920000, whose cubicall *side* is $\frac{295}{60}$, that is $4\frac{11}{12}$, the nearest to the true *side* that can be.

To find the nearest Cubick root in a surd number.

For the extraction of all sorts of Roots, *the Tables of Logarithms set forth by Mr* Briggs *are most excellent and ready.*

FINIS.

A Table *of* Board *and* Timber-measure *more perfect then ever hath been made, shewing also the Squares between* 4 *and* 37 *from quarter to quarter, calculated by* Robert Hartwell.

Board-measure.	Inches and quarters.	Squares.	Timber-measure.	Board-measure.	Inches & quarters.	Squares.	Timber-measure.
36.0.0	4	16	108.0.0	16.0.0	9	81	21.3.3
33.8.8	1	18	96. 0.0	15.5.7	1	85	20.3.3
32.0.0	2	20	86. 4.0	15.1.6	2	90	19.2.0
30.3.1	3	22	78. 5.4	14.7.7	3	95	18.1.9
28.8.0	5	25	69. 1.2	14.4.0	10	100	17.2.8
27.4.3	1	27	64. 0.0	14.0.2	1	105	16.4.6
26.1.8	2	30	57. 6.0	13.7.1	2	110	15.7.1
25.0.4	3	33	52. 3.6	13.3.9	3	115	15.0.3
24.0.0	6	36	48. 0.0	13.0.9	11	121	14.2.8
23.0.4	1	39	44. 3.0	12.8.0	1	126	13.7.1
22.1.5	2	42	41. 1.4	12.5.2	2	132	13.0.9
21.3.3	3	45	38. 4.0	12.2.5	3	138	12.5.2
20.5.7	7	49	35. 2.6	12.0.0	12	144	12.0.0
19.8.6	1	52	33. 2.3	11.7.5	1	150	11.5.2
19.2.0	2	56	30. 8.6	11.5.2	2	156	11.0.8
18.5.8	3	60	28. 8.0	11.2.9	3	162	10.6.7
18.0.0	8	64	27. 0.0	11.0.7	13	169	10.2.2
17.4.5	1	68	25. 4.1	10.8.7	1	175	9.8.7
16.9.4	2	72	24. 0.0	10.6.7	2	182	9.4.9
16.4.6	3	76	22. 7.4	10.4.7	3	189	9.1.4

Board-

Board-measure.	Inches and quarters.	Squares.	Timber-measure.	Board-measure.	Inches and quarters.	Squares.	Timber-measure.
10.2.8	14	169	8.8.1	6.8.6	21	441	3.9.2
10.1.0	1	203	8.5.1	6.7.7	1	451	3.8.3
9.9.3	2	210	8.2.3	6.6.9	2	462	3.7.4
9.7.6	3	217	7.9.6	6.6.2	3	473	3.6.5
9.6.0	15	225	7.6.8	6.5.4	22	484	3.5.7
9.4.4	1	232	7.4.4	6.4.7	1	495	3.4.9
9.2.9	2	240	7.2.0	6.4.0	2	506	3.4.1
9.1.4	3	248	6.9.7	6.3.3	3	517	3.3.4
9.0.0	16	256	6.7.5	6.2.6	23	529	3.2.7
8.8.6	1	264	6.5.4	6.1.9	1	540	3.2.0
8.7.3	2	272	6.3.5	6.1.2	2	552	3.1.3
8.6.0	3	280	6.1.6	6.0.6	3	564	3.0.6
8.4.7	17	289	5.9.8	6.0.0	24	576	3.0.0
8.3.5	1	297	5.8.1	5.9.4	1	588	2.9.4
8.2.3	2	306	5.6.4	5.8.8	2	600	2.8.8
8.1.1	3	315	5.4.8	5.8.2	3	612	2.8.2
8.0.0	18	324	5.3.3	5.7.6	25	625	2.7.6
7.8.9	1	333	5.1.9	5.7.0	1	637	2.7.1
7.7.8	2	342	5.0.5	5.6.5	2	650	2.6.5
7.6.8	3	351	4.9.2	5.5.9	3	662	2.6.1
7.5.8	19	361	4.7.9	5.5.4	26	676	2.5.5
7.4.8	1	370	4.6.7	5.4.8	1	689	2.5.1
7.3.9	2	380	4.5.5	5.4.3	2	702	2.4.6
7.2.9	3	390	4.4.3	5.3.8	3	715	2.4.2
7.2.0	20	400	4.3.2	5.3.3	27	729	2.3.7
7.1.1	1	410	4.2.1	5.2.8	1	742	2.3.2
7.0.2	2	420	4.1.1	5.2.3	2	756	2.2.8
6.9.4	3	431	4.0.1	5.1.9	3	767	2.2.5

Board-

Board-measure.	Inches and quarters.	Squares.	Timber-measure.	Board-measure.	Inches & quarters.	Squares.	Timber-measure.
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.2	4.3.0	2	1122	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1139	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1173	1.4.7
4.8.8	2	870	1.9.8	4.1.8	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1208	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1242	1.3.9
4.7.2	2	930	1.8.6	4.0.5	2	1260	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1278	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	976	1.7.7	3.9.8	1	1313	1.3.1
4.5.7	2	992	1.7.4	3.9.4	2	1331	1.2.9
4.5.3	3	1008	1.7.1	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	37	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.3	2	1056	1.6.4	3.8.4	2	1406	1.2.2
4.4.0	3	1072	1.6.1	3.8.1	3	1425	1.2.1

The use of this former Table.

IF upon a *Scale* or *Ruler* you divide one inch into ten equal parts or primes, and again by *Diagonals* and parallel-lines you subdivide each of them into ten equal parts or seconds with your Compasses, you may take a more exact running measure for *Board* and *Timber* then by any other means whatsoever, and so

ſo place the ſame, or this *Table* if you will, upon any *Ruler*.

Alſo by means of the Columns of Squares you may readily find a Square equal to any Parallelipipedon, or piece of timber which is thicker then it is broad. As for example, ſuppoſe a piece of timber to be ten inches thick, and 9 inches broad; if I multiply thoſe ſides one by another, they will produce 290: then I ſeek the column of Squares for 290, which I find not, but I find 289, the neareſt number to 290, to ſtand againſt 17: therefore I ſay, 17 inches *fere* will make a Square equal to ſuch an unlike-ſquared piece: then looking in the column of *Timber-meaſure* againſt 17, you ſhall find that 5 inches, 9 primes or $\frac{9}{10}$, and 8 ſeconds or $\frac{28}{100}$ of an inch in length, of that piece, will make a foot of timber.

Likewiſe for *Board-meaſure*, you may find how much in length or breadth of board muſt be in one foot.

By the like means, ſuppoſe for example that a board appointed to be meaſured is 15 inches $\frac{3}{4}$ broad, if I deſire to know how much in length thereof will make a foot, I ſeek in the columns that ſtand under unites and quarters for 15 $\frac{3}{4}$, and alſo againſt the ſame in the column under the title of *Board-meaſure*, where I find 9 inches, 1 prime or tenth of an inch, and 4 ſeconds or hundreds of an inch, will make a foot at that breadth. The like may be practiſed for any other breadth of board whatſoever.

Cer-

Certain Tables *ſhewing the* Intereſt *of any ſumme of mony whatſoever unto* 40 *years ; how much* Annuities *reſpited or forborn come unto ; and for* buying *or* ſelling *of* Annuities *for the ſaid time, and alſo the ſame in* reverſion *after any number of years unto* 30, *what they may be worth in preſent ready mony. By* R. C. *and now diligently corrected and amended by* Robert Hartwell.

Definition of Intereſt.

PRincipal *is the ſumme from which the* Intereſt *is reckoned.*

2 Intereſt *is the ſumme reckoned for the lending or forbearance of the* Principal *for any term or time.*

3 Intereſt ſimple *is that which is counted from the* Principal onely.

4 Intereſt compound *is that which is counted from the* Principal *together with the* Arrerage.

5 Intereſt profitable *is that which is added to the* Principal.

6 Intereſt damageable *is that which is to be ſubtracted from the* Principal.

The uſe *per annum* of	1 li.	is	2 s.
	10 s.		12 d.
	5 s.		6 d.
	2 s. 6d.		3 d.
	1 s.		1 d. $\frac{1}{5}$ of a peny.

A Table

A Table *shewing what* 1 li. *with interest, and interest upon interest, after* 10 *in the* 100, *comes to every year under* 41 *years, as followeth.*

years.	li.	s.	d.	li.	s.	d.	years.
1	1	2	0	7	8	0	21
2	1	4	2	8	0	9	22
3	1	6	7	8	19	1	23
4	1	9	3	9	16	11	24
5	1	12	1	10	16	8	25
6	1	15	5	11	18	4	26
7	1	18	11	13	2	2	27
8	2	2	10	14	8	5	28
9	2	7	1	15	17	3	29
10	2	11	10	17	8	11	30
11	2	17	1	19	3	10	31
12	3	2	9	21	2	3	32
13	3	9	0	23	4	6	33
14	3	15	11	25	10	11	34
15	4	3	6	28	2	0	35
16	4	11	10	30	18	0	36
17	5	1	1	34	0	0	37
18	5	11	2	37	8	1	38
19	6	2	3	41	2	10	39
20	6	14	6	45	5	2	40

By the former *Table* if you desire to know what 1 li. cometh to with interest, and interest upon interest, after 10 in the 100, for any number of years unto 40, look in the row or margent over which is written *years*, and against it on the right hand close unto it in the row or margent of *pounds*, *shillings* and *pence*, (which is titled thus, li. s. d.) you shall finde your desire.

Example.

I would know what 1 li. *with interest, and interest upon interest, cometh to in* 7 *years.*

I look in the row of *years* for the number 7: and against it on the right hand I finde 1 li. 18 s. 11 d. Also what it cometh unto in 13 years. I seek among the *years* for 13, and against it I finde 3 li. 9 s.

Again, for 21 years; I look for 21 among the *years*, and I finde 7 li. 8 s. 0 d. But if you would know for a greater summe then 1 li, then multiply your summe by that summe of 1 li. in the *Table* for any of those years, and you shall easily finde it. As thus, I would know what 10 li. cometh to for 7 years with interest, &c. I see that 1 li. cometh to 1 li. 18 s. 11 d. in that time. Then say I, that 10 li. must be ten times as much in that space, which is 19 li. 9 s. 2 d. Also of 10 li. in 13 years; I see that 1 li. in that time cometh unto 3 li. 9 s. Then must 10 li. be ten times as much in that space, which is 34 li. 10 s. Also what 10 li. cometh to in 21 years: I finde first, that 1 li. in that space cometh to 7 li. 8 s. Then I say, 10 must be 10 times as much, which is 74 li. Lastly, I would know what 100 li. cometh to in 7 years. I see it must be 100 times as much as 1 li. cometh to in that

that space, which is 194 li. 11 s. 8 d. Hereby you see the common saying is not true, that 100 li. doth double it self in 7 years, for it wants thereof 5 li. 8 s. 4 d. But in 8 years 100 li. cometh to 210 li. 8 s. 4 d, which you see is more then double it self by 10 li. 8 s. 4 d. And in this sort may any that can but cast with Counters, or indeed by memory, finde the increase of any summe whatsoever for any of the number of years in the foresaid *Table*, after they have found what 1 li. cometh unto for that time, as before is specified.

A Table shewing, if 1 li. Annuity *to endure for any number of years under* 41 *be all* respited *or forborn untill the last payment grow due, and then all be received together with interest, and interest upon interest, after* 10 *in the* 100 per annum, *what they will amount unto by any of the said number of years, as followeth.*

years.	l.	s.	d.	li.	s.	d.	years.
1	1	0	0	64	0	0	21
2	2	2	0	71	8	0	22
3	3	6	2	79	10	10	23
4	4	10	10	88	9	11	24
5	6	2	1	98	6	11	25
6	7	14	3	109	3	7	26
7	9	9	8	121	1	11	27
8	11	8	8	134	4	2	28
9	13	11	7	148	12	7	29
10	15	18	8	164	9	10	30
11	18	10	7	181	18	10	31
12	21	7	8	201	2	9	32
13	24	10	5	221	5	0	33
14	27	19	5	245	9	6	34
15	31	15	6	271	0	5	35
16	35	18	11	299	2	6	36
17	40	10	10	330	0	9	37
18	45	11	11	364	0	10	38
19	51	3	2	401	8	11	39
20	57	0	6	442	11	10	40

By this *Table* you may know what any Annuity being reſpited or forborn for any number of years unto 41, with intereſt upon intereſt, after 10 in the 100, will come unto: firſt ſeeking in the *Table* what 1 li. will come unto in that time; and that being found, multiplying it by the ſumme you deſire to know.

Example.

Example.

First, I would know what 1 li. Annuity, being forborn or respited for 14 years, cometh unto.

I look in this last *Table*, (which is for the purpose) and I finde 27 li. 19 s. 5 d.

Again, what 1 li. Annuity respited for 21 years cometh to; I look in the said *Table* for 21 years, and I finde 64 li. Also the like for 1 li. for 30 years respited; I look, and finde it to be 164 li. 9 s. 10 d, as by the said *Table* may appear. Now for greater Annuities, as 30 li. *per annum*, respited or forborn, what it amounteth to in 16 years; I seek first for 1 li. in this last *Table* before for 16 years, and against it I finde 35 li. 18 s. 11 d. Then say I, that 30 li. *per annum* being respited for that time will come to 30 times as much, which is 1078 li. 7 s. 6 d. Also if there be an Annuity of 45 li. due and unpaid for 12 years, I look in the said *Table* what 1 li. cometh to, 12 years being respited, and I finde it is 21 li. 7 s. 8 d. Then I conclude that 5 li. must be 45 times as much, which is 962 li. 5 s.

Lastly, I have an Annuity of 50 li. *per annum*, which hath been behind for 16 *years*, and must be answered unto me with interest, and interest upon interest, all at one payment: what shall or ought I to receive in all at the 16 years end?

I seek what 1 li. comes unto in that time, (as before taught) and I finde 35 li. 18 s. 11 d. Then must my 50 li. *per annum*, forborn for that time, come to 50 times as much, which is 1797 li. 5 s. 10 d. And thus may you finde any other summe, great or small, for any number of years contained in the aforesaid *Table*, without the help of Arithmetick, if

you can but use your Counters, or by memory count well.

A Table *shewing, if* 1 li. Annuity (*to endure for any number of years unto* 41.) *be to be sold for* present ready money, *how much ought that ready money to be, reckoning* 10 per 100 per annum, *abating interest, and interest upon interest, as followeth.*

years.	l.	s.	d.	li.	s.	d.	years.
1	0	18	2	8	12	11	21
2	1	14	8	8	15	5	22
3	2	9	8	8	17	7	23
4	3	3	4	8	19	8	24
5	3	15	9	9	1	6	25
6	4	7	1	9	3	2	26
7	4	18	4	9	4	8	27
8	5	6	8	9	6	1	28
9	5	15	2	9	7	4	29
10	6	2	10	9	8	6	30
11	6	9	9	9	9	7	31
12	6	16	3	9	10	6	32
13	7	2	0	9	11	4	33
14	7	7	4	9	12	2	34
15	7	12	1	9	12	10	35
16	7	16	5	9	13	6	36
17	8	0	5	9	14	1	37
18	8	4	0	9	14	7	38
19	8	7	3	9	15	1	39
20	8	10	3	9	15	6	40

This

This *Table* before last specified is very necessary and commodious for all Gentlemen or others that shall have cause to buy or sell *Annuities*, or such like; for by this they shall know what they doe, whether they demand or take too little or too much after the rate of ten in the 100, by which proportion all these *Tables* are ruled.

As for example, I am to buy an Annuity of 16 li. per annum *for* 12 *years, and am demanded for it ready money* 120 li. *I would know, if I give this rate, whether I give too much or too little, according to the proportion of ten in the* 100 per annum, *&c.*

I look in the *Table* last before what 1 li. is worth for 12 years, and I finde against 12 this summe 6 li. 16 s. 3 d. Now I say that 16 li. Annuity for that time, and after that proportion, cometh to 16 times as much, which is 109 li. So that I see the party demanded of me 11 *li.* too much, after the rate of ten in the 100 *per annum*, and therefore I must draw him to a lower price, or leave it.

Again, I am offered an Annuity of 20 li. per annum *of* 14 *years for* 130 li. *I would know, if I give it, whether I give too much or too little, according to the proportion aforesaid.*

I seek first what 1 li. Annuity is worth for 14 years, and I finde in the said last *Table* 7 li. 7 s. 4 d. Then say I that the Annuity of 20 li. *per annum* will come to 20 times as much, and will be worth 147 li. 6 s. 8 d, according to the proportion before mentioned, and is more then his demand by 17 li. 6 s. 8 d. So that I see, if I accept of it, I shall have a good bargain. And thus may you know readily by looking in your *Table*, and finding what 1 li. is worth for

any time therein contained, how much any greater summe will come unto, if you multiply it by that summe of 1 li, as before is sufficiently shewed.

But suppose this, I have 300 li. *ready money, and would bestow the same for a valuable Annuity answerable thereunto according to the proportion aforesaid. I would know what Annuity to endure* 21 *years this* 300 li. *will buy.*

I look in the former *Table* what 1 li. Annuity will cost for that time, and I finde 8 li. 12 s. 11 d. Then I say by the *Rule of proportion*, if 8 li. 12 s. 11 d. will buy 1 li. Annuity for 21 years, what Annuity shall 300 li. buy or be worth for that time? I reduce the summs to the least denomination, (which is pence) and I finde 34 li. 13 s. 11 d. And after this manner (by the help of this *Rule*) may you finde all other summs for any time contained in the foresaid last *Table*.

4 Ta-

A Table *shewing what* 1 li. in reversion *for any number of years under* 31 *is worth in ready money, the buier staying untill the thing be faln in hand.*

years.	li.	s.	d.	li.	s.	d.	years.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	9	1	3	29
15	0	4	9	0	1	1	30

This last *Table* differeth from, and is contrary to the other three before mentioned: For whereas the others increased more and more, according to the number of years specified, this doth grow and diminish less and less, as the number of years increaseth. As for example,

There

There is a Tenement, the fee-simple whereof after 7 years will be worth 40 li; *what am I to give for it in ready money now, staying untill it fall in hand?*

To know this, I look in this last *Table* for 7 years, and against it I finde 10 s. 3 d. So that a thing that after 7 years will be worth 1 li, is worth now in ready money but 10 s. 3 d. Then say I, that the foresaid Tenement, which after 7 years will be worth 40 li, is now worth 40 times 10 s. 3 d, which is 20 li. 10 s.

Again, there is a Farm which after 9 *years will be worth the Fee-simple* 420 li: *what is it now worth in ready money, staying untill it fall in hand?*

I look in the said *Table* what 1 li. is worth in reversion after 9 years, and I finde 8 s. 5 d. Then say I, that the Farm of 420 li. so long in Reversion will be now worth in ready money 420 times as much, which is 176 li. 15 s.

Lastly, there is a Lordship to be sold, the Fee-simple whereof after 14 *years will be worth* 7500 li: *I would know what the same is now worth in ready money for the Reversion.*

I look in this last *Table* for 14 years, and against it I finde 5 s. 3 d: so much 1 li. is worth in reversion after 14 years. Then say I, that 7500 li. is worth no more in reversion for that time then 7500 times 5 s. 3 d, which is 1968 li. 15 s. And after this manner may you finde out any other summe whatsoever. And though some men of their own experience can aim (as they think) near enough the mark to serve their own turns: yet I dare undertake they shall never so exactly doe it, nor justifie what they doe, as if they did it by Art.

FINIS.

New *Tables of Interest* at 8 *per centum per annum*, exactly calculated for 30 years, by *Robert Hartwel*, with necessary *Questions* for the use of them.

The first Table *expressing the increase of one pound principall put out and forborn for any number of years under* 31, *at* 8 per centum per annum.

years	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	6	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	0	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	1	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The

The deſcription and uſe of the Tables *of* Intereſt *at* 8 per centum per annum, *being profitable.*

The firſt of them.

THeſe *Tables* conſiſt of four Columns; in the firſt and fourth whereof is written over the head *years*, and under the firſt the number of years deſcending from 1 to 15; likewiſe in the fourth the number of years deſcending from 16 to 30; and againſt every year in the ſecond Column, toward the right hand, the *pounds, ſhillings, pence* and *farthings*, which one pound or 20 s. principal will amount unto, being put forth and forborn for the number of years ſet againſt it; (but the *pounds, ſhillings, pence*, &c. in the third Column, belong to the years ſet in the laſt Column.)

1. Example.

Let it be required what one pound, or 20 *ſhillings, being put forth and forborn for* 12 *years, ariſeth to at* 8 per centum per annum, *intereſt upon intereſt.*

Seek in the firſt Column, under the title of *years*, for 12, the number of years propoſed in the queſtion, and right againſt it toward the right hand in the ſecond Column you ſhall find 2 li—10 s—4 d—1 q; which is the principal and increaſe thereof due for the time required.

2. Example.

If 100 li. *be put forth for* 17 *years according to the ſame intereſt, I demand what it will amount to in that time.*

Look in the Column under the title of *years* for 17, and right againſt it toward the left hand in the *Table* is found 3 li. —14 s.—0 d.—0 q; which is the increaſe

increase of 1 li: by which you may thus gather the increase of 100 li. or any other summe; a hundred times 3 li. is 300 li, then 100 times 14 shillings is 70 li; both which added together do make 370 li.—0 s.—0 d, which is the increase of 100 li. put forth and forborn 17 years, the solution to the question.

li.	s.	d.	q.
300	0	0	0
70	0	0	0
370	0	0	0

3. Example.

Suppose 60 li. be put forth for 19 years according to that rate, what will it increase to in that time?

Seek 19 under the title of *years*, and against it toward the left hand is found 4 li.—6 s.—3 d.—3 q: now say, 60 times 4 li. is 240, and 60 times 6 shillings is 360 shillings, or 18 li, and 60 times 3 d. is 180 d, or 15 shillings, and 60 times 3 farthings is 3 shillings 9 d; all added together make 258 li. 18 s. 9 d. the increase thereof demanded.

li.	s.	d.	q.
240	0	0	0
18	0	0	0
0	15	0	0
0	3	9	0
258	18	9	0

The

The second Table *shewing what one pound Annuity or yearly rent is worth at the end of any number of years under 31, being forborn, at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	1	0	0	0	30	6	5	3	16
2	2	1	7	0	33	15	0	0	17
3	3	4	6	0	37	9	0	0	18
4	4	10	1	1	41	8	11	[illegible]	19
5	5	12	2	3	45	15	2	3	20
6	7	6	8	0	50	8	5	2	21
7	8	18	5	1	55	9	1	2	22
8	10	12	8	3	60	17	10	1	23
9	12	9	9	0	66	15	3	2	24
10	14	9	8	3	73	2	3	[illegible]	25
11	16	12	10	3	79	19	1	0	26
12	18	19	0	2	87	7	8	0	27
13	21	9	10	3	95	6	1	2	28
14	24	4	3	2	103	19	8	3	29
15	27	3	0	2	113	5	7	3	30

The use of the second Table, *whose disposition is altogether like the former, according to the title thereof, being profitable.*

1. Example.

1. Example.

There is a Lease worth 28 li. per annum, *to endure 14 years. I demand what it will rise unto at the end of those years, being all forborn with the interest upon interest at the rate prescribed in this* Table.

Look in the third *Table* for 14 years, against which, toward the right hand, you shall finde 24 li. —4 s.—3 d.—2 q. Now multiply 28 li. by 24, there ariseth 672 li: then 28 li. by 4 s. yieldeth 112 s, or 5 li. 12 s.

Again, 28 li. by 3d. produceth 84 d, or 7 s. Finally, 28. by 2 farthings yieldeth 56 farthings, or 1 s. 2 d. All which added together make 678 li. 0 s. 2 d. to be received at the end of 14 years, the same Rent or Annuity being respited.

li.	s.	d.	q.
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

2. Example.

If 60 li. *yearly Rent or Annuity be forborn* 20 *years, I demand how much it will increase at the end of the said term.*

In the Table I finde that 1 pound in 20 years will arise to 45 li.—15s.—2d—3q; therefore 60 li. in the like term will yield 60 times as much: which I will reckon thus; 60 times 45 li. is 2700 li, 60 times 15 s. is 900 s, or 45 l, 60 times

li.	s.	d.	q.
2700	0	0	0
45	0	0	0
	10	0	0
	3	9	0
2745	13	9	0

times 2 d. is 120 d, or 10 s, last of all, 60 times 3 q. is 180 farthings, or 3 s.—9 d: all which together amount unto 2745 li.—13 s.—9 d, the value thereof to be received at the end of the term.

3. Example.

The yearly rent of 6 li.—13 s.—4 d *being behind and unpaid the space of 7 years, at the end of which term the Tenant is compelled to pay the same, with the interest thereof, according to the above-named rate; I demand what the payment ought to be.*

The increase of 1 li. yearly rent answering to 7 years is 8 li. 18 s. 5 d. 1 q, which for 6 li. rent taken 6 times ariseth to 53 li. 10 s. 7 d. 2 q. Now because 13 s. 4 d. is two third parts of 1 li, therefore I take ⅔ of 8 li. 18 s. 5 d. 1 q, which is the the increase of 1 li. forborn for 7 years, that is 5 li. 18 s. 11 d. 1 q, which together make 59 li. 9 s. 7 d. 0 q, the summe to be received, as was required.

li.	s.	d.	q.
53	10	7	2
5	18	11	2
59	9	7	0

The

The third Table, *declaring what one pound due at the end of any number of years under* 31 *is worth ready mony at* 8 per centum per annum.

years.	li.	s.	d.	q.
1	0	18	7	0
2	0	17	10	3
3	0	15	1	2
4	0	14	8	1
5	0	13	7	1
6	0	12	7	0
7	0	11	8	1
8	0	10	9	2
9	0	10	0	0
10	0	9	3	0
11	0	8	6	3
12	0	7	11	1
13	0	7	4	0
14	0	6	9	2
15	0	6	3	3

li.	s.	d.	q.	years.
0	5	10	0	16
0	5	4	3	17
0	5	0	0	18
0	4	7	2	19
0	4	3	1	20
0	3	11	2	21
0	3	8	0	22
0	3	4	3	23
0	3	1	3	24
0	2	11	0	25
0	2	8	1	26
0	2	5	0	27
0	2	2	3	28
0	2	1	3	29
0	1	11	3	30

This third *Table* is disposed as the first, the use, according to the title thereof, being damageable.

1. Example.

Suppose there is 750 *li. due to be paid at the end of* 9 *years, the Creditor would sell this Debt for present mony; what ought that mony to be at the rate described in the* Table?

Seek in this third *Table* for 9 years at the left side of the *Table*, and right against it towards the right hand you shall find 10 shillings, which multiplied or taken 750 times yieldeth 7500 shillings, which is 375 li, the value of that Debt in present mony.

2. Example.

There is a Lease worth 500 li. *after the end of* 7 *years; what is it worth present mony, according to the rate described in the* Table, *staying till it fall?*

I seek in the *Table* for the 7 years, and right against it I find 11 s.—8 d: now I multiply 500 by 11, it yieldeth 5500 shillings, or 275 li: then 500 times 8d. maketh 4000 d, which is 16 li. 13 s. 4, which added together is 291 li. 13 s. 4 d, the value of the Lease to be paid before it fall in hand.

li.	s.	d.	q.
275	0	0	0
16	13	4	0
291	13	4	0

The fourth Table, *expressing what one pound yearly Rent or Annuity for any number of years not exceeding* 30 *is worth ready mony at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	9	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	16	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The fourth *Table* is diſpoſed altogether as the former, the uſe thereof in like ſort being damageable.

1. Example.

There is an Annuity or Rent of 20 s. per annum *to endure* 25 *years; it is required what it is worth ready mony.*

Look in the *Table* for 25 years, and right againſt it you ſhall find 10 li. 13 s. 5 d. 3 q, which is the ſolution.

2. Example.

What is the Leaſe of certain Land valued at 140 li. per annum *to begin preſently, and endure* 18 *years, worth ready mony?*

Search in the *Table* for 18 years, the term named in the Queſtion, and right againſt it toward the left hand you ſhall find 9 li. 7 s. 5 d. 1 q, which expreſſeth that one pound Rent to be bought for that term is worth ſo much; therefore that ſumme 140 times is the value required. Now 140 times 9 li. is 1260 li, and 140 times 7 s. is 980 s, or 49 li; likewiſe 140 times 5 d. is 700 d, or 2 li. 18 s. 4 d, and 140 farthings is 2 s. 11 d: all which added together make 1312 li. 1 s. 3 d. for the value of the ſaid Leaſe, paying no Rent.

li.	s.	d.
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3. Example.

3. Example.

A Lease taken for 21 *years at* 13 li. 6 s. 8 d. per annum, *after* 5 *years expired of which, the Tenant is desirous to give a Fine, and bring the Rent down to* 8 li. per annum *for the rest of the term; the demand is, what fine is to be paid?*

Subtract 5 years from 21, the remainer 16 is the time unexpired; likewise from the present Rent abate 8 li, the rest will be 5 li. 6 s. 8 d. Now the drift of the question is, what 5 li. 6 s. 8 d. yearly Rent or Annuity to endure 16 years is worth present mony.

The value of 1 li. Rent or Annuity answering to 16 years is 8 li. 17 s. 0 d. 1 q. Now 5 times 8 li. is 40 li, and 5 times 17 s. 4 li. 5 s, and 5 times one farthing is 1 d. 1 q; and because 6 s. 8 d. is $\frac{1}{3}$ of 1 li, I take $\frac{1}{3}$ of 8 li. 17 s. 0 d. 1 q, which is 2 li. 19 s. 0 d, all which added together make 47 li. 4 s. 0 d. 1 q, which is the Fine that ought to be paid to bring the Rent to 8 li. *per annum.*

li.	s.	d.	q.
40	0	0	0
4	5	0	0
2	19	0	1
47	4	0	1

The fourth *Table* is disposed altogether as the former, the use thereof in like sort being damageable.

1. Example.

There is an Annuity or Rent of 20 s. per annum *to endure* 25 *years; it is required what it is worth ready mony.*

Look in the *Table* for 25 years, and right against it you shall find 10 li. 13 s. 5 d. 3 q, which is the solution.

2. Example.

What is the Lease of certain Land valued at 140 li. per annum *to begin presently, and endure* 18 *years, worth ready mony?*

Search in the *Table* for 18 years, the term named in the Question, and right against it toward the left hand you shall find 9 li. 7 s. 5 d. 1 q, which expresseth that one pound Rent to be bought for that term is worth so much; therefore that summe 140 times is the value required. Now 140 times 9 li. is 1260 li, and 140 times 7 s. is 980 s, or 49 li; likewise 140 times 5 d. is 700 d, or 2 li. 18 s. 4 d, and 140 farthings is 2 s. 11 d: all which added together make 1312 li. 1 s. 3 d. for the value of the said Lease, paying no Rent.

li.	s.	d.
1260	0	0
49	0	0
2	18	4
	2	11
1312	1	3

3. Example.

3. Example.

A Lease taken for 21 *years at* 13 li. 6 s. 8 d. per annum, *after* 5 *years expired of which, the Tenant is desirous to give a Fine, and bring the Rent down to* 8 li. per annum *for the rest of the term; the demand is, what fine is to be paid?*

Subtract 5 years from 21, the remainer 16 is the time unexpired; likewise from the present Rent abate 8 li, the rest will be 5 li. 6 s. 8 d. Now the drift of the question is, what 5 li. 6 s. 8 d. yearly Rent or Annuity to endure 16 years is worth present mony.

The value of 1 li. Rent or Annuity answering to 16 years is 8 li. 17 s. 0 d. 1 q. Now 5 times 8 li. is 40 li, and 5 times 17 s. 4 li. 5 s, and 5 times one farthing is 1 d. 1 q; and because 6 s. 8 d. is $\frac{1}{3}$ of 1 li, I take $\frac{1}{3}$ of 8 li. 17 s. 0 d. 1 q, which is 2 li. 19 s. 0 d, all which added together make 47 li. 4 s. 0 d. 1 q, which is the Fine that ought to be paid to bring the Rent to 8 li. *per annum*.

li.	s.	d.	q.
40	0	0	0
4	5	0	0
2	19	0	1
47	4	0	1

The fifth Table, *declaring what yearly Rent or Annuity of one pound ready mony will purchase for any number of years under* 31 *at* 8 per centum per annum.

years.	li.	s.	d.	q.	li.	s.	d.	q.	years.
1	0	18	7	0	0	2	9	3	16
2	0	14	c	2	0	2	8	3	17
3	0	9	8	2	0	2	8	0	18
4	0	7	6	0	0	2	7	0	19
5	0	6	3	0	0	2	6	2	20
6	0	5	4	3	0	2	5	3	21
7	0	4	9	2	0	2	5	1	22
8	0	4	4	0	0	2	4	3	23
9	0	4	0	0	0	2	4	1	24
10	0	3	8	2	0	2	4	0	25
11	0	3	6	0	0	2	0	3	26
12	0	3	3	3	0	2	3	1	27
13	0	3	1	3	0	2	3	c	28
14	0	3	0	1	0	2	2	3	29
15	0	2	11	0	0	2	2	2	30

In

In the fifth *Table* the Numbers and Columns are all disposed as the former *Tables*, and it needeth no farther explanation but onely Examples.

1. Example.

The Table *declareth at first sight what yearly Rent or Annuity one pound ready mony will purchase for any term in the* Table *expressed.*

But if the ready mony be above one pound, then if any value or Rent set down in this *Table* be multiplied by the number belonging to the years in question, the Product will shew what yearly Rent or Annuity that ready mony will purchase for the time proposed.

2. Example.

A certain man hath 750 li. *to purchase an Annuity to endure* 27 *years, so as it may yield him the like profit as if it were put out according to the rate in the* Table *expressed; it is required what that Annuity ought to be.*

Because the Annuity is to endure 27 years, seek out the value or Rent set against 27 years in this fifth *Table*, which is 2 s——3 d——1 q. Now this being the Annuity which 20 s. ready mony will purchase for that term, it must be multiplied by 750 li. as followeth: because 2 s. is the tenth part of 20 s. therefore take the tenth part of 750 li. which is 75 li,

li.	s.	d.	q.
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	2

which set first down; then 750 times 3 d. is 9 li. 7 s. 6 d, which set under the former; last of all 750 farthings is 15 s. 7 d. 2 q. All which added together produce 85 li. 3 s. 1 d. 2 q, the yearly Annuity required.

Deo soli omnis laus, honor & gloria tribuatur.

AMEN.

FINIS.

Compendious Tables *of Interest-mony forborn any number of Days, Weeks, Months, or Years under* 22, *exactly calculated at* 6 li. per cent. per annum.

Days.	Numbers.
I	100016
II	100032
III	100048
IV	100064
V	100080
VI	100096
Weeks.	
I	100112
II	100224
III	100336
Months.	
I	100487
II	100976
III	101467
IV	101961
V	102458
VI	102956
VII	103457
VIII	103961
IX	104467
X	104976
XI	105486

Years.	Numbers.
I	106000
II	112360
III	119102
IV	126248
V	133822
VI	141852
VII	150363
VIII	159385
IX	168948
X	179085
XI	189830
XII	201220
XIII	213293
XIV	226090
XV	239656
XVI	254035
XVII	269277
XVIII	285434
XIX	302560
XX	320714
XXI	339956

The

The use of this Table *of proportionate numbers; the* Radius 100000.

This *Table* is divided into 4 columns, and the first in 3 parts descending from 1 *day* unto 6, secondly, from 1 *week* to three, thirdly, from one *month* proceeding to 11: the third column comprehends the *years* inclusive from 1 to 21, as by their numeral letters doth appear: the second and fourth columns are proportional numbers in arithmetical characters, respectively answering the time or times compounded of Days, Weeks, Months, and Years to 21, as by examples shall be evidenced.

An explanation.

What will 50 li. amount unto, if forborn 21 years, the Interest allowed at 6 li. *per cent. per An.*

Look in the column of *years* for the term of forbearance propounded, in this 21, whose *Decimall* is 339956, which multiplied by the principal forborn, *viz.* 50 li, the Product will be 169|97800, to be divided by the *Radius* 100000; therefore cut off 5 places (as in the margent) from the right hand, and there will appear 169 li: the remainder 97800 multiply by 20 s. or by 2, and annex a cypher at the right hand, the Product is 19|56000; cut off 5 places as before: on the left hand you will find 19 s, the remainder 56000, which increased by 12 d. produceth

	339956
	50
li.	169|97800
s.	19|56000
	112000
d.	6|72000
q.	2|88|000
	100

ceth 6|72000: strike off 5 places, you will discover the 6; remainder 72000 multiplied by 4 q, work as before, you shall find 2 $\frac{22}{25}$. q. So 50 l. forborn 21 years at 6 l. *per cent. per Ann.* will amount unto the summe of 169 l. 19 s. 6 d. 2 $\frac{22}{25}$ q, the demand solv'd.

If a question consists of several denominations, *viz.* pounds, shillings, pence, see the rules of Practice in the propositions of Interest: if mixt, as in respect of time, *viz.* Months, Weeks, Days, &c. find first the increase for the term of years, and multiply all that by the proportional number found in the former *Table*, for the parts of a year, the result will answer your expectation. For *Decimal Tables* (or what these Authors have not treated of) I referre the Reader unto my Books of *Natural and Artificial Arithmetick*, or my *Scales of Commerce and Trade*, this place being not convenient to enlarge my self, or build upon another's ground; neither will I lessen their works (by me now corrected) whereby to magnifie my own, but have inserted this *Table*: and here I subscribe my name; and hoping my labours may give you ease, I rest,

Your friend, although unknown,

Thomas Willsford.

To every young Arithmetician, or Practitioner in Numbers, who shall peruse these Books.

Candid Reader,

YOU will here receive an old Arithmetick *from the authority of* Record, *entail'd upon the People, ratified and signed by the approbation of Time, with multitudes of surviving Witnesses; so expect not from me to confer Encomiums, when I had written nothing here, but that through mistakes and oversights of former Correctors Errours have appeared, like infirmities incident to decrepit Age, involved within the sheets, as if prepared for a Funeral, the Authour's Senses departed, or in a trance, confused and ambiguous, their names remaining like Inscriptions on a Tombe, where corruptions of one have demonstrated generations of others.*

Some places I found obliterated, other parts dislocated, or false numbers had usurped their rooms; and those established by sundry Impressi-

Impressions may animate some (ignorantly) to plead Prescription for them, although multiplied into a numerous and adulterous off-spring, which deterred many young beginners from their progress in these Numbers, and invoked me to their assistance in the restitution of the Authour: notwithstanding I have published some Books of this Subject already, differing in form, the scope I aimed at being both Speculation and Practice, my intentions dedicating that and this to the publick good, whereby purblind Suspicion and fond Affection (the parents of Partiality) are expulsed and vanished, and I elected as an impartial Corrector of this Treatise, where many of the Tables *and* Rules *of direction were directly divided from the first Composer's meaning, the grounds of Truth, or ways of Art; some of which Deviations I have rectified, subtracted others, and totally cancelled many, adding numbers in their places accommodated and reduced to the Authour's sense, the Questions stated, and the pristin Copies, ushered from the Press again by Mr.* Mellys, *attended by* Hartwell: *and all these I expose to publick view, drest (without disguise) in their old attires; otherwise it would seem as absurd, as to see grave Antiquity vested in* French *habit.*

Humanum

Humanum est errare; *so I will not promise that all the old errours are corrected, (although above* 1000) *nor yet engage the Press shall commence no new: but so faithfully as I could, these Authours recovered are here presented to your view; which is all that was required of me, and from you (Courteous* Reader,) *a friendly acceptance, (in recompence of my labours) desiring to be number'd amongst the Coadjutors of my Countrey-men; in testimony whereof I here subscribe my name,*

Thomas Willsford.

FINIS.

Thomas

Enoch Noyes 1827

69
27
—
42

199
33
—
191

1827
1668
—
159

159 years old in 1827